Experimental observation of an interaction of vortex filaments

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A steady-state double-spiral structure of interacting vortex threads of the same sign has been observed experimentally. It is described.

One of the most urgent and promising directions in research on the problem of describing turbulence is to study coherent structures (or organized motions).[1] It has been known for a long time that the turbulence in shear flows is not a purely stochastic process and instead includes numerous, well-defined structures: "spots," "horseshoes," "spindles," etc.[1–5] The formation and evolution of such structures depend strongly on the interaction and decay of vortices of various scales.

Particularly surprisingly, a turbulent flow may contain extended longitudinal vortices: genuine vortex filaments, with a longitudinal dimension tens or even hundreds of times their diameter. These threads apparently play a special role in turbulent motion. In the hydrodynamics of superfluid helium, it is generally accepted that turbulence is an interaction of quantized vortex threads.[6] It would be extremely important to clarify the role played by vortex filaments in viscous liquids. However, the time variation, the 3D nature, and the small scales of the coherent structures have held back research in this field to the stage of a buildup of empirical information, despite an extremely rapid increase in the number of studies. In this letter we are reporting the acquisition of some new experimental data which may serve as the starting point for a detailed study of the interaction of vortex filaments in viscous liquids.

In planning the experiment, we were guided by the following factors. A narrowing of the exit aperture in a vortex or cyclone chamber leads to a concentration of the vorticity near the chamber axis and to the formation of a vortex whose radius is much smaller than the dimensions of the chamber.[7,8] In other words, a large-scale vortex filament is generated. Any asymmetry, e.g., a displacement of the exit aperture with respect to the geometric axis of the chamber, causes the filament to bend and, by virtue of a self-induced motion, to assume the shape of a vortex line fixed in space.[8] Kinematic analysis of twisted flow with a spiral axis suggests the existence of a flow with two intertwined axes and thus makes it possible to model the interaction of two vortex filaments at a macroscopic level. Numerous attempts to develop such a regime have been successful in the version of a vortex chamber with a central exit aperture and a two-inclined-plane end. The results of the experimental study are reported below.

The experiments were carried out in a hydraulic vortex chamber of square cross section, 188×188 mm, with a height up to 625 mm (Fig. 1a). In a study of vortex filaments, the shape of the cross section does not play a key role, as was shown in experiments with cylindrical inserts. The liquid was admitted, and the twisted flow was

