Domain-wall velocities above the Walker limit

V. G. Bar'yakhtar and B. A. Ivanov
Donetsk Physicotechnical Institute, Academy of Sciences of the Ukrainian SSR

(Submitted 15 December 1981)

Pis'ma Zh. Eksp. Teor. Fiz. **35**, No. 2, 85–87 (20 January 1982)

The motion of a domain wall at velocities above the Walker limit is discussed. This type of motion is accompanied by a Cerenkov emission of spin waves. The energy loss due to this emission is found. The dependence of the wall velocity on the external field is discussed.

PACS numbers: 75.60.Ch

Experiments by the Chetkin group[1] on the motion of domain walls at velocities above the Walker limit v_c in orthoferrites have attracted interest to this type of motion of domain walls. This motion is clearly of a time-varying nature, and its analytic description at the "soliton" level runs into serious difficulties. Certain aspects, however, can be described qualitatively. It was mentioned in Ref. 2, for example, that motion above the Walker limit is accompanied by the Čerenkov emission of magnetization oscillations (by the emission of magnons or magnetic solitons).

An important aspect of the problem is the circumstance that a domain wall, either moving or at rest, is a topologically stable formation, whose destruction will require the surmounting of a potential barrier[1)] whose energy is proportional to the volume of the object. In calculations, accordingly, the domain wall can be taken as a given feature, and its interaction with the magnetization field can be described by the Hamiltonian

$$\mathcal{H} = \frac{1}{\sqrt{\Omega}} \sum_{\mathbf{k}} U_{\mathbf{k}} c_k e^{-ikvt} + \text{H.a.} \qquad (1)$$

Here Ω is the volume of the system, c_k and c_k^+ are magnon operators, and v is the velocity of the domain wall. We will assume that the wall is a plane wall and is homogeneous in its plane; we assume that the amplitude U_k, which is determined by

 0021-3640/82/02101-04$01.00

the Fourier component of the magnetization at the domain wall, is nonzero only if $k \parallel \mathrm{v}$. Strictly speaking, $\mathscr{H}$ contains terms which are nonlinear in the operators c_k and $c_k^\dagger$ and which are not indicated in expression (1), but expression (1) will be sufficient for the purposes of the present analysis.

Since we do not know the exact structure of the wall at $\mathrm{v} > \mathrm{v}_c$, we will calculate U_k by working from a model according to which the domain wall in a rare-earth orthoferrite is a jog of width Δ. Here

$$U_k \cong \Delta\sqrt{\epsilon_0 E_0}\begin{cases} 1, & k\Delta << 1 \\ \exp(-k\Delta), & \Delta >> 1 \end{cases}, \tag{2}$$

where ϵ_0 is the magnon activation energy, and E_0 is the energy of the domain wall at rest.

When we incorporate $\mathscr{H}_1$, we find fundamentally different results at $\mathrm{v} < \mathrm{v}_c$ and $\mathrm{v} > \mathrm{v}_c$. To clarify the situation, we will calculate the rate of change of the magnon energy, $\dot{Q}$. The value of $\dot{Q}$ corresponding to a unit area of a domain wall is

$$\dot{Q} = 2\pi \mathrm{v} \int dk \mid U_k \mid^2 k\delta(\epsilon_k - k\mathrm{v}) = 2\pi \mathrm{v} \sum_a k_a \mid U_{ka} \mid^2 \mid \frac{\partial \epsilon}{\partial k_a} - \mathrm{v} \mid^{-1}, \tag{3}$$

where ϵ_k is the energy of a magnon with momentum k. It is easy to see that we have $\dot{Q} \neq 0$ only if there exists a real root for the equation

$$\epsilon_k = k\mathrm{v} \quad \text{or} \quad \mathrm{v} = \mathrm{v}_\varphi(k). \tag{4}$$

The index α in the second equation in (3) designates the particular root of Eq. (4), and $\mathrm{v}_\varphi(k) = \epsilon_k / k$ is the magnon phase velocity.

The existence of a root of (4) can also be seen in an evaluation of the expectation value of the magnon magnetization ΔM, which is expressed in terms of $\langle c_k \rangle \langle c_k^\dagger \rangle$; here the angle brackets denote an average over the density matrix of the system. It turns out that at $\dot{Q} = 0$ the quantity ΔM is localized near the domain wall: $\Delta M(x - \mathrm{v}t) \to 0$ as $|x - \mathrm{v}t| \to \infty$. In this case, ΔM actually describes a correction to the magnetization at the wall. If, on the other hand, Eq. (4) has even a single real root, then we would have $\dot{Q} \neq 0$, and ΔM would not vanish far from the wall.

The various types of motion of domain walls can be classified on the basis of $\dot{Q}$. The motion of domain walls with $\dot{Q} \neq 0$, which is accompanied by the Čerenkov emission of magnons, can be called "above-limit motion." Let us analyze the condition for Čerenkov emission, (4). We know that the quasiparticle energy ϵ_k is a bounded periodic function of the quasimomentum k, so that in the limit $k \to \infty$ we have $\mathrm{v}_\varphi(k) \to 0$ (Fig. 1). Strictly speaking, then, the motion of a domain wall at an arbitrarily small velocity v is accompanied by the Čerenkov emission of magnons, but at low velocities we have $k_\alpha \cong 1/a$, where a is the lattice constant. The value of $\dot{Q}$ or the amplitude ΔM is thus exponentially small at low velocities and can be ignored. This result means that it is meaningful to talk in terms of an above-limit motion only if $k_\alpha \Delta \lesssim 1$. To find such solutions of Eq. (3), it is sufficient to use the long-wave approximation for the ϵ_k spectrum. For rare-earth orthoferrites, for example, we have $\epsilon_k^2 = \epsilon_0^2 + c^2 k^2$, and motion at $\mathrm{v} > c$ should be considered above-limit motion.

