The quantum oscillations in the velocity and attenuation of sound under magnetic-breakdown conditions have different mechanisms

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It is shown experimentally and theoretically that the quantum oscillations in the velocity and attenuation of sound associated with magnetic breakdown have qualitatively different mechanisms, in contradiction to the conventional semiclassical interpretation.

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1. The quantum magnetic-breakdown oscillations of the kinetic coefficients in a magnetic field H are qualitatively different from the semiclassical oscillations.[1] They result from an interference of semiclassical waves which are scattered by magnetic-breakdown centers, and they do not (in general) reduce to oscillations in the number density of states, in contrast with the semiclassical oscillations. The interference nature of the magnetic-breakdown kinetic oscillations is particularly clear in the common case in which the magnetic-breakdown configuration (a system of classical orbits in p space which are coupled by the magnetic breakdown) consists of orbits with the ordinary dimensions p_F of the characteristic Fermi momentum, connected by anomalously small orbits (Fig. 1). A small orbit, which is an interference (quantum) "gate," whose transmission is periodic with a period of 2π in the semiclassical phase $\varphi_M = cS(E,p_z)/e\hbar H$ [S is the area of the small orbit, E is the electron energy, and p_z is the projection of the electron momentum on $H = (0,0,1)$] "controls" the motion of the electron in large orbits. Because of this control effect, the matrix elements of the physical quantities oscillate over φ_M with a period of 2π. The amplitude of
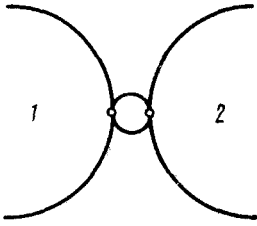
FIG. 1. Coupling of two large orbits (1 and 2) through a small orbit. The points are the points of magnetic breakdown.

these oscillations remains finite even when the relative time spent by an electron in a small orbit, β_M, tends toward zero. As a result, there are distinctive "giant" magnetic-breakdown oscillations in various kinetic coefficients [with a period Δ_M; $1/H = eh/cS_0$, where S_0 is the extreme value of $S(E_F, p_z)$, and E_F is the Fermi energy]. In particular, there are unusual oscillations in the resonant (collisionless) attenuation of sound. There are two quantitatively different mechanisms leading to the control effect in the magnetic-breakdown oscillations of the attenuation. One mechanism stems from the influence of the control effect on the matrix elements of the electron-phonon interaction operator (as has been shown theoretically[2] and experimentally[3]). The other, and subtler, mechanism, observed theoretically and experimentally in Ref. 4 (the experiments involved the attenuation of longitudinal sound in tin), stems from the influence of the control effect on the frequency of the spatial resonance; this second mechanism has an anomalously sharp anisotropy at small values of the angle (θ) between H and the symmetry axis of the metal, n [in tin, $n = (0,0,1)$]. Our purpose in this study is to determine the role played by the small orbits in the magnetic-breakdown oscillations of the sound velocity s.

2. The conditions in the present experiments corresponded to the situation discussed in Ref. 4. We studied the magnetic-breakdown oscillations in s and in the attenuation of longitudinal sound for oscillations having a period corresponding to the small orbit of the third zone. The sample was a tin single crystal with a residual resistance $\sim 2 \times 10^{-5}\,\Omega$. The measurements were made by a pulsed method[5] at a frequency $\omega/2\pi = 250$ MHz at fields $H \sim 30$ kOe, where there is a pronounced magnetic breakdown. As in Ref. 4, the angle between the wave vector (q) of the sound and the n axis was no greater than $0.5°$. The measurements were carried out at $T = 4.2$ K. Comparison of Figs. 2a, 2b, 3a, and 3b shows that the behavior of s as a function of H and as a function of the angle $\theta \ll 1$ is qualitatively different from the behavior of Γ as a function of the same quantities. These results explicitly contradict the conventional semiclassical interpretation, according to which the oscillatory behavior of s is of the same form as that of Γ (aside from a phase shift); furthermore, it can even be shown that such a difference between the velocity and attenuation curves contradicts the dispersion relations which relate s and Γ. As will be shown below, there actually is no contradiction here, and the difference in the behavior of the oscillations amplitudes of s and Γ is found because the magnetic-breakdown oscillations of s and Γ are qualitatively different in nature.

