Sensitivity spectrum of a superconducting attometer

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Measurements of the noise spectrum of a device for measuring small displacements and small vibration amplitudes are compared with the spectrum of thermal fluctuation vibrations of the sensitive element. This measurement device, a superconducting attometer, uses a superconducting resonator and has a sensitivity of 6×10^{-18} m (6 attometers) at an averaging time of 2.5×10^3 s in the vibration frequency range 10^3–3×10^4 Hz.

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Experimental methods for measuring extremely small displacements and vibration amplitudes are being developed rapidly in connection with the construction of devices for detecting gravitational waves[1-4] and also in connection with research on weak piezostrictive effects, magnetostrictive effects, and phase transitions in samples of small dimensions at low temperatures.[5-7]

One of the most successful methods for measuring small displacements and small vibration amplitudes, which dates back to Ref. 7, uses a coaxial resonator with a capacitive load,[2-4,6,7] which increases the microwave electromagnetic energy density in a small volume near the surface of the vibrating object. Since the sensitivity of such devices is approaching fundamental limits, it is worthwhile to determine the actual noise spectrum of the measuring device and to compare this spectrum with the theoretical spectrum of natural fluctuations.

1. Design and operation of the superconducting attometer. Figure 1 shows a block diagram of the measurement apparatus. The superconducting measuring resonator, of the type described in Ref. 7, is made of bulk niobium; the bottom of this resonator is a niobium membrane M with a mass $m = 85$ g. A quality factor $Q_M = 6 \times 10^2$ at the fundamental vibration mode ($f_M = 55$ kHz) is achieved by mounting the membrane in a region of reduced thickness along its perimeter and by using a

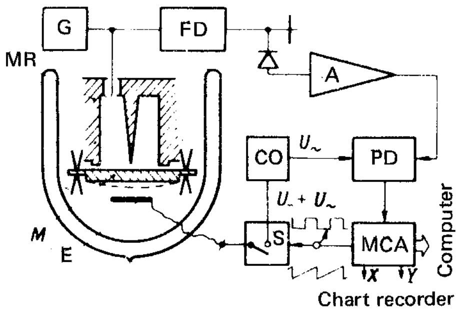

FIG. 1. The measurement apparatus. G—Gunn oscillator; MR—measuring resonator; M—membrane; E—electrode for electrostatic calibration; FD—frequency detector using a cylindrical superconducting resonator; A—Unipan 233 turned amplifier; PD—phase detector; CO—source of an alternating calibration voltage $U_\sim$ with an adjustable average level U_- (HP 3310A); S—electronic switch; MCA—NTA 1024 multichannel analyzer.

contactless connection of the membrane to the resonator. The superconducting resonator is in an evacuated chamber in a cryostat, held at $T = 4.2$ K. A frequency-pulling circuit connects the resonator to a low-noise Gunn oscillator[8] and simultaneously reduces the frequency fluctuations of the source by a factor ~ 200 in the frequency band $\sim$0-10^5 Hz. The resonant frequency of the resonator is $\mu_0 = 9300$ MHz; its quality factor is $Q = 2 \times 10^5$; the working gap between the central rod of the resonator and the membrane is $\Delta \approx 5$ μm; and the derivative of the frequency with respect to the gap width is $d\nu_0/d\Delta \approx 10^{14}$ Hz/m.

The frequency modulation caused in the output signal of the source G by the vibrations of the membrane is converted into an amplitude modulation by the frequency discriminator FD (which uses a superconducting resonator). The amplitude-modulated signal is detected by a low-noise diode detector, amplified by a tuned amplifier, and synchronously detected.

2. *Measurement of the noise spectrum of the attometer.* Stimulated vibrations of the membrane at the frequency of the signal ($U_\sim$) from a calibration oscillator CO are excited by the attractive electrostatic force proportional to $U_- + U_\sim$ (Fig. 1), in a manner similar to that of Refs. 3, 4, and 9. The signal is extracted from the

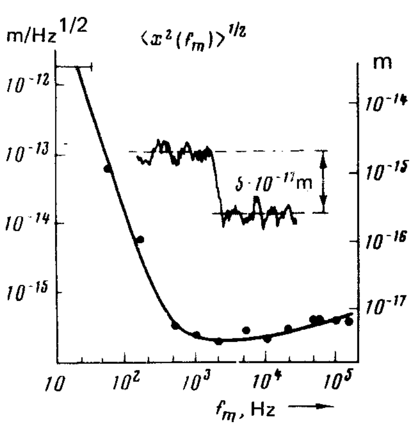

FIG. 2. Sensitivity of the attometer vs the membrane vibration frequency. The ordinate at the left gives the noise in a 1-Hz band, while that at the right gives the noise after averaging over a time of 2.5×10^3 s. The inset shows a sample recording of the stimulated vibration of the membrane at 15 kHz with an amplitude of 5×10^{-17} m (the averaging time is 2.5×10^3 s).

