Stability of the critical state of a type-II superconductor subjected to a finite temperature perturbation

V. R. Romanovskiĭ
Kurchatov Institute Russian Science Center, 123182 Moscow, Russia

(Submitted 7 April 1994)
Pis'ma Zh. Eksp. Teor. Fiz. **59**, No. 10, 671–677 (25 May 1994)

On the basis of a critical-state model it is shown that finite temperature perturbations strongly influence the conditions for the occurrence of a thermomagnetic instability in a type-II superconductor. General conditions for adiabatic stability of the screening currents induced by an external magnetic field are formulated.

A basic feature of type-II superconductors is that dissipative processes occur in them because of a thermally activated motion of vortices. The resulting dissipation of the energy stored by the screening currents flowing along the superconductor may be accompanied by a disruption of the critical state of these currents in response to an external force which gives rise to an electric field.[1] This process is manifested in so-called instabilities, which are initiated by numerous external perturbations of various types. One of the first instabilities to be seen experimentally, the thermomagnetic instability, stems from a redistribution of magnetic flux in the sample. This instability leads to an increase in the temperature of the superconductor, with the result that the latter may undergo a transition to the normal state.

The onset and evolution of an instability in a superconductor are typical examples of nonequilibrium transient processes in a two-phase dissipative structure. Collective processes which depend only slightly on the microscopic properties of the medium play a governing role here. It is thus possible to formulate several extremely important conditions for the stability of the critical state on the basis of macroscopic models. For example, a stable magnetic-field distribution in a superconducting plate (stable in the face of infinitely small perturbations) is described by the so-called adiabatic-stability condition[2–6]

$$\frac{\mu_0 a^2 J_{c0}^2}{C(T_c - T_0)} < 3, \tag{1}$$

where $C=\text{const}$ is the specific heat at constant volume of the superconductor, a is the half-width of the plate, T_0 is the coolant temperature, and J_{c0} and T_c are critical parameters of the superconductor.

However, most of the theoretical work which has been carried out has been based on a study of the initial stage of the penetration of magnetic flux into the superconductor. Such studies simplify the derivation of stability conditions, but they ignore physical features of the time evolution of the electric and magnetic fields in the superconductor. They accordingly overlook the formation of the final state of the superconductor, toward which the latter tends as a result of the onset of the instability. An approach of this sort

 0021-3640/94/100709-07$10.00

has been conducive in the development of a theory for the thermomagnetic instability in the approximation that only an insignificant heating is permissible. On the other hand, in a theory of the thermal stabilization of superconductors, which is based solely on an analysis of variations in the temperature profile of the sample, it has been shown both experimentally and theoretically that a sample can undergo a significant heating during the application of temperature perturbations without undergoing a transition to the normal state.[6-11] In addition, as was shown in Ref. 12, there is a nontrivial relationship between permissible nonzero rise in the temperature of the superconductor and the rate at which transport current flows into the superconductor. This relationship nonetheless does not destroy the superconductivity of the sample. It is most obvious when the perturbation develops adiabatically. Can there be therefore a range of a permissible increase in the temperature of the superconductor without incurring the loss of its critical state? The solution of this problem will add to the class of effects which can be studied in the phenomenological electrodynamics of superconductors. However, previous studies[13,14] have been restricted to the existing ideas regarding the role of the temperature factor under conditions corresponding to the onset of the thermomagnetic instability. These previous studies have not been capable of formulating a general concept regarding the effect of the temperature on the processes which occur in superconductors during the application of an alternating magnetic field or current.

Our purpose in this letter is to analyze the conditions for the onset of the thermomagnetic instability in a type-II superconductor, with allowance for a nonzero temperature perturbation of the original state of the superconductor and for the corresponding dynamics of the temperature profile produced as a result of the instability.