3. Expressing the oscillatory part of the sound velocity, s, in terms of the polarization operator $\Pi(\omega, q)$ of the retarded single-phonon Green's function, and using the dispersion relations which relate $\mathrm{Re}\Pi$ and $\mathrm{IM}\Pi$, we can write $\tilde{s}$ as a power series
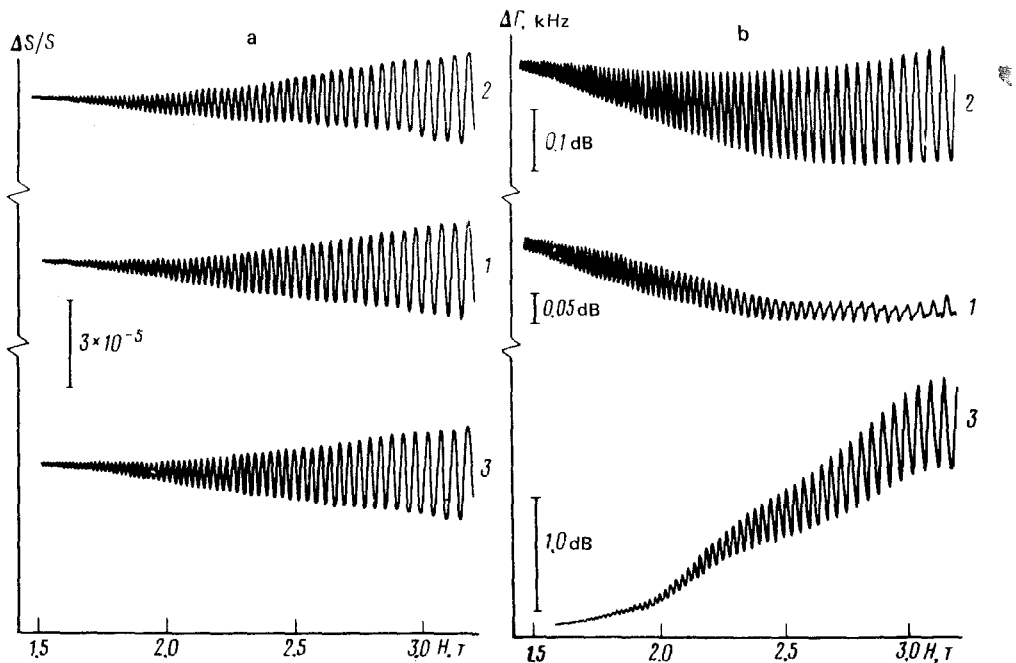
FIG. 2. Dependence of the sound velocity (a) and the sound attenuation (b) on H at various angles θ. $1-\theta = -0.4°$; $2--1.5°$; $3--3.2°$.
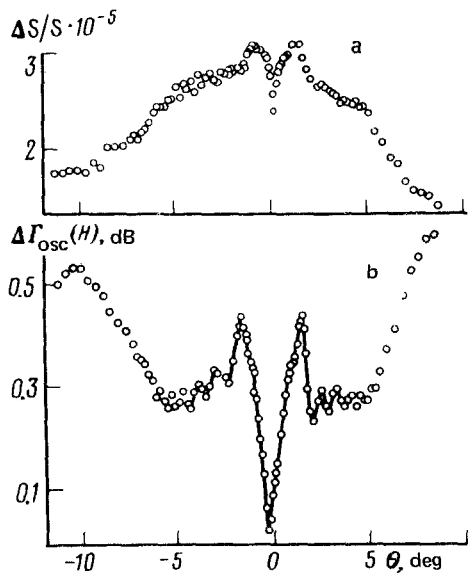

FIG. 3. Dependence of the amplitude of the oscillation in the sound velocity (a) and that in the sound attenuation (b) on the angle θ for $H = 30.6$ kOe.

in ω, the smallest of the frequencies of the problem: $\tilde{s} = \tilde{s}_0 + \tilde{s}_1 + \ldots$, where

$$\frac{\tilde{s}_0}{s} = \frac{1}{\pi} \int \frac{d\omega'}{\omega'} \, \tilde{\gamma}(\omega, q) \; ; \quad \frac{\tilde{s}_1}{s} = \frac{\omega^2}{\pi} \int \frac{d\omega'}{\omega'} \, \frac{\partial^2 \tilde{\gamma}(\omega' q)}{\partial \omega'^2} \; ; \qquad (1)$$

s is the sound velocity at $H = 0$; and γ is the oscillatory part of the expression

$$\gamma = \frac{1}{V\rho s^2} \sum_{\eta\eta'} \delta(E_\eta - E_{\eta'} - \hbar\omega) \, [f_0(E_\eta) - f_0(E_{\eta'})] \, | \langle \eta | \Lambda e^{i\mathbf{q}\mathbf{r}} | \eta' \rangle |^2 . \qquad (2)$$

