Instantons in quark plasmas. Role of zeroth fermion modes

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The single-loop increment in the action of an instanton in a plasma of light quarks is calculated by the method of exact Green's functions. Off-diagonal elements of the particle density matrix are taken into account. A temperature-independent term, which suppresses large instantons, is found.

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The quark plasma is recognized as a convenient model for applications of quantum chromodynamics, since a high density automatically puts the system in the region of asymptotic freedom. An interesting problem which has not yet been completely resolved, is that of the behavior of instantons[1] in a plasma.[2]

In a previous paper[3] it was suggested that the thermodynamic functions might be studied by a simultaneous perturbation theory in the charge and mass of the light quarks. In the single-loop approximation, the quarks do not interact with pseudoparticles, and the diagrams of the next order must be calculated.[4]

An error was made here, as was pointed out in Ref. 5, since at a finite chemical potential the zeroth fermion modes begin to grow exponentially at the temporal infinity.

A more detailed analysis has shown that the wave functions of the particles in a self-dual field were found correctly, but the off-diagonal elements of the density matrix were ignored. The net result was that the analysis was restricted to functions which are orthogonal to the zeroth fermion modes.

The incorporation of transitions in the field of instantons involving a change in the quark 4-momentum can cancel the undesired growth of the zeroth modes. Furthermore, the presence of off-diagonal matrix elements means that quarks are scattered by the instanton, so that the interaction of the particles and pseudoparticles is manifested even in the single-loop approximation.

The calculated results may be summarized as follows: The density of the quark plasma decreases as instantons become immersed in it. The medium repels pseudoparticles, and the action of each pseudoparticle increases by an amount

$$\Delta S_{inst} = N_f \mu^2 \rho^2,$$

where N_f is the number of quark species, μ is the chemical potential of the plasma, and ρ is the instanton radius.

This result does not depend on the temperature of the system. Since ΔS is nonzero in first order, the corrections of higher order in g_2 turn out to be inconsequential.

The integral over the dimensions of the pseudoparticles now contains a cutoff factor $\exp[-\Delta S_{\text{inst}}(\mu\rho)]$, so that the corrections to the thermodynamic functions of the plasma can be calculated. With a reasonable choice of physical parameters ($N_f = 3$, $m_u = 4$ MeV, $m_d = 7$ MeV, $m_s = 150$ MeV, and $\Lambda = 100$ MeV) in three-color quantum chromodynamics it is found that the contribution of the instantons to the thermodynamics is small wherever the coupling constant α is small: $\alpha = g^2/4\pi < 1$.

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