Spectroscopy of vibrations of the earth by means of gravitational-wave interferometers

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The possibility of using large-baseline, free-mass gravitational-wave antennas as highly sensitive tiltmeters to study global geodynamic characteristics is discussed.

Funding has now started for the construction of two laser free-mass gravitational-wave antennas with baselines of several kilometers: the LIGO project[1] in the U.S. and the Franco–Italian project VIRGO.[2] These installations were designed to detect metric perturbations from $h = 10^{-21}$ to $h = 10^{-23}$ as the measurement apparatus is refined. The reception bandwidth is to be $\simeq 10^3$ Hz; the range of frequencies which can be received is to be[1–3] from $\simeq 10$ Hz to 2 kHz. Realizing these extreme characteristics in practice will require some nontrivial technical facilities,[1,2] so these projects are extremely expensive undertakings. They are justified by the fundamental nature of only one ultimate goal: to master a new gravitational-wave channel for astrophysical information.

In this letter we wish to call attention to the possibility in principle of another, independent application of these installations: for solving problems of interest in geophysics, in particular, for studying the global characteristics of the earth's core–mantle system. Below we point out some specific geodynamic effects, explain the idea of measuring them by means of gravitational-wave interferometers, and discuss some instrumental and background limitations on the sensitivity.

1. Integral physical characteristics of the earth such as elastic coefficients (Love numbers), the magnitude of the losses, and the spectra of the losses are measured by measuring the response of the earth to pulsed or periodic excitations of a global nature. The pulsed excitation accompanying strong earthquakes makes it possible to detect natural modes of the earth at frequencies above[4,5] 10^{-3} Hz. Oscillations of the core correspond to lower frequencies, at which periodic excitation is provided by the tidal influence of the moon–sun system. The amplitude–phase response of the viscoelastic earth to tidal excitation is described by a sum of harmonic vibrations of the type $k_i A_i \sin(\omega_i t + \phi_i + \kappa_i)$, where k_i is the dimensionless factor by which the viscoelastic earth increases the amplitude (A_i) of the tidal harmonic of frequency ω_i and initial phase ϕ_i, and κ_i is the phase correction for damping.[5] To give the reader an idea of the scale of the numbers involved here, we note that at a latitude of 47° the total vertical displacement of the earth's core due to tides is 35–40 cm, the change in the gravitational force is 100–200 μGal [$(1\text{–}2) \times 10^{-7}$ g], and the deflection of a plumb line is 0.01–0.03 arc sec. The amplitude factors k_i measured by gravimeters and tiltmeters are by convention denoted by δ_i and γ_i. They are related to the Love numbers by simple linear equations.[6] The feasibility of a spectroscopy of forced vibrations of the earth (the possibility of measuring the amplitudes and phases of tidal harmonics), like the feasibility of observing natural vibrations of the core, depends on the instrumental sensitivity and on whether it is possible to filter the observed effect from the noisy geophysical background. The intrinsic noise of cryogenic gravimeters[7] in the diurnal and semidiurnal bands is a few nanogals $[(2\text{–}5)\times 10^{-12}$ g] or 0.1 μGal/Hz$^{1/2}$. The relative error in the determination of the amplitude factors δ_i is $\sim 0.1\%$. At best, tiltmeters offer a sensitivity ~ 0.0001 (arc sec)/Hz$^{1/2}$ and a relative error of less than 1% in the determination of γ_i (Ref. 8). Instruments of both types have been used successfully to detect the fundamental effect predicted in Ref. 9—the "resonance of the liquid core"—through an accumulation of data over many years (see Ref. 10 regarding gravimeters and Ref. 11 regarding tiltmeters). However, an increase in the relative accuracy by even an order of magnitude will make it possible to study the effect of Ref. 9 in detail and to refine the shape of the core–metal boundary and the Q factor of the core. We might also mention the need for high resolution in the spectrum of natural vibrations of the earth[4] and in the observation of natural vibrations of the core with a main period of approximately[7] 14 h and of the inner core with a period of 3 h 18 min, because of oscillations of the barycenter of the earth–moon system.[12]

2. A laser free-mass gravitational antenna can be thought of in principle as a two-coordinate tiltmeter with a potential sensitivity several orders of magnitude better than that which has been achieved to date. Operation of the gravitational antenna in a long-term "watch" would of course make it possible to accumulate data over many years. Let us explain the operating principle of such an antenna as a tiltmeter, and let us estimate its sensitivity.

