Rotation of a ray by a magnetic field

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An experiment in which the application of a magnetic field H along the light propagation direction causes a rotation $\propto H$ of the output intensity pattern is proposed. A case is developed to satisfy Faraday's claim "of the magnetization of the ray," i.e., the transversely localized intensity burst, and not just the rotation of the polarization direction.

It is generally assumed that a magnetic field applied to the transparent medium influences the state of polarization only, so that the azimuth of the latter is rotated at a rate

$$d\psi/dz = VH \cos\theta. \tag{1}$$

Here V is the Verdet constant, i.e., the constant of the Faraday effect,[1] and θ is the angle between the wave vector **k** and the magnetic field vector **H**. Some transverse effects were also discussed: the deviation of the group velocity vector $\mathbf{v}/v$ from $\mathbf{k}/k$ at an angle $\delta\theta(\mathbf{v},\mathbf{k}) = -\sigma k^{-1} VH \sin\theta$ (see Sec. 101 in Ref. 2), where σ is the sign of the polarization circularity: $\mathbf{E} \propto (\mathbf{e}_x + i\sigma\mathbf{e}_y)\exp(ikz - i\omega t)$. However, in a typical case of moderate $HVL \sim 1$ (where L is the propagation distance) those effects are small. For example, if L corresponds to the Fresnel length, $L = ka_0^2$, of a beam with a finite transverse dimension a_0, then the transverse shift of a beam is $\delta x = L\,\delta\theta \approx (\sigma H a_0 V) a_0 \ll a_0$, since usually $HVa_0 \ll 1$.

In this letter we suggest an experiment in which the longitudinal magnetic field leads to some kind of "rotation,"

$$d\varphi_{\mathrm{eff}}/dz \sim VH, \tag{2}$$

of the azimuth, φ_{eff}, of the output intensity pattern. In this connection, let us consider an axially symmetric optical waveguide which has only three propagating modes $M_{ij}(x,y)$ in the scalar approximation (see, e.g., Ref. 3):

$$M_{00} = f(r)e^{i\beta_0 z}, \quad M_{01} = xg(r)e^{i\beta_1 z}, \quad M_{10} = yg(r)e^{i\beta_1 z}, \tag{3}$$

where z is the axis of the fiber, $x = r\cos\varphi$, and $y = r\sin\varphi$. Only the axial symmetry guarantees the degeneracy of the M_{01} and M_{10} modes, i.e., the coincidence of the propagation constants β_{01} and β_{10}. Therefore, x and y can be combined into $r\exp(i\varphi)$ and $r\exp(-i\varphi)$. The vector nature of the (nearly transverse) electromagnetic field leads to the splitting of these modes into the following six combinations (for $\sigma = \pm 1$; see Ref. 3):

$$\mathrm{M}_{00\sigma} = (\mathbf{e}_x + i\sigma\mathbf{e}_y) f(r)\exp(i\beta_0 z),$$

$$\mathrm{M}_{0+1+1} = (\mathbf{e}_x + i\mathbf{e}_y) g(r) r e^{i\varphi} \exp(i\beta_1 z + iDz),$$

$$\mathrm{M}_{0-1-1} = (\mathbf{e}_x - i\mathbf{e}_y) g(r) r e^{i\varphi} \exp(i\beta_1 z + iDz), \tag{4}$$

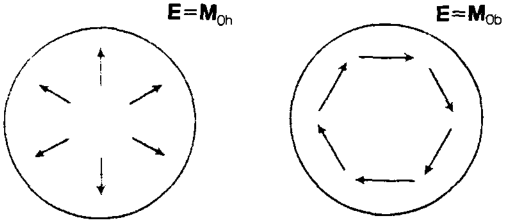

FIG. 1. The electric field of a hedgehog mode $\mathbf{E}=\mathbf{M}_{0h}\ (x,y)$ and a bagel mode $\mathbf{E}=\mathbf{M}_{0b}\ (x,y)$.

$$M_{0h}=(e_x \cos\varphi+e_y \sin\varphi)g(r)r \exp(i\beta_1 z+ihz),$$

$$M_{0b}=(-e_x \sin\varphi+e_y \cos\varphi)g(r)r \exp(i\beta_1 z+ibz).$$

The degeneracy of the eigenvalues for $\mathbf{M}_{00\pm1}$ modes follows from the reflection ($x\to -x,\ y\to +y$) symmetry of the waveguide which is made of isotropic material; the same is true for the $\mathbf{M}_{0+1+1}$ and $\mathbf{M}_{0-1-1}$ modes. The modes $\mathbf{M}_{0h}$ (hedgehog) and $\mathbf{M}_{0b}$ (bagel) are shown in Fig. 1. These modes are generally split, i.e., h is generally not equal to b. The latter two modes can be interpreted as the remnants of an ensemble of meridional rays with all possible values of the azimuth φ_0; such a ray for a fixed φ_0 is shown in Fig. 2. At the same time, all other modes more or less correspond to the sagittal rays. Therefore, the phase shift of the total internal reflection process is not accumulated for any particular direction of polarization, and the latter may be chosen to be circular. For a more accurate interpretation of the vector mode structure in an optical waveguide see Ref. 3.

