Determination of the specularity coefficient for surface reflection of electrons from doppleron oscillations

I. F. Voloshin, N. A. Podlevskikh, V. G. Skobov, L. M. Fisher, and A. S. Chernov
V. I. Lenin All-Union Electrical Engineering Institute

(Submitted 2 February 1982)

Pis'ma Zh. Eksp. Teor. Fiz. **35**, No. 6, 235–238 (20 March 1982)

A general expression is obtained for the impedance of a metallic plate in a strong magnetic field **H**, perpendicular to the surface, for an arbitrary value of the coefficient of reflection for electrons p. A method, which is based on the use of this expression, is proposed for determining p from the shape of doppleron oscillations. With its help, the specularity coefficient is determined for octahedron holes from measurements of the impedance of a tungsten plate in the case **H**|| [100].

PACS numbers: 73.25. + i

Several methods have recently been proposed for determining the specularity coefficient p for reflection of electrons from the surface of a metal. The method for measuring p directly, proposed by Tsoĭ,[1] is very effective for values of p that are not very small.

Another method is based on measuring the anharmonicity of Sondheimer vibrations,[2] which was predicted in Ref. 3. In this paper, we describe a method which makes use of the anharmonicity of doppleron vibrations in the impedance of a metal plate. It differs from the method in Ref. 2 by the fact that it is more sensitive and does not require a knowledge of the explicit form of the nonlocal conductivity. In addition, in contrast to Refs. 1 and 2, the method proposed is contact-free.

The field distribution in a plate in strong magnetic fields $[(H/H_L)^3 \gg 1$, where H_L is the threshold doppleron field] for arbitrary value of p can be expressed in terms of the field $e_0(\zeta)$ in a semi-infinite metal for $p=0$,

$$e_0(\zeta) = e^{iq_1\zeta} + f(\zeta), \tag{1}$$

where $\zeta = 2\pi z/u$, z is the distance from the surface, u is the maximum displacement of electrons over a cyclotron period, $q_1 = k_1 u/2\pi$, k_1 is the propagation vector of the long-wavelength component, $f(\zeta)$ is the short-wavelength part of the field, which consists of a sum of the doppleron and Gantmakher-Kaner component fields (see, for example, Ref. 4). In the range of magnetic fields being examined, the inequalities $q_1 \ll 1$ and $|f(\zeta)| \ll 1$ are satisfied, while the dimensionless wave vector q, corresponding to the asymptotic form of $f(\zeta)$ for $\zeta \gg 1$, is close to -1.

The efficiency of the interaction of electrons, which are responsible for the Doppler-shifted cyclotron resonance, with the long-wavelength component of the field in the presence of specular reflection is small. A specularly reflected electron acquires a momentum that is nearly equal in magnitude and opposite in sign to the momentum it would acquire in approaching the surface. As a result, the ratio of the momenta acquired with specular and diffuse reflection turns out to be equal to $-2q_1$. For this reason, for $1-p \gg |q_1|$, the effect of specularly reflected electrons on the excitation of the short-wavelength component can be ignored and the field in the semi-infinite metal is described by the equation

$$e_p(\zeta) = e^{iq_1\zeta} + (1-p)f(\zeta). \tag{2}$$

For antisymmetric excitation of the plate, the long wavelength component of the field, normalized to unity, has the form

$$\left[e^{iq_1\zeta} - e^{iq_1(L-\zeta)}\right](1-e^{iq_1L})^{-1},$$

while the corresponding short-wavelength component is equal to $(1-p)[f(\zeta)-f(L-\zeta)]$, where $L=2\pi d/u$ and d is the thickness of the plate. In addition, the presence of specularly reflected electrons leads to the fact that the short-wavelength component, having reached the surface, is reflected with coefficient p and propagates in the opposite direction. For this reason, $f(\zeta)$ must be replaced by the sries $f(\zeta)+pf(2L-\zeta)+p^2f(2L+\zeta)\cdots$. As a result, the distribution of the field in the plate has the form

$$E(\zeta) = [e^{iq_1\zeta} - e^{iq_1(L-\zeta)}](1-e^{iq_1L})^{-1} + (1-p)\sum_{n=0}^{\infty}(-p)^n\left\{f(\zeta+nL) - f[(n+1)L-\zeta]\right\}. \tag{3}$$

Using (3), we find the impedance

$$\mathcal{Z} = a \frac{iE(0)}{E'(0)} = a\left[a\mathcal{Z}_p^{-1} - (1-p)^2 \sum_{n=1}^{\infty} (-p)^{n-1} f(nL)\right]^{-1}, \tag{4}$$

$$\mathcal{Z}_p = a\left[q_1 \frac{1+e^{iq_1L}}{1-e^{iq_1L}} - if'(0)(1-p)\right]^{-1}, \quad a = 4\omega u/c^2, \tag{5}$$

where ω is the frequency of the exciting field, and the prime indicates differentiation with respect to ζ. In obtaining (4)-(5), we took into account the relation $f'(L) = -if(L)$, which follows from the inequality $L \gg 1$.