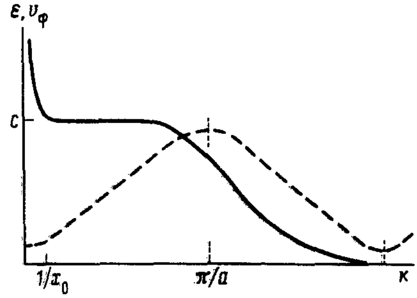

FIG. 1. Solid curve—$v_\varphi(k)$; dashed curve—ϵ_k, both for a rare-earth orthoferrite.

The Čerenkov emission of spin waves leads to an additional damping force. Under the condition $v > c$ we can easily derive the following expression for F_{em}:

$$F_{em} = \left(\frac{E_0}{x_0}\right)\left(\frac{\Delta}{x_0}\right)\left(\frac{c}{v}\right)^2 \equiv F_m\left(\frac{c}{v}\right)^2 . \qquad (5)$$

We thus see that the damping force (as well as ΔM) vanishes in the limit $(v/c) \to \infty$. This vanishing of the damping force justifies our use of the linear theory, i.e., Eq. (1). Let us assume that Eq. (5) also holds, in order of magnitude, at $v \cong c$. The damping force F_{rel} due to ordinary relaxation processes in the spin system was found in Ref. 4 for motion at velocities below the Walker limit. At $v > c$, we can estimate F_{rel} from $F_{rel} = B(x_0/\Delta)v$, where B is the mobility coefficient at $v < c$. Knowing the dependence of the resultant damping force $F = F_{em} + F_{rel}$ on the domain wall velocity v, we can qualitatively construct the function $v(H)$. Figure 2 shows two version of the function $v(H)$; curve 1 corresponds to small values of B $[B < (F_m\Delta/cx_0)]$ and has a region in which $v(H)$ is unstable; curve 2 corresponds to large values of B. The dependence $v(H)$ observed in the experiments of Ref. 1 can be attributed to a transient transition of the domain wall through values $v \cong c$.

We would like to point out that in addition to the emission of spin waves there may be a Čerenkov emission of other types of collective oscillations of the system (sound,[5] surface waves, optical spin waves, etc.). The corresponding energy loss by the domain wall will lead to knees of various sizes on the curve of $v(H)$ at wall velocities near the phase velocity of the long-wave quasiparticles, as has actually been observed experimentally.[1,6,7]

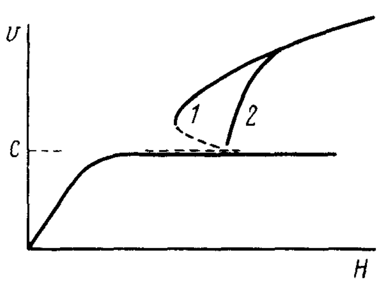

FIG. 2. The function $v(H)$. Dashed region—Instability region; 1—slight relaxation; 2—major relaxation.

[1]We are assuming that the external field is not too strong, so that domain walls with different magnetizations can exist.

1. M. V. Chetkin and A. I. Akhutkina, Zh. Eksp. Teor. Fiz. 78, 761 (1980) [Sov. Phys. JETP 51, 383 (1980)].
2. V. G. Bar'yakhtar, B. A. Ivanov, and A. L. Sukstanskiĭ, Pis'ma Zh. Tekh. Fiz. 5, 853 (1979) [Sov. Tech. Phys. Lett. 5, 351 (1979)].
3. V. M. Eleonskiĭ, N. N. Kirova, and N. E. Kulazin, Zh. Eksp. Teor. Fiz. 79, 321 (1980) [Sov. Phys. JETP 52, 162 (1980)].
4. V. G. Bar'yakhtar, B. A. Ivanov, and A. L. Sukstanskiĭ, Zh. Eksp. Teor. Fiz. 78, 1509 (1980) [Sov. Phys. JETP 51, 757 (1980)].
5. V. G. Bar'yakhtar, B. A. Ivanov, and A. L. Sustanskiĭ, Zh. Eksp. Teor. Fiz. 75, 2183 (1978) [Sov. Phys. JETP 48, 1100 (1978)].
6. M. V. Chetkin, Zh. I. Byňzarov, and S. N. Gadetskiĭ, Tezisy XV Vsesoyuznoĭ konferentsii po fizike magnitnykh yavleniĭ (Proceedings of the Fifteenth All-Union Conference on the Physics of Magnetic Phenomena), Perm', 1981, pp. 32–33.
7. P. D. Kim and D. C. Huang, Tezisy XV Vsesoyuznoĭ konferentsii po fizike magnitnykh yavleniĭ (Proceedings of the Fifteenth All-Union Conference on the Physics of Magnetic Phenomena), Perm', 1981, pp. 26–27.

Translated by Dave Parsons
Edited by S. J. Amoretty