We consider the system of equations

$$\frac{\partial^2 E}{\partial x^2}=\mu_0\frac{\partial J_c}{\partial t}, \tag{2}$$

$$C(T)\frac{\partial T}{\partial t}=\frac{\partial}{\partial x}\left(\lambda\frac{\partial T}{\partial x}\right)+EJ_c(T), \tag{3}$$

$$J_c=J_{c0}\,\frac{T_c-T}{T_c-T_0}$$

with the boundary conditions[4-6]

$$\frac{\partial T}{\partial x}(0,t)=0,\quad \frac{\partial T}{\partial x}(a,t)=0,\quad E(0,t)=0,\quad \frac{\partial E}{\partial x}(a,t)=0 \tag{4}$$

and the initial condition

$$T(x,0)=T_i>T_0. \tag{5}$$

These equations describe the change in the temperature T and the electric field E inside a plane-parallel, thermally insulated superconducting plate with a thermal conductivity λ as the result of a complete penetration of magnetic field into the sample. The temperature of the sample differs from that of the coolant because of some perturbation.

As was shown in Refs. 4–6, system (2), (3) follows from the general system of Fourier's and Maxwell's equations under the assumption that the resistive component of the total current flowing through the superconductor has only a small effect. This system of equations describes dissipative processes in a type-II superconductor in the limiting case in which the ratio of the coefficients of thermal and magnetic diffusion is much smaller than one, and the instability develops under adiabatic conditions. For simplicity, we write the equation of the critical state in accordance with Bean's model.

From (2) we easily find the electric field distribution along the cross section of the plate:

$$E=\mu_0\frac{\partial J_c}{\partial t}\left(\frac{x^2}{2}-ax\right).$$

Since the new equilibrium state of the superconductor, toward which the latter tends as a result of the instability, is determined primarily by the amount of energy stored in the superconductor, we can ignore the spatial nonuniformity of the temperature profile along the cross section of the plate in deriving the criterial relations. (A corresponding numerical solution of the problem of nonisothermal diffusion of magnetic flux in a superconductor in a more general formulation—incorporating the changes in the temperature, the electric field, and the magnetic field in space and time—confirms the results found below. The only change is in the dynamics of the intermediate states which precede the final equilibrium temperature distribution in the superconductor.)

We integrate Eq. (3) from 0 to a and switch to an equation averaged over the spatial coordinate:

$$C(T)\frac{dT}{dt}=\frac{J_c(T)}{a}\int_0^a E\,dx.$$

Substituting $E(x)$ into this equation, we put the latter in the form

$$C(T)\frac{dT}{dt}=-\frac{\mu_0 a^2}{6}\frac{dJ_c^2}{dt},\quad T(0)=T_i. \tag{6}$$

Since the critical current density may fall to zero as the temperature rises, the solutions of this equation depend on the final thermal state which results from the instability.

For the case in which the final temperature exceeds the corresponding transition temperature, we find the following equation for determining it from (6):

$$\int_{T_i}^{T_k} C\,dT=\frac{\mu_0 a^2}{6}J_{c0}^2\left(\frac{T_c-T_i}{T_c-T_0}\right)^2. \tag{7}$$

At $T_k=T_c$, this equation determines boundary values of the original parameters for which the thermomagnetic instability is always accompanied by a transition of the superconductor to its normal state. In particular, for all

$$a>a_k'=\frac{1}{J_{c0}}\frac{T_c-T_0}{T_c-T_i}\sqrt{\frac{6}{\mu_0}\int_{T_i}^{T_c} C\,dT}, \tag{8}$$

the critical state is therefore unstable. Making use of the relationship between the magnetic induction at the surface of the plate and the intital critical parameters of the superconductor, we write an equation which sets an upper bound on the permissible magnetic-field drop in the sample:

$$B_m > B_m' \sqrt{6\mu_0 \int_{T_i}^{T_c} C dT}. \tag{9}$$

After this upper bound is reached, spontaneous heating of the superconductor is accompanied by a destruction of the superconducting state. Equation (7) also leads to the existence of a characteristic value of the initial temperature of the superconductor, T_x', which is the solution of the equation

$$\int_{T_x'}^{T_c} C dT = \frac{\mu_0 a^2}{6} J_{c0}^2 \left(\frac{T_c - T_x'}{T_c - T_0} \right)^2.$$

Since the critical-current density falls off with increasing temperature of the initial perturbation, and since the energy of the screening currents therefore decreases, the physical meaning of T_x' is that boundary value of T_i above which the superconductor does not go into its normal state.

