A theoretical interpretation of the β-stability line of nuclear matter

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The β-stability line is deduced from mean-field theory and Hartree–Fock approximation. The spherical surface distribution of protons in stable nuclei or stable nuclear matter is assumed to fit the empirical nonsymmetric coefficient. The theoretical relative error is smaller than 3%. Some conclusions are drawn from the discussion.

The empirical formula relating the proton number to the total number of nucleons in stable nuclei or nuclear matter, which is expressed as

$$Z=0.5A-0.3\times 10^{-2}A^{5/3}, \tag{1}$$

has been used for a long time. Until now, however, no theoretical model has been developed to interpret it successfully. In order to achieve this goal, a new idea for the proton distribution in stable nuclei or nuclear matter must be proposed and the mean-field theory and Hartree–Fock approximation must be used.

According to the mean-field theory of nuclear matter and the first-order Hartree–Fock approximation, the single-particle spectrum is[1,2]

$$\begin{aligned}E_i(k)=&\frac{\hbar^2k^2}{2m_i}-\mu_{i0}+\sum_{k'\sigma'\tau'}\langle k\sigma\tau,k'\sigma'\tau'|V_{NN}|k\sigma\tau,k'\sigma'\tau'\rangle n_{k'\sigma'\tau'}\\&-\sum_{k'\sigma'\tau'}\langle k\sigma\tau,k'\sigma'\tau'|V_{NN}|k'\sigma'\tau',k\sigma\tau\rangle n_{k'\sigma'\tau'}\\=&\frac{\hbar^2k^2}{2m_i^*}+\epsilon_{i0}-\mu_{i0}\qquad(i=p,n),\end{aligned} \tag{2}$$

where V_{NN} is the effective nucleon–nucleon interaction.

Using the Skyrme-type interactions for V_{NN} in Eq. (2), we obtain[3]

$$\begin{aligned}\epsilon_{p0}=&\left[t_0(2+x_0)+\frac{t_3}{6}(2+x_3)\rho^d\right]\frac{\rho}{2}-\left[i_0(1+2x_0)+\frac{t_3}{6}(2+x_3)\rho^d\right]\frac{\rho_p}{2}\\&-3k_BT\left[\frac{1}{m_p^*\lambda_p^3}\frac{\partial m_p^*}{\partial\rho_p}f_{5/2}(\tilde{Z}_p)-\frac{1}{m_n^*\lambda_n^3}\frac{\partial m_n^*}{\partial\rho_p}f_{5/2}(\tilde{Z}_n)\right],\\\epsilon_{n0}=&\epsilon_{p0}(p\to n;\ n\to p),\end{aligned} \tag{3}$$

where

$$\bar{Z}_i = \exp[\beta(\mu_{i0} - \epsilon_{i0})] \qquad (i = p,n). \tag{4}$$

In stable nuclear matter, the β equilibrium must be achieved via the weak force,[1] so the chemical potential relation between protons and neutrons is

$$\mu_C + \mu_{p0} = \mu_{n0}, \tag{5}$$

where μ_C is the Coulomb chemical potential of protons.

Many authors have considered protons distributed uniformly in nuclei or nuclear matter in discussing the phase transitions.[3,5–7] Also, the shielding Coulomb interaction has been used to discuss the equation of state of nuclear matter.[8] It is important to note, however, that when the protons are distributed near the surface, the system has the lowest energy; hence it is in the most stable state. We can assume, therefore, that stable nuclei have a spherical surface distribution of positive charges. We then easily obtain

$$\mu_C = Ze^2 / \left(\frac{3A}{4\pi\rho_0} \right)^{1/3}, \tag{6}$$

where $\rho_0 \approx 0.17\ \mathrm{fm}^{-3}$ is the nucleon saturation density in the liquid-drop model.

In another way, it is easy to prove that the rearrangement chemical potentials[9] for protons and neutrons are equal and this result has been used by some authors.[3,7,10,11] The result of Eq. (5) therefore does not change after the rearrangement chemical potential effect is considered.

