Mesoscopic conductance fluctuations in an electron billiard

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Mesoscopic fluctuations of the conductance of a 2D electron gas in a periodic array of antidots have been studied experimentally, for the first time. These fluctuations have an anomalous temperature dependence. It results from an unusual distribution of interfering trajectories with respect to area. The results indicate that mesoscopic conductivity fluctuations are not a universal effect in an electron billiard, in contrast with the situation in a disordered conductor.

Two-dimensional (2D) electron systems with a periodic array of artificial scatterers acquire certain magnetotransport properties because the cyclotron diameter is commensurate with the period of array.[1–4] These features are of a classical nature and are determined by the stochastic nature of the motion of an electron in these systems, which are essentially a type of Sinaĭ billiard.[5,6]

In addition to these classical effects, such systems exhibit several interesting quantum properties, which stem from an interference of electron waves. In particular, we have already reported observing Aharonov–Bohm oscillations in a periodic array of antidots in a weak magnetic field.[7] Similar periodic oscillations of the magnetoresistance have been observed[8,9] in stronger magnetic fields ($2R_c \approx d$). In addition, the observation of an aperiodic oscillation of the conductance in samples of small dimensions, containing 40–80 antidots, has been reported,[10,11] but the physical nature of these oscillations has not been analyzed. We should also point out that the information reported on the electron transport associated with quantum interference effects in those papers was incomplete, because the temperature dependence of the effects was not studied.

In this letter we are reporting an experimental study of quasiperiodic and aperiodic oscillations of the magnetoresistance of samples with a periodic array of antidots in magnetic fields from 0 to 0.6 T. The results show that, within the experimental errors, the oscillation amplitude and the correlation magnetic field of the oscillations are essentially independent of the temperature at temperatures between 10 mK and 1.7 K. We show that this behavior of the fluctuations is due to an anomalous distribution of interfering trajectories over area in the electron billiard.

The test samples were prepared from a GaAs/AlGaAs heterojunction, in which the mobility of the 2D electron gas was $\mu = 1.5 \times 10^5$ cm^2/(V·s), and the electron density was $n_s = 6.7 \times 10^{11}$ cm^{-2}. The periodic array of antidots was produced by electron lithography and reactive ion etching. The period of the array was $d = 500$ nm,

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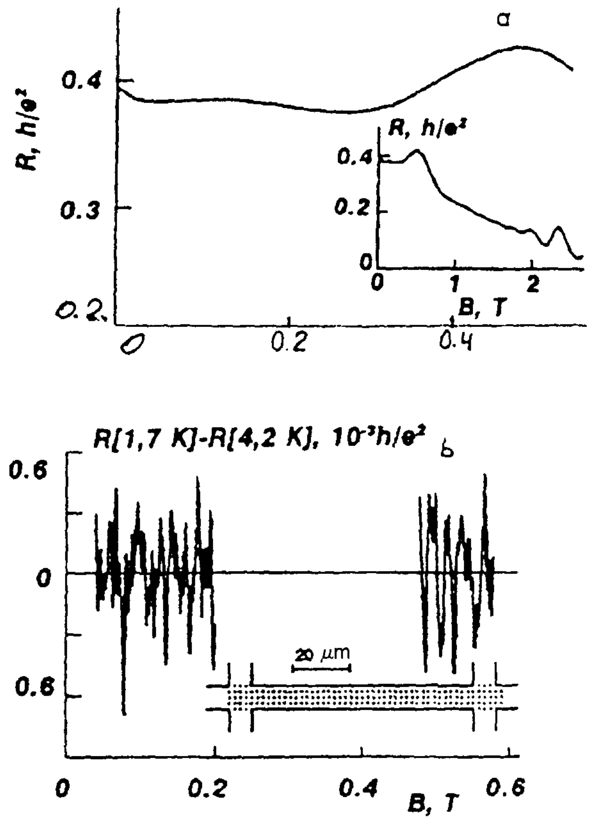

FIG. 1. a: Resistance of a sample at 4.2 K (R) versus the magnetic field B. b: Difference between the resistances of the sample at 1.7 K [R(1.7 K)] and at 4.2 K [R(4.2 K)] versus the magnetic field. At these magnetic fields the condition $2R_c > d$ holds. The inset in part a shows the resistance of the sample at $T=4.2$ K over a broad magnetic-field range, up to 2.7 T. The inset in part b is a schematic diagram of the geometry of the test samples.

and the diameter of the antidots was $2a=220$ nm, with the depletion layers being taken into account. The test samples had *macroscopic* dimensions: 10×100 μm. The geometry of the samples is shown in the inset in Fig. 1b. After the array of antidots was fabricated, the resistance of the samples at $B=0$ T increased by about an order of magnitude. This increase means that the electron transport in these structures was determined entirely by scattering by antidots.