We wish to stress the important role played by the temperature when the superconducting state is stable, according to our analysis. A difference between the initial temperature of the superconductor and the coolant temperature leads to not only a quantitative change in the existing isothermal criteria, but also a qualitatively new result: the existence of a characteristic perturbation temperature which separates a region in which the superconductor unavoidably goes into its normal state from a region in which the superconductor is heated to a temperature below its transition temperature. The criteria formulated above answer a question of fundamental importance: Under what conditions is the critical state destroyed by infinitely small perturbations? [Only under conditions (8) and (9), for all $T_0 < T_i < T_x'$.] In the temperature interval $T_x' < T_i < T_c$ the critical state does change, but the sample remains superconducting. Accordingly, if the requirement is that the condition $T_x' = T_0$ be satisfied, then for all parameter values which satisfy the condition

$$6 \int_{T_0}^{T_c} C dT > \mu_0 a^2 J_{c0}^2 \tag{10}$$

an arbitrary external change in the temperature of the superconductor, up to its transition temperature, will not lead to a subsequent transition to the normal state, despite the onset of an instability in the superconductor. In terms of the theory of the thermal stabilization of superconductors,[6] condition (10) means that the superconducting state is stable with respect to a temperature perturbation between T_0 and T_c. The assumption that the temperature of the superconductor can rise by a finite amount thus links the theory of the thermomagnetic instability with the theory of thermal stabilization.

These conclusions broaden the range of permissible parameter values—those for which the test sample retains its superconducting properties. On the other hand, the role played by a temperature perturbation in the stability of the critical state of the superconductor is also slightly unexpected from the standpoint of thermal stabilization of the superconductor. The reason is that the range of stable states lies in a region of temperature perturbations adjacent to the transition temperature.

Let us examine the changes in the conditions for the onset of the thermomagnetic instability for all $T_i > T_x'$, with $T_k < T_c$. In this case we easily find from (6) the following expression for the final temperature to which the superconductor is heated:

$$\int_{T_i}^{T_k} C dT = \frac{\mu_0 a^2 J_{c0}^2}{6(T_c - T_0)^2} (T_k - T_i)(2T_c - T_k - T_i). \tag{11}$$

It is clear from simple physical considerations that there exist states such that the left side of Eq. (11) may be greater than its right side. As the initial temperature of the superconductor is raised, the energy of the screening currents decreases. Accordingly, beginning at some value of T_i, any change in the enthalpy will be greater than the energy released as a result of the instability, so no change will occur in the initial thermal state. This conclusion means that a thermomagnetic instability does not occur. Taking the limit $T_i \to T_k$ in (11), we write the condition under which there is no instability,

$$\frac{\mu_0 a^2 J_{c0}^2}{C(T_i)(T_c - T_0)} \frac{T_c - T_i}{T_c - T_0} < 3. \tag{12}$$

[This inequality could be derived more rigorously from (11), through an analysis of the condition for a change in the sign of the derivative dT_k/dT_i.]

Inequality (12) yields some limitations on the parameters of the superconductor if the critical state is to be completely stable. Let us formulate some of these limitations. The magnetic-field drop in the sample, for example, must satisfy the condition

$$B_m < B_m'' = \sqrt{3\mu_0 C(T_i)(T_c - T_i)}. \tag{13}$$

This condition holds if the half-thickness of the plate satisfies

$$a < a_k'' = \frac{T_c - T_0}{J_{c0}} \sqrt{\frac{3}{\mu_0} \frac{C(T_i)}{T_c - T_i}}.$$

If the initial temperature of the superconductor is in the interval $T_x'' < T_i < T_c$, where T_x'' is the solution of the equation

$$3C(T_x'')(T_c - T_0)^2 - \mu_0 a^2 J_{c0}^2 (T_c - T_x'') = 0,$$

then for the given parameter values of the superconductor the latter does not undergo a spontaneous heating. From the condition $T_x'' = T_0$ we thus find the condition

$$\frac{\mu_0 a^2 J_{c0}^2}{C(T_0)(T_c - T_0)} < 3.$$

This condition means that not only is the critical state completely stable, as follows from isothermal condition (1), but also that the superconductor is thermally stable with respect to a temperature perturbation between T_0 and T_c.