Using Eqs. (3)–(6), we calculate the theoretical β-stability line in zero temperature:

$$Z = \frac{A}{2} \frac{1}{1 + \xi A^{2/3}} \approx 0.5A - \frac{\xi}{2} A^{5/3}, \tag{7}$$

where

$$\xi = -e^2 \left(\frac{3}{4\pi\rho_0} \right)^{-1/3} \rho_0^{-1} \left[t_0(1 + 2x_0) + \frac{t_3}{6}(1 + 2x_3)\rho_0^{d} \right]^{-1}. \tag{8}$$

The calculated nonsymmetric coefficient from the Skyrme parameters in Ref. 3 is

$$\frac{\xi}{2} = \begin{cases} 0.206 \times 10^{-2} & \mathrm{Sk1} \\ 0.395 \times 10^{-2} & \mathrm{Sk3} \\ 0.318 \times 10^{-2} & \mathrm{Sk}M^*. \end{cases} \tag{9}$$

Comparing these results with the standard formula which has been supported by experiments, we see that the theoretical β-stability line has the same form as the empirical formula, but all the coefficients calculated from Skyrme-type interactions, except Skyrme M^*, apparently drift off the exact value. This shows that the proposed theoretical model is only qualitatively correct. The big error probably comes from the difference in the **n–n**, **n–p**, and **p–p** strong interactions, which are considered in the Skyrme-type interactions. It is expected to give a better theoretical result by using the Gogny interaction, because it involves the nonsymmetric effects of the nucleon–nucleon interactions, and because it was found to be successful in describing many nuclear properties.[12–14]

The Gogny interaction without spin–orbit coupling is[7,8]

$$V_{NN}=\sum_{i=1}^{2}(W_i+B_iP_\sigma-H_iP_\tau-M_iP_\sigma P_\tau)e^{-(\mathbf{r}_1-\mathbf{r}_2)^2/\mu_i^2}+t_0(1+x_0P_\sigma)\rho^d\delta(\mathbf{r}_1-\mathbf{r}_2). \tag{10}$$

We can therefore easily find the single-particle spectrum, from which ϵ_{q0} can be calculated:

$$\epsilon_{q0}=t_0\rho^d\left[\rho-\frac{1}{2}\rho_q+\left(\frac{1}{2}\rho-\rho_q\right)x_0\right]+\sum_{i=1}^{2}(\sqrt{\pi}\mu_i)^3\left[\left(W_i+\frac{1}{2}B_i\right)\rho\right.$$

$$\left.-\left(H_i+\frac{1}{2}M_i\right)\rho_q\right]+\sum_{i=1}^{2}(\sqrt{\pi}\mu_i)^3\int[(H_i+2M_i)(n_{pk'}+n_{nk'})$$

$$-(W_i+2B_i)n_{qk'}]e^{-(1/4)\mu_i^2k_i'^2}\frac{d_{k'}^3}{(2\pi)^3},\quad(q=p,n). \tag{11}$$

Using the same method which was used above, we can easily calculate the constant ξ in Eq. (7):

$$\xi=-e^2\left(\frac{3}{4\pi\rho_0}\right)^{-1/3}\rho_0^{-1}\left[t_0\rho_0^d(1+2x_0)+\sum_{i=1}^{2}(\sqrt{\pi}\mu_i)^3(W_i+2B_i+2H_i+M_i)\right]^{-1}. \tag{12}$$

Using the parameters in Refs. 7 and 8, we obtain

$$\xi/2=0.308\times10^{-2}. \tag{13}$$

This calculated nonsymmetric coefficient coincides well with the empirical value. The relative error is only 2.6%. The small error comes partly from the high-order terms of the expansion series in Eq. (7) and partly from the Hartree–Fock approximation and the Gogny effective interaction. The success of the mean-field theory, especially the Gogny interaction in describing the strong nucleon–nucleon interaction, is thus confirmed further.

Finally, we can draw two conclusions from the deduction and from the calculated results above.

First, the difference in the number of neutrons and protons in stable nuclei or nuclear matter is attributable to the weak equilibrium and determined mainly by the Coulomb interactions of protons. The nonsymmetric character of strong nucleon–nucleon interactions has a subsidiary effect on it.

Second, the protons in stable nuclei tend to be near the surface, while neutrons tend to be at the center, because not only the total energy of the system is lowest in that state, but also the chemical potential caused by the Coulomb repulsion force has an important contribution to the weak equilibrium.

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