The antenna used in the studies[1,2] is a Michelson interferometer whose arms contain high-Q Fabry–Perot resonators. These resonators are formed by plane input mirrors and spherical output mirrors. These mirrors are also low-frequency mechanical pendulums with periods on the order of 1 s. For frequencies below 10 Hz, a null intensity is maintained at the output of the Michelson interferometer by tracking systems. The information of interest—on slow motions of the mirrors—is thus given by error signals in feedback circuits. Individual tracking systems are responsible for "frequency detuning" (for maintaining a fixed baseline of the Fabry–Perot resonators) and for "adjusting the axial alignment (for keeping the axis of the beam coincident with the axis of the resonator). There are at least three independent photodetectors for the output signals: one at the output from the Michelson interferometer and two behind the spherical end mirrors of the Fabry–Perot arms. The operation of the tracking systems which maintain the adjustment of the Fabry–Perot resonators is based on detection of the first nonaxial mode, TEM_{10}, against the background of the fundamental working mode, TEM_{00}. Here use is made of an additional modulation at the difference frequency of the modes, and the Fabry–Perot photodetectors have a quadratic spatial structure (this is the so-called Anderson technique[13]). Significantly, the error signal generated by tilts of a plane mirror (displacements normal to the beam are degenerate) is $\pi/2$ out of phase with the error signal generated by large shifts of the spherical mirror (pure tilts are degenerate). The plane mirror can thus be thought of as the test mass of a plumb-line tiltmeter, and the spherical mirror can be thought of as a reference point. The two arms of the Michelson interferometer form a two-coordinate tiltmeter. ("Recycling"—"blocking" the outgoing light in the interferometer by placing an additional semitransparent mirror in the path of the incoming beam—mixes the perturbations of the arms, but it does not rule out the possibility of a separation of the "error signals" by virtue of a different phase coloring.[2])

The shot noise of the photocurrent and the thermal noise of the mirror supports are natural factors which limit the accuracy of the systems that track the inclinations of the mirrors.

A. Limitations on the fluctuations of the photocurrent

In the Anderson technique,[13] the maladjustment signal corresponding to a mirror deflection $\Delta\alpha$ is given by

$$\Delta i_s=\sqrt{\frac{2}{\pi}}\,\frac{4\pi e\eta\lambda}{\hbar c}\,P_0TJ_0(m)J_1(m)\frac{\Delta\alpha}{\alpha_0},\quad T=\frac{2}{1-R_1}(1-R_2^2), \tag{1}$$

which contains, in addition to constants (the charge of an electron, e, the velocity of light c, and Planck's constant $\hbar$), some parameters of the apparatus: the wavelength, $\lambda=1.04\times10^{-5}$ cm; the quantum yield of the photocathode, $\eta=0.8$; the reflection coefficient of the plane mirror, $R_1=0.98$; and that of the spherical mirror, $R_2=0.99995$. The nominal power incident on the photodiode is $P_{\text{pd}}=P_0T=5$ V. The quantities $J_0(m)$ and $J_1(m)$ are Bessel functions of the fundamental and first nonaxial modes (for a modulation depth $m=0.2$, their values are $J_0\simeq1.0$, and $J_1\simeq0.1$). A fundamental parameter, which determines the steepness of the conversion of angular displacements into error current, is the beam divergence $\alpha_0=1.04\times10^{-5}$, which is calculated from $\alpha_0=\lambda/(\pi D)$, where D is the beam diameter at the plane mirror. The average current through the photodiode and the noise component of this current are

$$\langle i\rangle=\frac{2\pi\eta\lambda e}{\hbar c}\,P_0TJ_0^2\simeq0.25\text{A},\quad \langle\delta i^2\rangle^{1/2}=\{2\langle i\rangle e\}^{1/2}\simeq3\times10^{-10}\ \text{A/Hz}^{1/2}. \tag{2}$$