The adiabatic conditions for stability of the critical state of a type-II superconductor thus depend strongly on the temperature of the initial perturbation. Because of this dependence, there is a region of thermal stability of the screening currents induced by the external magnetic field. The existence of this region thus makes it possible to relate the theory of the thermomagnetic instability to the theory of the stability of the superconducting state with respect to thermal perturbations, with an expansion of the range of permissible states.

This study also makes it possible to evaluate the existing experimental and theoretical results from a new standpoint. First, we know that the numerous experiments carried out to determine the field of a jump in magnetic flux have yielded results with a substantial scatter. Taking account of a possible change in the thermal state of the superconductor in the course of an experiment, we should use nonisothermal models to explain these discrepancies, along with the factors discussed in Ref. 6. Second, the effect of the thermal history of the sample on the stability conditions requires a correct determination of the sample temperature preceding the flux jump. Accordingly, an *a priori* specification of the initial temperature of the superconductor in a determination of the permissible parameter values of the superconductor may distort the final result. According to the isothermal model,6 the permissible increase in the external magnetic induction as $T_0 \rightarrow 0$ also tends toward zero. At the same time, it follows from (13) that an arbitrary temperature perturbation of the initial state of the superconductor (such a perturbation is unavoidable if the field or current is varied) will not lead to a limiting transition of this sort. Consequently, in a nonisothermal analysis of the field of the flux jump at ultralow temperatures, the permissible values of the magnetic induction will be higher than the corresponding isothermal values. Furthermore, the isothermal theory explains the experimental observation that flux jumps may be absent at a coolant temperature close to the transition temperature of the superconductor. The explanation is based on the circumstance that in this region the field of the flux jump is higher than the corresponding value of the magnetic induction at which the field penetrates completely into the superconductor. However, the existence of $T_x'' < T_c$ leads to a different interpretation of this effect: A thermomagnetic instability does not occur, because of a thermal stabilization of the superconductor in this region. Third, the existing theory does not fully explain the behavior of the flux-jump field as a function of the rate of change of the external magnetic field. In particular, the stability conditions formulated for a superconductor with a nonlinear current–voltage characteristic $E \propto \exp(J/J_\delta + T/T_\delta)$ (J_δ and T_δ are given parameters) loses its physical meaning when we take the limit of an ideal current–voltage characteristic ($J_\delta, T_\delta \rightarrow 0$), since they lead to the conclusion that superconductors are not stable in an alternating magnetic field in the critical-state model.[14] On the other hand, the nonisothermal model yields an explanation of this relationship, regardless of the type of current–voltage characteristic, since a variation of the magnetic field changes the flux-jump field, according to the plot of the permissible temperature increase versus the rate of change of the field.

[1] C. P. Bean, Rev. Mod. Phys. **36**, 31 (1964).
[2] S. L. Wipf, Phys. Rev. **161**, 404 (1967).
[3] P. S. Swartz and C. P. Bean, J. Appl. Phys. **39**, 4991 (1968).
[4] M. G. Kremlev, JETP Lett. **17**, 223 (1973).
[5] R. G. Mints and A. L. Rakhmanov, Usp. Fiz. Nauk **121**, 499 (1977) [Sov. Phys. Usp. **20**, 249 (1977)].
[6] R. G. Mints and A. L. Rakhmanov, *Physics of Composite Superconductors* [in Russian] (Nauka, Moscow, 1987).
[7] C. Schmidt and G. Pasztor, IEEE Trans. Magn. **13**, 116 (1977).

[8] W. Y. Chen and J. R. Purcell, J. Appl. Phys. **49**, 3546 (1978).
[9] W. Nick *et al.*, IEEE Trans. Magn. **15**, 359 (1979).
[10] V. R. Romanovski, Dokl. Akad. Nauk SSSR **279**, 884 (1984) [Sov. Phys. Dokl. **29**, 1053 (1984)].
[11] N. N. Martovetsky, IEEE Trans. Magn. **25**, 1692 (1988).
[12] V. E. Keilin and V. R. Romanovskii, IEEE Trans. Magn. **28**, 771 (1992).
[13] A. L. Rakhmanov, Cryogenics **23**, 487 (1983).
[14] E. Yu. Klimenko and N. N. Martovetsky, IEEE Trans. Magn. **25**, 2109 (1988).

Translated by D. Parsons