Nonlinear spin relaxation in superfluid ^{3}He-B in the absence of an external magnetic field

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It is shown that spin relaxation in ^{3}He-B with the external field switched off is characterized by the polarization of the motion of the spin vector and of the order parameter. For the constant $\Gamma_{\|}/\Omega_L$ equal to 0.1, the switched-off field must be of the order of $\gtrsim$ 19 G.

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This paper is concerned with the study of spin dynamics in superfluid ^{3}He-B within the scope of Leggett and Tagagi's intrinsic relaxation mechanism.[1,4] It is assumed that the system is spatially uniform and the magnetic field is switched off over a time that is much less than the relaxation time. Then the LT equations in the absence of an external field have the form

$$\frac{d\mathbf{S}}{dt} = -\frac{dU}{d\theta}\mathbf{c}$$

$$\frac{d\mathbf{c}}{dt} = -\frac{1}{2}\gamma^2\chi^{-1}\cot\frac{\theta}{2}(\mathbf{S}\cdot\mathbf{c})\mathbf{c} + \frac{1}{2}\gamma^2\chi^{-1}\cot\frac{\theta}{2}\mathbf{S} + \frac{1}{2}\gamma^2\chi^{-1}(\mathbf{S}\times\mathbf{c})$$

$$\frac{d\theta}{dt} = \gamma^2\chi^{-1}(\mathbf{S}\cdot\mathbf{c}) - \mu\frac{dU}{d\theta}.$$

Here θ and $\mathbf{c}$ are the angle and axis of rotation of the order parameter matrix. The dissipation-free Leggett equations,[1] corresponding to frequencies ω such that $\omega\tau \ll 1$, where τ is the quasiparticle relaxation time, 10^{-7} sec, are obtained for $\mu = 0$.

The study of the system of LT equations is based on systematic application of computer analysis and asymptotic methods. Numerical integration was carried out on a computer with the help of a fourth-order Runge-Kutta algorithm. In order to avoid computational errors, double precision and a variable step in the difference scheme were used. The computational results agree with the characteristics obtained from the averaged LT equations for large values of the magnetization. The calculations were carried out assuming that the Leggett frequency Ω_L is of the order of 10^5 rad/s and that the D mode regime corresponds to the frequency ω_0 of the order of 10^6 rad/s or magnetic fields of the order of 100 G. The dissipative constant for longitudinal NMR, $\Gamma_{\|}$, was assumed to be of the order of 10^{-4} and 10^{-3} s^{-1}, which corresponds to values of the constant $a = \Gamma_{\|}/\Omega_L$ equal to 0.1 and 0.01.[1] It was found that the attractor regime, indicated in Ref. 2, is characterized by a polarization of the order parameter and of the spin vector: for characteristic time intervals of the order of $1/\omega_0$, the vector $\mathbf{c}$ of the axis of rotation of the order parameter rotates in a plane for which the coordinates of the unit normal, L_i, are

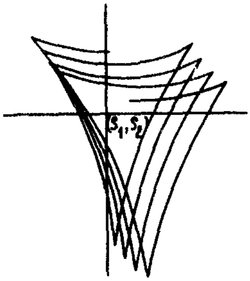

FIG. 1. The motion of the spin vector in coordinates S_1 and S_2 for high magnetization.

given by the values of the integrals of the dissipation-free Leggett equations (see the review by Brinkman and Cross in Ref. 3)

$$L_i = \sin\frac{\theta}{2}\left[\cot\frac{\theta}{2}\,(\mathbf{S}\times\mathbf{c})_i - S_i + (\mathbf{S}\cdot\mathbf{c})c_i\right] / \ (\mathbf{S}^2 - (\mathbf{S}\cdot\mathbf{c})^2)^{1/2}.$$

Within the same period, the vector **S** describes in a plane parallel to the plane of **c**, a curve (see Fig. 1) with characteristic dimension 0, $3\gamma^{-2}\chi\Omega_L^2/(\omega_0 S)$. In this case, the frequencies for the motion of the vectors **S**, **c**, and the angle θ satisfy the relation $2\omega_S = \omega_C = \omega_\theta$ and are of the order of ω_0. Over the time of a single period of the vector **c**, the normal **L** is deflected by dissipation by not more than $\delta(\mathbf{L}) = 0.005$ rad. The lifetime on the attractor, estimated by the condition $\delta(\mathbf{L}) \leqslant 0.005$, depends linearly on the square of the magnetization and attains several 0.1 s for $a = 0.1$ and, in addition, the angle of precession of the normal, over the entire lifetime on the attractor, does not exceed 0.05 rad. The time for the system to reach the attractor, τ_B, estimated from the condition that the criterion $\delta(\mathbf{L}) \leqslant 0.005$ is satisfied during a single period, is several ms; for example, for $a = 0.1$, an initial field of 100 G, and an angle $\theta_0 = 0.1$, the computed value of τ_B was equal to 2 ms.

Figure 2 shows the motion of the spin vector on a scale of the order of $|S|$, i.e., much greater than the size of the triangle in Fig. 1. When the attractor is reached from the region of values of dynamic variables, characterized by an angle θ less than $\pi/2$, the direction of rotation of the average spin vector **S** is reversed. When the attractor is reached from the region $\theta > \pi/2$, the direction of rotation does not change. The attractor regime corresponds to an angle of $\pi/4$ between the vectors **L** and **S** (to within 0.04 rad). For $a = 0.1$, the attractor regime continues up to magnetization values corresponding to fields of 19 G and, in addition, it remains stable relative to perturbations of the input data.

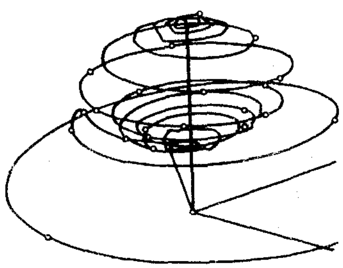

FIG. 2. The average motion of the spin vector in S_1, S_2, and S_3 space. The upper curve corresponds to leaving the region $\theta > \pi/2$ and the lower curve corresponds to leaving the region $\theta < \pi/2$. The average value of the vector **L** almost coincides with the OS_3 axis. The breaks in the curve are caused by the small number of points drawn by the plotter.

The frequency of rotation of the vector c over the entire duration of the relaxation regime, from high values of magnetization to the lowest values, decreases monotonically, in contrast to frequencies ω_S and ω_θ, which drop steeply for fields of the order of 10 G.

It follows from the results of this paper that the attractor regime, characterized by a polarization of the spin vector and of the order parameter, is realized for small values of switchable magnetic fields. Suitable experimental conditions have been achieved in studies on identification of superfluid phases of ^{3}He in the *P*-paired state[5] and in the WP mode.[3,6] Presumably, the characteristics of the attractor regime described in this paper make it possible to observe it experimentally.

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