Let us examine a situation when the Doppler oscillations greatly exceed the Gantmakher-Kaner oscillations (GKO) and $f(L) \cong b_0 \exp(iq_2L)$, where $q_2 = q_2' + iq_2''$ is the reduced doppleron wave vector. In this case, the sum in (4) becomes a geometric progression and the expression for $\mathcal{Z}$ with the help of identity transformations assumes the form

$$\mathcal{Z} = \mathcal{Z}_p + \frac{A_1 \exp(iq_2'L)}{1 + \lambda \exp(iq_2'L)}, \tag{6}$$

$$\lambda = pe^{-q_2''L} - A_1/\mathcal{Z}_p, \quad A_1 = (1-p)^2 \frac{b_0}{a} \mathcal{Z}_p^2 e^{-q_2''L}. \tag{7}$$

The quantities $R_p = \mathrm{Re}\mathcal{Z}$ and $X_p = \mathrm{Im}\mathcal{Z}_p$ represent the smooth parts of the surface resistance and reactance of the plate; the real and imaginary parts of the second term in (6) describe their oscillations. It is evident that these oscillations are not harmonic and that the degree of anharmonicity is determined by the parameter $\eta = |\lambda|$. For this reason, it is possible to find the specularity coefficient p from measurements of the impedance.

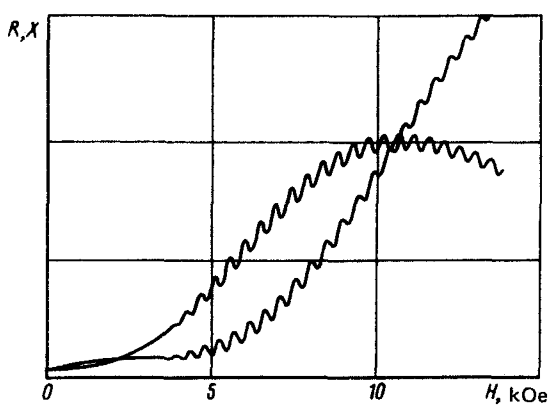

FIG. 1. The functions $R_+(H)$ (curve 1) and $X_+(H)$ (curve 2) for a tungsten plate. $\mathbf{H} \| \mathbf{n} \| [100]$, $(\omega/2\pi) = 330$ kHz, $d = 0.43$ mm, and $T = 4.2$ K.

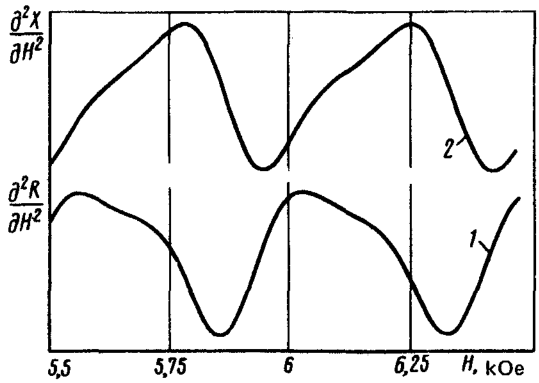

FIG. 2. The traces of d^2R_+/dH^2 (1) and d^2X_+/dH^2 (2) on an enlarged scale.

The total range of the oscillations $2r$, where

$$2r = \max(R - R_p) - \min(R - R_p) = \max(X - X_p) - \min(X - X_p)$$

is related to A_1 by the relation

$$|A_1| = r(1 - \eta^2). \tag{8}$$

Using Eqs. (7) and (8), as well as the fact that $b_0 < 0$, we shall express the specularity coefficient p in terms of experimentally determined quantities:

$$pe^{-q''_2 L} = \left\{ \eta^2 - \left[\frac{rX_p}{R_p^2 + X_p^2} (1-\eta^2) \right]^2 \right\}^{1/2} - \frac{rR_p}{R_p^2 + X_p^2} (1-\eta^2). \tag{9}$$

We emphasize that the form of this equation does not depend on the nonlocal conductivity and is universal.