Comparing (1) and (2), we find an estimate of the angular sensitivity:

$$\Delta\alpha_{\min}\geqslant5\times10^{-14}\ \text{rad/Hz}^{1/2}\quad(\text{or }1\times10^{-8}(\text{arcsec})/\text{Hz}^{1/2}. \tag{3}$$

B. Limitations on the thermal noise of the mirrors

Since the measurements are to be carried out at frequencies well below the resonant frequency of the supports, we use the following formula to estimate the thermal angular fluctuations:

$$\Delta\alpha_{\min}\geqslant(\kappa T/I\omega_0^2Q)^{1/2}\simeq10^{-15}\ \text{rad/Hz}^{1/2}. \tag{4}$$

This estimate incorporates the moment of inertia $I=10$ kg · m^2 and the Q value of the support, $Q=10^6$; here $\kappa T=4.2\times10^{-23}$ J. The thermal noise in (4) is weaker than the photodetection noise, so one can predict a potential angular sensitivity four orders of magnitude better than the typical level in tiltmeter technology. Experiments[13] on the self-adjustment of the mirrors of a laser gyroscope at frequencies of 0.1 Hz and up suggest $\sim 10^{-10}$ rad/Hz$^{1/2}$ at a low power level $P_0T = 160\ \mu$W. Converting this figure to a power level of 5 W, we find a sensitivity on the order of 10^{-12} rad/Hz$^{1/2}$. This figure is two orders of magnitude better than the level in tiltmeter technology today. The loss of two orders of magnitude in comparison with (3) characterizes the level of the flicker noise in the experiments of Ref. 13.

3. Detecting such subtle effects as natural core modes and an approximately diurnal resonance of the liquid core depends strongly on the capability to distinguish these effects from the more intense background of seismic and thermal noise, which can reach $\simeq 10^{-3}$ arc sec or more. It is difficult to predict precisely whether this capability can be achieved. We will present a few arguments in favor of an optimistic prediction.

First, observations over many years will lower the background, as in Refs. 10 and 11. With several (at least three) independent channels generating output data, the conditions for cancelling the noise of undesired color will be satisfied. (We note in this connection that a gravitational-wave laser interferometer, thought of as a geophysical instrument, is not simply the two-coordinate tiltmeter which we have been discussing here; it also includes two-coordinate horizontal and vertical deformation meters.) The long baseline of the instrument will help remove local perturbations. Using correlation methods on recordings of variations of the temperature, atmospheric pressure, and the microseismic background in the analysis of the data will help lower the background. In the case of natural vibration modes of the earth and its core, the frequency coloring will make it possible to reduce the background by narrowing the reception band. This comment does not apply to the approximately diurnal resonance of the earth tides, for which thermal and barometric effects will induce a coherent noise. Here one can hope that the joint operation of two installations in different hemispheres, with a sampling of correlated signals, will significantly reduce this noise. However, the decisive factor would be to install the antennas several meters below the surface of the earth. It would be even better to install them in the tunnels of old mines or in neutrino observatories, as has been done with the interferometer of the Shternberg State Astronomical Institute of Moscow State University at the Baksan Neutrino Observatory of the Russian Academy of Sciences.[14] This measure would sharply reduce the geophysical background noise by virtue of the natural passive temperature regulation and the weakening of surface microseisms. We believe that the increased construction cost would be recouped in the ability to use the instrument for a variety of purposes.

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