Gluon distribution as a function of F_2 and $dF_2/d\ln Q^2$ at small x

A. V. Kotikov[1)]

Laboratory of Particle Physics, Joint Institute for Nuclear Research, 141980 Dubna (Moscow Region), Russia

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The paper presents a formula to extract the gluon distribution from deep inelastic structure function F_2 and its derivative $dF_2/d\ln Q^2$ at small x in the leading order of perturbation theory. A detailed analysis is given for new data of the H1 group from HERA. The values of gluon distribution are found at $10^{-3} \leq x \leq 2\times 10^{-2}$ and $Q^2=20$ GeV^2.

Recently the small-x behavior of the structure functions (SF) of deep inelastic scattering (DIS) was considered in connection with the possibility of carrying out experimental studies on the new, powerful colliders HERA[1] and LEP·LHC.[2] Analysis of SF gives the main information about the behavior of the parton (quark and gluon) distributions (PD) of a nucleon. The knowledge of PD is a basis for studying other processes.

We introduce the standard parametrizations of the singlet quark $s(x, Q^2)$ and gluon $g(x,Q^2)$ PD[2)] (see, for example, Ref. 3)

$$s(x,Q^2)=A_s x^{-\delta}(1-x)^{\nu_s}(1+\epsilon_s\sqrt{x}+\gamma_s x)\equiv x^{-\delta}\tilde{s}(x,Q^2),$$

$$g(x,Q^2)=A_g x^{-\delta}(1-x)^{\nu_g}(1+\gamma_g x)\equiv x^{-\delta}\tilde{g}(x,Q^2), \tag{1}$$

with the Q^2 dependent parameters in the r.h.s.. We use a similar small-x behavior for the gluon and sea quark PD that follows from the form of the kernel of the Gribov–Lipatov–Altarelli–Parisi (GLAP) equation (see also the recent fits of experimental data in Ref. 4).

The "conventional" choice is $\delta=0$, which leads to nonsingular behavior of PD (see the D_0' fit in Ref. 3) when $x\rightarrow 0$. Another value, $\delta\sim\frac{1}{2}$, was obtained in the studies[5] as the sum of the leading powers of ln $(1/x)$ in all orders of perturbation theory (PT) (see also the D_-' fit in Ref. 3). Recent NMC data[6] agree with the small values of δ. This choice corresponds to the present experimental data for pp and $\bar{p}$p total cross sections (see Ref. 7) and the model of Landshoff and Nachtmann pomeron with the exchange of a pair of nonperturbative gluons, yielding $\delta=0.086$. However, the new H1 data[9] from HERA prefer $\delta\sim 0.5$. With the help GLAP equation some attempts[10] were undertaken to obtain an agreement between the results of NMC at small Q^2 and H1 group at large Q^2.

In the present letter we are studying the behavior of gluon PD at small x, using the new H1 data and the method (see Ref. 11) of replacement of the Mellin convolution by ordinary products.

1. Assuming the Regge-like behavior for the gluon and singlet quark PD [see Eq. (1)], we obtain the following equation for the Q^2 derivative of the SF F_2:[3)]

$$\frac{dF_2(x,Q^2)}{d\ln Q^2} = -\frac{\alpha(Q^2)}{2}\,\delta_s x^{-\delta} \sum_{p=s,g} [\tilde{\gamma}_{sp}^{1+\delta}(\alpha)\tilde{p}(0,Q^2)+\tilde{\gamma}_{sp}^{\delta}(\alpha)x\tilde{p}'(0,Q^2)]$$

$$+O(x^{2-\delta}), \tag{2}$$

where $\tilde{\gamma}_{sp}^{n}(\alpha)$ are some combinations of the Wilson coefficients and anomalous dimensions of the n "moment" of the Wilson operators (i.e., the corresponding variables expand from integer values of the argument to the noninteger values) and

$$\tilde{p}'(0,Q^2) \equiv \frac{d}{dx}\tilde{p}(x,Q^2) \quad \text{at} \quad x=0.$$

Here δ_s is the coefficient which depends on the process and on the number of quarks f: $\delta_s=5/18$ for the ep collision, where $f=4$.

Further, we restrict the analysis to the leading order (LO) of perturbation theory [where $F_2(x,Q^2) \equiv \delta_s s(x,Q^2)$ and the $\tilde{\gamma}_{sp}^{n}(\alpha)$ are equal to the LO anomalous dimension γ_{sp}^{n}] and to the case $\delta=0.5$ which corresponds to the Lipatov pomeron which is supported by the H1 data. Below we will take into account the case $\delta=0$, which corresponds to the standard pomeron, and we will extend this analysis to the next-to-leading (NLO) order to perturbation theory.

For the gluon part from r.h.s. of Eq. (2), within accuracy of $O(x^2)$, we have the form

$$\gamma_{sg}^{3/2}\tilde{g}(x/\xi_{sg},Q^2) \quad \text{with} \quad \xi_{sg}=\gamma_{sg}^{3/2}/\gamma_{sg}^{1/2}. \tag{3}$$