Now the application of the longitudinal magnetic field $\mathbf{H}=He_z$ does not break the axial symmetry; however, it breaks the reflection symmetry ($x\to -x,\ y\to +y$). As is well known (see, e.g., the crystal optics theory in Refs. 1 and 2), the circularly polarized σ modes acquire an addition $\delta\beta=-VH\sigma$ to the propagation constant, while the linearly polarized modes hold their β values constant. We therefore suggest the following scheme for the experiment. Let us put two e_x-transmitting polarizers at the input and the output of the fiber, and let us illuminate its input by a more or less arbitrary field which contains a combination of M_{00}, M_{01}, and M_{10} modes:

$$\mathbf{E}_{in}=\mathbf{E}(r,\varphi,z=0)=e_x(C_0M_{00}+C_xM_{01}+C_yM_{10}). \tag{5}$$

The output intensity pattern $I_{out}(r,\varphi)=|(\mathbf{E}(r,\varphi,z)\cdot e_x)|^2$ exhibits a distortion upon the application of a moderate ($VHz\sim1$) magnetic field.

The main assertion of the present letter is that a part of this distortion can be interpreted as a "ray rotation," $\delta\varphi\propto H$. A direct calculation of the intensity profile I_{out},

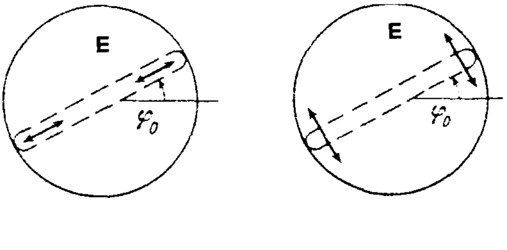

FIG. 2. Interpretation of the splitting $\beta_h-\beta_b$ in terms of the meridional rays (the dashed line) with different polarizations, which acquire slightly different phase shifts for the total internal reflection at the core–cladding boundary.

with allowance for the propagation laws, gives a very large number of H-dependent terms even in this oversimplified model (three modes only, $e_x \to e_x$). The abundance of these terms constitutes the main difficulty of the painstaking process of the determination of the necessary terms, i.e., those terms which may be qualitatively interpreted as a result of the "pattern rotation." These terms are

$$\delta I_{\rm out}(r,\varphi)=\frac{1}{8}\, r^2 g^2 \,{\rm Re}[(C_x-iC_y)(C_x^*-iC_y^*)e^{2i(\varphi-VHz)}]$$

$$+\frac{1}{2}\, rfg\, \cos(VHz){\rm Re}[C_0^* e^{i(\beta_1+D-\beta_0)z}[(C_x-iC_y)e^{i(\varphi-VHz)}$$

$$+(C_x+iC_y)e^{-i(\varphi-VHz)}]]+\frac{1}{8}\, r^2 g^2\, {\rm Re}[(C_x-iC_y)$$

$$\times(C_x^* e^{-ihz}-iC_y^* e^{-ihz})e^{iDz}e^{2i(\varphi-VHz/2)}]+\cdots\ .$$

Important consequences are that (1) there is a kind of "rotation" of the output speckle pattern at an angle $\delta\varphi_{\rm eff}\sim VHz$, (2) the sign of that rotation changes upon switching the magnetic field $\mathbf{H}\to-\mathbf{H}$, and (3) the sign of the "ray rotation" coincides with the sign of the "polarization rotation" due to the Faraday effect in the same medium. Similar results were obtained for the scheme in which the input and the output polarizers are circular with *the same* sign of circularity. In addition, the effect occurs in a truly multimode waveguide, but the analytical expressions are unwieldy in this case.

By analogy with the optical Magnus effect,[4,5] the magnetic "rotation of the ray" is based on the spin–orbit coupling of a photon in an inhomogeneous medium, i.e., it is based on the coupling between the propagation and the polarization. However, in contrast with the Magnus case, here the participation of the hedgehog and bagel modes is essential for the existence of the "ray-rotation" effect. We wonder if there are any magnetic effects of this kind in a homogeneous medium. (An effect like the Magnus effect in a homogeneous medium was recently suggested in Ref. 6 and observed in Ref. 7.

In summary, we predicted the rotation $\delta\varphi\sim VHz$ of the speckle pattern of the intensity at the output of the multimode fiber due to an external longitudinal magnetic field H. We thus hope to fulfill the claim made by M. Faraday "to magnetize the ray."

The effect was recently observed in an experimental study.[8]

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