Figure 1 shows the experimental traces of $R_+(H)$ and $X_+(H)$ for tungsten plate with a resistivity ratio $\rho_{300}/\rho_{4.2K} = 35\,000$ (in tungsten, the resonant carriers are holes and, for this reason, all the equations presented above refer to positive circular polarization). In preparing the specimen, its surface was mechanically ground and chemically polished. Figure 2 shows fragments of the traces of d^2R_+/dH^2 and d^2X_+/dH^2 on a large scale, obtained by using a modulation technique. We note that these curves, recorded under conditions when the amplitude of the modulation is commensurable with the period of the oscillations, strictly speaking, are not exact derivatives (see, for example, Ref. 6). It is more clearly evident from traces of the derivatives that the oscillations indeed are not sinusoidal. The quantity η is easily found from the characteristic points on the curve $R(H)$ or $X(H)$ with the help of elementary relations following from (6). The value of η obtained is not large ($\eta = 0.1$-0.2). It is possible to determine η with high accuracy from the ratio of the amplitudes of the harmonics of d^2R/dH^2 or d^2X/dH^2. For this, it is necessary to expand the experimental curve over a single period in a Fourier series.

Analysis of the experimental data using the method described above for the fifth through tenth periods gives values of the quantity $p\exp(-q''_2 L)$ within the range

0.08 ± 0.01. The values of this quantity for subsequent periods decrease, since the amplitude of the doppleron oscillations becomes comparable to the GKO amplitude, whose form is much less appreciably distorted. For the first four periods, this quantity is also found to be smaller, since its damping increases as the doppleron threshold is approached. For the periods presented above, the quantity $q_2'' L$ is approximately equal to d/l, where l is the mean free path of resonance electrons. The mean free path of electrons in the same group was determined in Ref. 6 for specimens with a resistivity ratio equal to 5×10^4. It turned out to be close to 1 mm. For this reason, for the given specimen, we shall assume that $l = 0.8$ mm. As a result, we obtain the value $p = 0.15 \pm 0.05$, which agrees with the data in Ref. 7. On the other hand, this value is much smaller than that obtained by Tsoĭ, who studied specimens which were prepared by using a different process.

In conclusion, we shall present the equation for GKO with "minus" polarization

$$\mathcal{Z}_- - \mathcal{Z}_p = -\mathcal{Z}_p \left\{ 1 + \frac{a}{(1-p)^2 \mathcal{Z}_p} \left[\frac{1}{2\pi i} \int_{C_-} \frac{q\,dq}{D_-(q)} \, \frac{e^{iqL}}{1 + p e^{iqL}} \right]^{-1} \right\}^{-1}$$

where $D_-(q)$ is the left side of the dispersion equation $D_-(q)$; C is the contour that circumscribes in the counterclockwise direction a cut, drawn in the q plane from the branch point of the nonlocal conductivity on the right side to ∞. Analysis of the GKO also allows measuring the quantity p, but only if the explicit form of the nonlocal conductivity is known. Using for tungsten the function $D_-(q)$ from Ref. 4 and comparing (10) with experimental data, we obatined $p = 0.15$.

1. V. S. Tsoĭ, Zh. Eksp. Teor. Fiz. **68**, 1849 (1975) [Sov. Phys. JETP **41**, 927 (1975)].
2. A. M. Grishin, P. P. Lutsishin, Yu. S. Ostroukhov, and O. A. Panchenko. Zh. Eksp. Teor. Fiz. **76**, 1325 (1979) [Sov. Phys. JETP **49**, 673 (1979)].
3. Yu. M. Goland, Fiz. Tverd. Tela **10**, 81 (1968) [Sov. Phys. Solid State **10**, 58 (1968)].
4. I. F. Voloshin, V. G. Skobov, L. M. Fisher, and A. S. Chernov, Zh. Eksp. Teor. Fiz. **80**, 183 (1981) [Sov. Phys. JETP **53**, 92 (1981)].
5. I. F. Voloshin, S. V. Medvedev, V. G. Skobov, L. M. Fisher, and A. S. Chernov, Zh. Eksp. Teor. Fiz. **71**, 1555 (1976) [Sov. Phys. JETP **44**, 814 (1976)].
6. I. F. Voloshin, V. G. Skobov, L. M. Fisher, and A. S. Chernov, Zh. Eksp. Teor. Fiz. **82**, 293 (1982) [Sov. Phys. JETP (to be published)].
7. I. I. Razgonov and V. S. Tsoĭ, Abstracts of Reports at the 19th All Union Conference on Low-Temperature Physics (NT-19), Minsk, 1976, p. 113.

Translated by M. E. Alferieff
Edited by S. J. Amoretty