Figure 1 shows the resistance of a sample versus the magnetic field at $T=4.2$ K. The magnetoresistance has a commensurate maximum at $B=0.53$ T, which stems from the onset of electron trajectories which run off along the rows of the array of antidots at $2R_c \approx d$, where R_c is the Larmor radius.[6] As the temperature is lowered to 1.7 K and below, additional fluctuations of the magnetoresistance, much smaller in amplitude, arise against the background of the commensurate oscillations. Figure 1b shows a recording of these oscillations for two ranges of magnetic fields: up to the

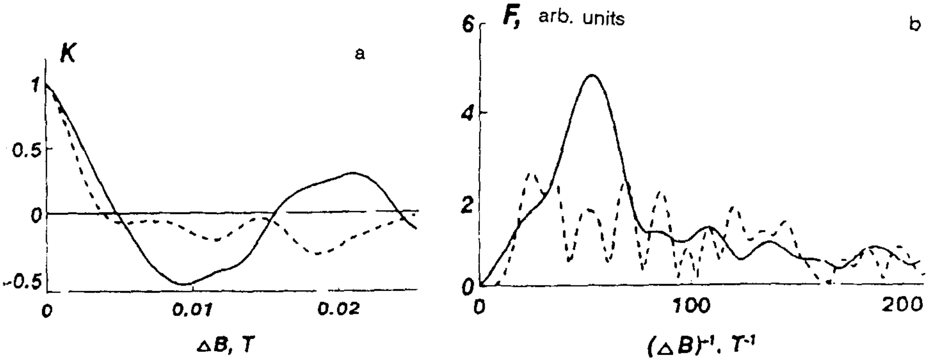

FIG. 2. Autocorrelation function (a) and amplitude of Fourier harmonics (b) of the magnetoresistance fluctuations at $T=1.7$ K. Solid curves—For quasiperiodic oscillations at 480 $<B<580$ mT; dashed curves—for aperiodic fluctuations at $36<B<200$ mT.

commensurate maximum ($B<0.2$ T) and against the background of the commensurate maximum (0.48 T $<B<0.58$ T).

At magnetic fields up to the commensurate maximum, the fluctuations of the resistance are aperiodic, as can be seen from the behavior of their autocorrelation function and Fourier transform (the dashed curves in Fig. 2). The average amplitude of the conductance oscillations at various values of the resistance of the samples is $\langle(\Delta G)^2\rangle^{1/2}\approx 10^{-3}\ e^2/h$, essentially independent of the temperature between 10 mK and 1.7 K (see the inset in Fig. 3). The correlation magnetic field $B_c\approx 1.7$ mT also remains constant.

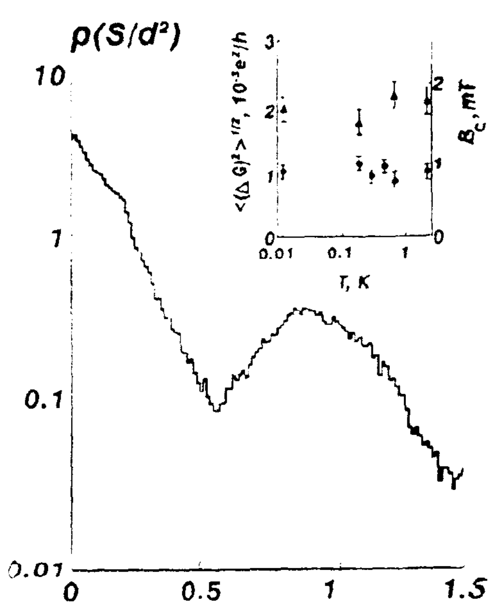

FIG. 3. Theoretical distribution of interfering trajectories with respect to the area covered for a periodic array of antidots with $a/d=0.22$. The inset shows the temperature dependence of the mean square amplitude of the conductance fluctuations (circles) and that of the correlation magnetic field (triangles).

At magnetic fields of 0.48–0.58 T, which correspond to the commensurate maximum, the fluctuations are quasiperiodic. The amplitude of the conductance oscillations remains constant, on the order of $10^{-3}\ e^2/h$, over the temperature range from 10 mK to 1.7 K. Figure 2 shows the autocorrelation function K and the Fourier transform of the oscillations of the resistance in this range of magnetic fields (the solid curves). The maximum in the Fourier spectrum corresponds to quantization of the magnetic flux through an area of $0.85d^2$. This area corresponds to the area of a unit cell of the array of antidots minus the area of an antidot.