Here V and ρ are the volume and density, respectively, of the metal; $f_0(E)$ is the Fermi distribution function; Λ is the strain energy of the longitudinal sound; the index η represents the complete set of stationary quantum states $|\eta\rangle$ of an electron in the magnetic field $(\eta = \{n, p_z, P_x\}$, where n is a discrete quantum number, and P_x is the x component of the generalized momentum); and E_η is the energy of the stationary states, a quasirandom function of η according to Ref. 1. Since the control effect strongly influences the matrix elements which appear in γ and also the energy difference $E_\eta - E_\eta$, we could expect important manifestations of the control effect in both $\tilde{s}_0$ and $\tilde{s}_1$. A detailed analysis based on a quasirandom magnetic-breakdown spectrum for an arbitrary magnetic-breakdown configuration by the formalism of Ref. 1 shows, however, that the control effect does not influence $\tilde{s}_0$: The amplitude of the $\tilde{s}_0$ oscillations associated with the small orbit vanishes (linearly) in the limit $\beta_M \to 0$ and is determined by a universal expression which contains the average of the strain energy over the revolution time in the small orbit, $\bar{\Lambda}_M$:

$$\tilde{s}_0 = s \, \frac{2e \, H\beta_M \Omega_H}{cp s^2 V (2\pi\hbar)^3} \int dp_z \, dE \, \bar{\Lambda}_M^2 \frac{\partial f_0(E)}{\partial E} \, (F + F^*) , \qquad (3)$$

where Ω_H is the characteristic cyclotron frequency, and

$$F \equiv F(E, p_z) = \frac{(1 - W) \exp\{ i \, (\varphi_M(E, p_z) + \delta)\}}{1 - (1 - w) \, \exp\{i(\varphi_M(E, p_z) + \delta)\}} .$$

Here $w(H_0/H)$ is the probability for magnetic breakdown, which is common to both magnetic-breakdown points of the small orbit (Fig. 1), $\delta(H_0/H)$ is the phase shift as the electron passes a magnetic-breakdown point, and H_0 is the breakdown field. The functions w and δ are described in Ref. 1. We have omitted from Eq. (3) all the oscillatory terms associated with the large orbits, assuming them to be negligibly small because of the thermal attenuation at $T = 4.2$.

In contrast with $\tilde{s}_0$, the small increment $\tilde{s}_1$ has a structure which is determined by the control effect and which depends on not only the form of the magnetic-breakdown configuration but also the specific mechanism for the manifestation of the control effect in the quantity $\bar{\gamma}$. In the case under consideration here it can be shown that the major contribution to $\tilde{s}_1$ comes from small values of ω', so that $\tilde{s}_1$ reproduces the behavior of Γ, causing a characteristic narrow spike on the curve in Fig. 3a, which is determined primarily by the quantity $\tilde{s}$, a smooth function of θ. The characteristic value of $\tilde{s}_1$ is on the order of $s(s/v_F)^2 (\theta/\kappa)$ $(\kappa = eH\hbar/cp_F^2$ is a parameter which is a measure of the deviation from the classical case and has the value $\kappa \sim 10^{-4}$;

v_F is the characteristic Fermi velocity), while the characteristic value of $\tilde{s}_0$ is $\tilde{s}_0$ $\sim s\sqrt{\kappa\beta_M}(\bar{\Lambda}_M/E_F)\sim 10|\tilde{s}_1|.$

It should be noted that the quantity $\tilde{s}_0$ is an oscillatory term of that part of the sound velocity which can be expressed in terms of thermodynamic quantities such as the compressibility. Accordingly, the thermodynamic properties of the metal *are not affected* by the control effect, even if the electron-phonon interaction is taken into account in them.

We thus conclude that under magnetic-breakdown conditions the oscillations of s and Γ are caused by qualitatively different mechanisms, in contrast with the semi-classical picture. To the best of our knowledge, this is the first observation of a "structural mismatch" of the quantum oscillations of the sound velocity and attenuation.

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