For the quark part a similar simple form is absent, because the corresponding anomalous dimensions $\gamma_{ss}^{3/2}$ and $\gamma_{sg}^{1/2}$ have the opposite signs. However, within accuracy of $O(x^2)$, it may be represented as a sum of two terms like Eq. (3), with a shift of some coefficients and arguments. Choosing the shifts as 1 and ξ_{sg}^{-1}, we can write the following representation for the quark part:

$$c_1\ \tilde{s}(x,Q^2)+c_2\ \tilde{s}(x/\xi_{sg},Q^2),$$

where

$$c_1=\frac{\gamma_{ss}^{3/2}\gamma_{sg}^{1/2}-\gamma_{ss}^{1/2}\gamma_{sg}^{3/2}}{\gamma_{sg}^{1/2}-\gamma_{sg}^{3/2}} \quad \text{and} \quad c_2=\gamma_{sg}^{3/2}\frac{\gamma_{ss}^{1/2}-\gamma_{ss}^{3/2}}{\gamma_{sg}^{1/2}-\gamma_{sg}^{3/2}}. \tag{4}$$

Using exact values of the anomalous dimensions, we thus find the following expression from Eqs. (2)–(4):

$$\frac{dF_2(x,Q^2)}{d\ln Q^2} = 8\alpha(Q^2)\times\left[\frac{\sqrt{253}}{30\sqrt{7}}\left(eg\left(\frac{77}{23}x,Q^2\right)+\frac{497}{81}F_2\left(\frac{77}{23}x,Q^2\right)\right)\right.$$

$$\left.-\frac{4}{3}\left(\frac{413}{360}-\ln 2\right)F_2(x,Q^2)\right]+O(x^{2-\delta}), \tag{5}$$

where $e = \Sigma_i e_i^2$ is the sum of the squares of quark charges. From Eq. (4) with the accuracy of $O(x^{2-\delta})$, we obtain the following expression for the gluon PD:

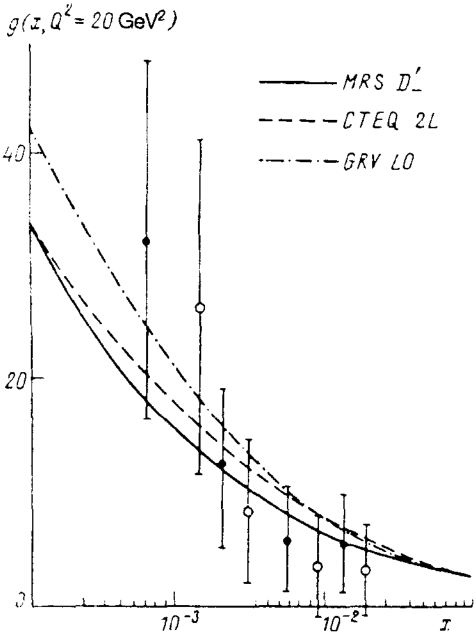

FIG. 1. The gluon distribution $g(x,Q^2)$ at $Q^2=20$ GeV^2. The white and black circles indicate the values extracted with the help of our equation [see Eq. (6)] and that of Prytz (see Ref. 13), respectively. Only the statistical errors are presented. The curves represent different parametrizations of $g(x,Q^2)$ (Refs. 3, 15, and 16). The *CTEQ* and *GRV* curves are the leading-order parametrization, and the *MRS* parametrization is given in the *DIS* renormalization scheme.

$$g(x,Q^2)=\frac{0.56}{\alpha(Q^2)}\frac{dF_2(0.3x,Q^2)}{d\ln Q^2}+2.72F_2(0.3x,Q^2)-5.52F_2(x,Q^2). \qquad (6)$$

2. Let us analyze the predictions inspired by Eq. (6). We use the values of SF F_2 and its Q^2 derivative found by the H1 collaboration (see Refs. 9 and 12, respectively). A similar analysis was carried out by the H1 group and reported in Ref. 12, where the results of Ref. 13 were used. Note that our basic formula, (2), coincides with the corresponding formula from Ref. 13 when we use the LO approximation, $\delta=1$, and ignore the singlet quark contribution. However, since it was studied in a recent preprint,[14] the result from Ref. 13, exact for $\delta=1$, is not a good approximation for δ in the interval $0\leq\delta\leq0.5$, especially at $\delta\sim0$. The addition of the NLO corrections strongly violates the Prytz results (see Ref. 14).

The extracted gluon PD values are shown in Fig. 1. These values are compared with those of Ref. 12. As in Ref. 12, we used the hypothesis concerning the approximate linear $\ln Q^2$ dependence of F_2 at small x and the value of QCD scale $\Lambda(f=4)/MS=200$ MeV^2. As can be seen in Fig. 1, we found the gluon PD values to be very similar to the results in Ref. 12. Some differences between our results and those of Ref. 12 are attributable to the singlet quark contribution, which is important for $x\leq10^{-2}$. Indeed, the singlet quark distribution reduces $g(x,Q^2)$ from several percent at $x\approx10^{-3}$ to 20% at $x\approx2\times10^{-2}$.

In summary, we used Eq. (2) to extract a gluon distribution at small x from SF F_2 and its Q^2 derivative. This equation generalizes the previous equation derived by Prytz (see Ref. 13) for the case of arbitrary values of the pomeron intercept and includes the singlet quark contribution. The addition of the NLO contribution to Eq. (2) can be done by analogy with Ref. 11.

Equation (6) was used in the analysis of H1 data from HERA. The values of gluon distribution for small x: $10^{-3} \leqslant x \leqslant 2 \times 10^{-2}$ were found. The extension of this analysis to the case $\delta \sim 0$, which is in agreement with the NMC data and the evaluation of the NLO contributions, will be done in the future.

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[1)]e-mail: KOTIKOV@SUNSE.JINR.DUBNA.SU.
[2)]We use PD multiplied by x and neglect the nonsinglet quark distribution at small x.
[3)]In contrast with the standard case, we use below $\alpha(Q^2) = \alpha_s(Q^2)/4\pi$.

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