A primary distinguishing feature of these results is that the amplitude of both the quasiperiodic and aperiodic fluctuations of the magnetoresistance remains constant over the broad temperature range from 10 mK to 1.7 K and then falls to essentially zero at $T > 3.5$ K. The absence of these fluctuations at high temperatures suggests that these fluctuations probably stem from a quantum interference. A quantum interference is responsible for the well-known mesoscopic fluctuations of the conductance in disordered conductors. The amplitude of these fluctuations, for a sample of length L and width W, is determined by the phase-coherence length L_φ (Ref. 12):

$$\langle(\Delta G)^2\rangle^{1/2} \sim \frac{e^2}{h}\left(\frac{W}{L_\varphi}\right)^{1/2}\left(\frac{L_\varphi}{L}\right)^{3/2}.$$

The temperature dependence of this amplitude is related to the T dependence of L_φ (usually a power law). The reason is that the interference correction to the conductivity is important for trajectories whose lengths are less than L_φ. In other words, as T is lowered (and as L_φ is thus increased), this correction becomes important for a progressively larger number of these trajectories. The typical area covered by two interfering trajectories, $\sim L_\varphi^2$, determines the magnitude of the correlation magnetic field, $B_c \sim \Phi_0/L_\varphi^2$ ($\Phi_0 = h/e$). The distribution of trajectories with respect to area covered is exponential[13] in the case of a disordered conductor (as in the case of a chaotic billiard):

$$p(S) \sim \exp(-\alpha S).$$

In other words, the probability [$p(S)$] for finding, among all pairs of interfering trajectories, those which cover areas between S and $S+dS$, decreases monotonically with increasing S. In our case (for a periodic array of antidots) this dependence becomes very nonmonotonic.

We have carried out a numerical calculation of $p(S)$ for an array of antidots for the ratio $a/d = 0.22$. In the calculations we assumed that the motion of the electron was classical, with elastic reflections from the antidots. We considered 10^5 pairs of electron trajectories, which had a common initial point and a common final point. The results are shown in Fig. 3. The plot has a huge maximum at $S = d^2$ against the background of an exponential decay (the vertical scale in this figure is logarithmic). Because of this maximum, as the temperature is lowered, and L_φ becomes greater than d, the interference correction to the conductivity comes to be governed by trajectories of length $\sim d$ with a covered area $\sim d^2$ (the number of which is anomalously large), instead of being governed by trajectories with a characteristic length L_φ, as in the case of a disordered conductor. This circumstance explains why no temperature dependence of the amplitude of the magnetoresistance fluctuations is seen experimentally. It also explains the correlation magnetic field at temperatures corresponding to $L_\varphi > d$. The maximum on the $p(S)$ distribution also agrees with the observation of Aharonov–Bohm oscillations corresponding to a quantization of the magnetic flux through an area of d^2, which have been seen experimentally in weak magnetic fields.[7]

Since the mesoscopic fluctuations of the conductance are independent of the temperature over the broad temperature range down to 10 mK, we are faced with the extremely important question of the behavior of the fluctuations in the limit $T \to 0$. We know that in disordered systems the amplitude of these fluctuations under the condition $L_\varphi \max(L,W)$ has a universal value $\approx e^2/h$. The results described above indicate that in an electron billiard the mesoscopic fluctuations of the conductance are determined in the limit $T \to 0$ not only by the value of e^2/h, but also by the geometric characteristics of the billiard which determine the $p(S)$ dependence. This interpretation means that the mesoscopic fluctuations of the conductance in an electron billiard may be of a nonuniversal nature.

The quasiperiodic conductance fluctuations observed near the commensurate maximum (in magnetic fields for which the condition $2R_c \approx d$ holds) require a separate analysis, since the picture drawn above does not explain the quasiperiodic nature of these fluctuations, although it does correctly describe the independence of these fluctuations from the temperature over the temperature range from 10 mK to 1.7 K. Both the autocorrelation function (Fig. 2a) and the Fourier transform (Fig. 2b) of these fluctuations clearly indicate the presence of a periodic component in them. The period of this component corresponds to a quantization of the magnetic flux through an area of $(d^2 - \pi a^2)$.

We suggest that the periodic component of the resistance fluctuations arises near the commensurate maximum because the $p(S)$ distribution changes in such a way in these magnetic fields that the maximum of the function $p(S)$, mentioned above, becomes narrower, and it shifts toward smaller areas. An exact quantitative analysis of this case will require a separate theoretical study. However, from a phenomenological standpoint one can distinguish three characteristic areas in this system: $d^2, \pi a^2$, and $(d^2 - \pi a^2)$. The area πa^2 is small in our case, and the oscillations corresponding to quantization of the magnetic flux through it would have a period comparable to the width of the commensurate maximum. Consequently, they could not be seen against the background of this maximum.

Oscillations corresponding to quantization of the magnetic flux through an area of d^2 were observed in Refs. 8 and 9. However, the distances between antidots in the arrays studied there were very small. As the ratio a/d increases (i.e., as the antidots are moved closer together), there is an increase in the period of the oscillations corresponding to quantization of the flux through the area $(d^2 - \pi a^2)$. Consequently, for the samples studied in Refs. 8 and 9, oscillations like those observed in the present study could not have been observed, since they would have had a period on the order of the width of the commensurate maximum.

In summary, the results of this study show that the behavior of mesoscopic fluctuations of the conductance in a 2D electron gas with a periodic array of antidots is quite different from the behavior in a disordered conductor. In addition, the results indicate that mesoscopic fluctuations of the conductance in an electron billiard are not universal. By this we mean that their amplitude in the limit $T \to 0$ may be governed not only by the value of e^2/h, but also by geometric characteristics of the billiard.

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