noise with a multichannel analyzer MCA. A switch in the circuit which applies the electrostatic signal is closed at the beginning of each scanning cycle of the multichannel analyzer and opened when half of the channels have been scanned. An N-fold coherent summation improves the signal-to-noise ratio $\propto\sqrt{N}$. The stored signal is transferred to an EMG-666 minicomputer for statistical processing.

The inset in Fig. 2 is an example of the measurement of the vibration, with an amplitude 5×10^{-17} m, excited at 15 kHz. The results of these measurements are fed from the multichannel analyzer to a chart recorder after 5×10^4 storage events (over 2.5×10^3 s). The dispersion, i.e., the sensitivity for this particular example, is 6×10^{-18} m, which corresponds to 3×10^{-16} m/Hz$^{1/2}$.

Figure 2 shows the dependence of the sensitivity of the device on the membrane vibration frequency. One of the ordinate axes corresponds to a frequency band of 1 Hz, while the other corresponds to the narrowest band achieved (the accumulation time is 2.5×10^3 s). The horizontal bar starting from the ordinate at the left (and corresponding to the frequency interval 0-40 Hz) shows the result obtained by applying only a constant voltage to the electrostatic system during the time required to scan half the channels (25 ms).

In the frequency interval 10^3-10^5 Hz the sensitivity is determined by the frequency noise of the calibration oscillator and by the noise of the diode. Below 10^3 Hz, there is another contributing factor: the vibrational noise of the apparatus, which was located in an area unfavorable from the microseismic standpoint.

3. *Fundamental sensitivity limits.* What are the classical and quantum-mechanical limits on the sensitivity of a small-displacement meter?

a) The thermal fluctuation oscillations of the membrane have a spectrum

$$< x^2(\omega) > = \frac{2kT}{\pi Q_M \gamma \omega_M \left\{\left[\left(\frac{\omega}{\omega_M}\right)^2 - 1\right]^2 + \left(\frac{\omega}{\omega_M Q_M}\right)^2\right\}} \qquad (1)$$

with a sharp maximum at the frequency of the mechanical resonance, ω_M. For measurements near ω_M in a 1-Hz band we would have

$$< x^2(\omega) >^{1/2} \cong \sqrt{\frac{2kT}{\pi\gamma}\frac{Q_M}{\omega_M}} \sim 4\cdot 10^{-18}\,\mathrm{m/Hz^{1/2}} \qquad (2)$$

and at frequencies $\omega \ll \omega_M$ we would have

$$< x^2(\omega) >^{1/2} \cong \sqrt{\frac{2kT}{\pi\gamma Q_M \omega_M}} \sim 2\cdot 10^{-22}\,\mathrm{m/Hz^{1/2}} \qquad (3)$$

for the membrane parameters listed above. Here $\gamma\cong 6\times 10^{11}$ dyn/cm is the stiffness of a membrane which is pinned along its edges[10] at $T=4.2$ K.

b) The quantum-mechanical uncertainty in the buckling of a membrane in a 1-Hz band is

$$\langle x^2(\omega_M)\rangle^{1/2} \sim \frac{1}{\omega_M}\sqrt{\frac{hQ_M}{2m}} \sim 10^{-20}\ \mathrm{m/Hz^{1/2}} \qquad (4)$$

and is, at least for now, beyond the noise range of measurement devices which have been reported.[1-7] On the other hand, the height of the resonance thermal-vibration peak in (2) is only ~75 times lower than the noise level (Fig. 2).

We note in conclusion that this attometer detects vibrations over the broad frequency range 0-10^5 Hz. This broad-band capability is achieved by using a low-Q frequency discriminator with a bandwidth ~10^5 Hz. A discriminator with a higher Q would be preferable for measurements in a narrower range of vibration frequencies. For this particular apparatus, a higher-Q discriminator would make it possible to reduce the sensitivity by two orders of magnitude, to 10^{-18} m/Hz$^{1/2}$ at vibration frequencies ~10^3 Hz.

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