$U(1)$ supergravity

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An explicitly covariant superfield description is offered for a new minimal version of 1-supergravity. The new description is based on a complex geometry which is inherent in this version.

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A new minimal version of 1-supergravity, which is like the ordinary version in that it contains 12 Fermi fields and 12 Bose fields but differs in the composition of these fields, has recently attracted widespread interest[2)] (Refs. 1–4). A remarkable property of this new version is that it contains a local chiral U(1) invariance; the gauge field for this invariance is an auxiliary field, rather than the physical field. If this new mechanism for gauge invariance also proves correct in the quantum theory, it will be extremely promising for deriving a superunification of all interactions on the basis of the $N=8$ supergravity.

In this letter we are proposing an explicitly covariant geometric approach to a new version of supergravity. This new approach is based on a complex geometry which is inherent in this new version. This approach has previously been applied successively to the usual minimal 1-supergravity[6,7] and also[8] to a nonminimal[9] 1-supergravity. We recall that in this approach one considers a superspace with four vector coordinates and four spinor coordinates,

$$C^{4,4} = \{ z_L \} = \{ x_L^m , \theta_L^\mu , \bar{\varphi}_L^{\dot{\mu}} \} , \tag{1}$$

and a group of analytic transformations of "triangular" structure in it,

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$$\delta x_L^m = \lambda^m (x_L, \theta_L)$$

$$\delta\theta_L^\mu = \lambda^\mu (x_L, \theta_L) \qquad (2)$$

$$\delta\bar{\varphi}_L^{\dot\mu} = \bar{\rho}^{\dot\mu}(x_L, \theta_L, \bar{\varphi}_L),$$

which leaves the complex chiral subspace invariant:

$$C^{4,2} = \{ \mathfrak{z}_L \} = \{ x_L^m , \theta_L^\mu \}. \qquad (3)$$

The next step is to define the real superspace

$$R^{4,4} = \{ z \}\{ x^m, \theta^\mu, \bar\theta^{\dot\mu} \} \qquad (4)$$

as a hypersurface in $C^{4,4}$:

$$x^m = \mathrm{Re}\, x_L^m , \quad \theta^\mu = \theta_L^\mu , \quad \bar\theta^{\dot\mu} = \bar\theta_R^{\dot\mu}$$

$$\mathcal{H}^m (x, \theta, \bar\theta) = \mathrm{Im}\, x_L^m , \quad \mathcal{H}^\mu = (x, \theta, \bar\theta) = \varphi_R^\mu - \theta_L^\mu,$$

$$\bar{\mathcal{H}}^{\dot\mu}(x, \theta, \bar\theta) = \bar\varphi_L^{\dot\mu} - \bar\theta_R^{\dot\mu}. \qquad (5)$$

The coordinates $C^{4,4}/R^{4,4}$ become the functions

$$\mathcal{H}^m(x, \theta, \bar\theta), \mathcal{H}^\mu(x, \theta, \bar\theta), \bar{\mathcal{H}}^{\dot\mu}(x, \theta, \bar\theta)$$

of the coordinates $R^{4,4}$.

These coordinates define a surface in $C^{4,4}$ and, simultaneously, the curved geometry of $R^{4,4}$. In this case the transformations in (2) correspond to a conformal supergravity. By constraining them in an appropriate manner, we can find the transformation groups of Einstein supergravities. Because of the triangular structure of group (2), the superdeterminants (berezinians) of the transformations in $C^{4,4}$ and $C^{4,2}$ have a multiplicative property. As a result, we can single out subgroups of group (2) by imposing the natural constraint

$$\left[\mathrm{Ber}\left(\frac{\partial z_L'}{\partial z_L}\right)\right]^{3n+1} = \left[\mathrm{Ber}\left(\frac{\partial \mathfrak{z}_L'}{\partial \mathfrak{z}_L}\right)\right]^{2n} . \qquad (6)$$

It turns out that each value of n corresponds to a nonminimal formulation of an Einstein 1-supergravity with 20 + 20 fields, with two exceptions: $n = -\frac{1}{3}$ and $n = 0$. At $n = -\frac{1}{3}$ the supervolume $C^{4,2}$ is conserved, and this case corresponds to the ordinary minimal 1-supergravity. In the case $n = 0$, the supervolume $C^{4,4}$ is conserved, and this case corresponds to a new version, U(1)-supergravity. By virtue of the conservation of the supervolume in the case $n = 0$, the left volume, $d^8 z_L$, and the right volume, $d^8 z_R$, are invariant. Another invariant is the superdeterminant of the transformation from the left volume to the right one, treated as a function of the coordinates $R^{4,4}$; it may be set equal to unity:

$$U(x, \theta, \bar\theta) = \mathrm{Ber}\left(\frac{\partial z_L}{\partial z_R}\right) = \frac{\det(\delta_n^m + i\partial_n \mathcal{H}^m)}{\det(\delta_{\dot\nu}^{\dot\mu} + \bar\Delta^{\dot\mu}\bar{\mathcal{H}}_{\dot\nu})} \frac{\det(\delta_\nu^\mu + \Delta_\nu \mathcal{H}^\mu)}{\det(\delta_n^m - i\partial_n \mathcal{H}^m)} = 1, \qquad (7)$$

where $\partial_n = \partial/\partial x^n$, and Δ^μ and $\bar{\Delta}^{\dot\mu}$ are the differential operators from Ref. 7. The superfield $U(x, \theta, \bar\theta)$ is unitary: $U = \exp[iu(x, \theta, \bar\theta)]$, where u is real. The invariant constraint in (7) thus causes 8 + 8 fields of the total 20 + 20 to vanish, and we are left with the necessary 12 + 12 fields. Their composition and transformation properties are analyzed in the Wess-Zumino gauge. Constraint (7) can be solved in this gauge, and we are left with graviton and gravitino fields and auxiliary fields: a gauge vector and an antisymmetric tensor. In this gauge it is clear how the local U(1) invariance in the case $n = 0$ arises. At this value of n (and at only this value), transformations (2) and (6) cannot fix the parameter of local γ_5 transformations. When the terms in $\mathcal{H}^\mu$, which are linear in θ, are fixed in the Wess-Zumino gauge, this parameter enters with a factor of $2n/(3n+1)$ and vanishes in the case $n = 0$.

In contrast with other supergravities, the action in this new version cannot be specified as an invariant volume[2] $R^{4,4}$, since in this version it is simply zero, when constraint (7) is taken into account (as in the version of an $N = 2$ supergravity which was studied in Ref. 10):

$$V_{inv} = \int d^8 z\, E = \int d^8 z_L = 0$$

$$E = \left[\frac{\det(\delta_n^m + i\partial_n \mathcal{H}^m)}{\det(\partial_{\dot\nu}^{\dot\mu} + \bar{\Delta}^{\dot\mu}\bar{\mathcal{H}}_{\dot\nu})} \quad \frac{\det(\delta_n^m - i\partial_n \mathcal{H}^m)}{\det(\delta_\nu^\mu + \Delta_\nu \mathcal{H}^\mu)}\right]^{1/2}. \tag{8}$$

In this version the invariant action integral should be defined as follows (cf. Ref. 2):

$$S = \frac{1}{\kappa^2} \int d^8 z E \ln F, \tag{9}$$

where

$$F = \det{}^{-1/4}\left(\frac{1}{4}[\Delta, \sigma_a \bar{\Delta}]\mathcal{H}^m\right) \det{}^{-1/8}(\delta_n^m + \partial_n \mathcal{H}^k \partial_k \mathcal{H}^m)$$
$$\times [\det(\delta_\nu^\mu + \Delta_\nu \mathcal{H}^\mu) \det(\delta_{\dot\nu}^{\dot\mu} + \Delta^{\dot\mu}\bar{\mathcal{H}}_{\dot\nu})]^{3/8} \tag{10}$$

is a factor which appears in the definition of the U(1) connectedness (it is equal to $\nabla_\alpha \ln F$) and which has the transformation law

$$\delta \ln F = \frac{\partial}{\partial x_L^m} v^m - \frac{\partial}{\partial \theta_L^\mu} v^\mu + \text{H.c.}\ . \tag{11}$$

The action is invariant because $\ln F$ acquires chiral increments [see (10]; it is then simple to see that $\delta S = 0$ by transformations to the left and right parametrizations. After substituting the solutions of constraint (7) into the Wess-Zumino gauge, we find an expression for the action in terms of fields which is the same as that found in Ref. 1. We note that constraint (7) may be relaxed slightly by replacing it by

$$\frac{\partial^2}{\partial \varphi_R^2} U = 0. \tag{12}$$

Now we have a description of 16 + 16 fields. The additional 4 + 4 fields form a multiplet with a superspin of $\frac{1}{2}$ in the interaction and a superspin of 0 on the mass shell—a super-analog of the notoff[11] (spin 1 in the interaction and spin 0 on the mass shell).

The questions touched on in this letter will be analyzed in more detail in a separate paper.

We have a few concluding remarks. At the moment, constraint (7) can be solved completely in the Wess-Zumino gauge in terms of the components, while in terms of the superfields this is possible only in the linearized approximation. It would be extremely important to find a complete solution of this constraint in superfields, particularly for use in extending the results which have been derived to expanded supergravities, as appears possible. What should we expect in these cases? The appearance of a local U(1) symmetry is clearly possible, Will SU(N) local symmetries appear? In the field approach, anomalies arise upon quantization. Will it be possible to construct an explicitly convariant superfield approach without anomalies?

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[2)] Its linear version was encountered in Ref. 5, published about five years ago.

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