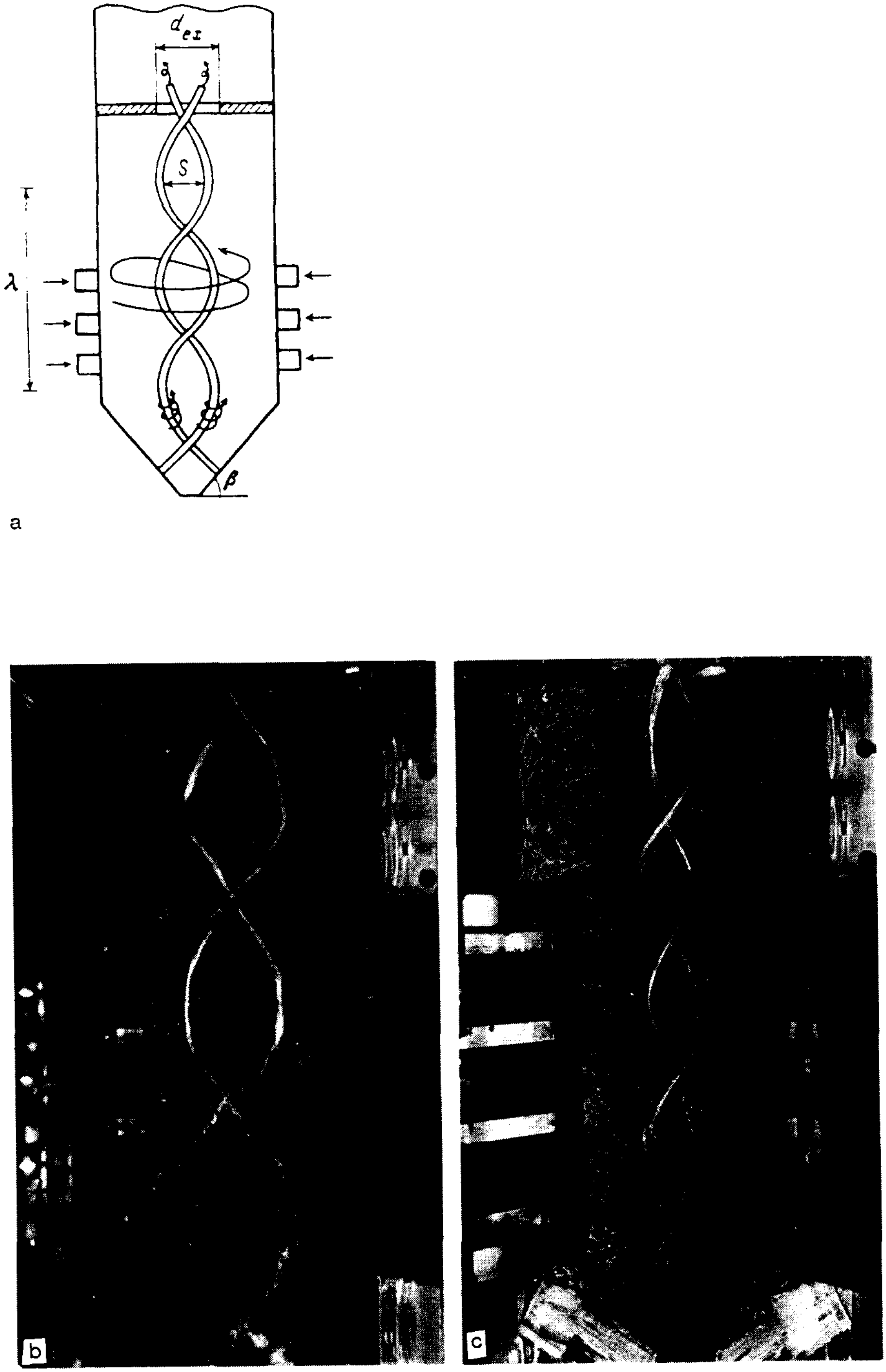

FIG. 1. Double-spiral interaction of vortex filaments. a—Experimental layout; b,c—visualization of the flow.

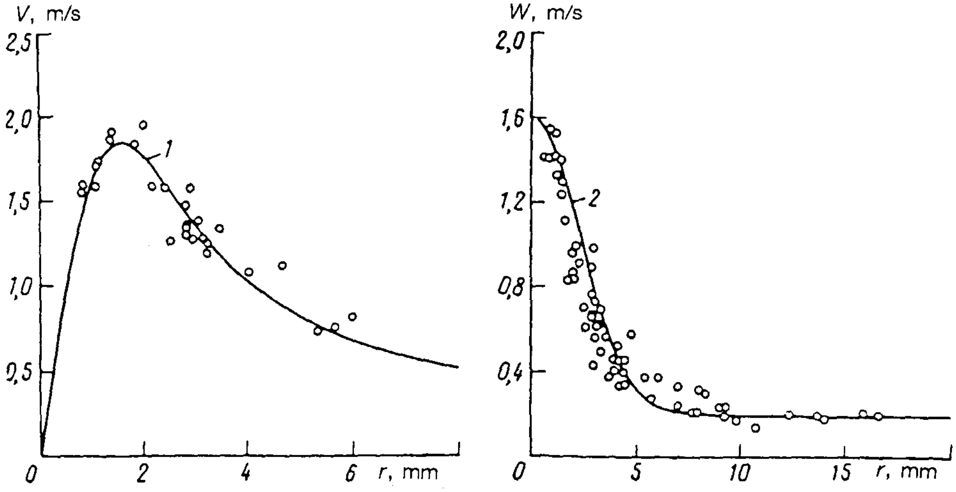

FIG. 2. Profiles of the tangential (V) and axial (W) components of the velocity near a rectilinear vortex filament. *1,2*— Approximate curves constructed from Eqs. (1) and (2), respectively.

organized, through twelve rotatable rectangular nozzles. The axes of these nozzles were directed along the tangent to a certain circle. By varying the nozzle directions in the transverse plane, we were able to control the extent to which the flow was twisted. The chamber was designed in such a way that the following parameters could be varied: the shape of the bottom (the lower end), the height H, and the diameter d_{ex} of the exit aperture. All these geometric parameters had a strong effect on the structure of the twisted flow. The relative Reynolds number of the flow, on the other hand, was self-similar. Water was used as working liquid. The rate of outflow from the nozzles reached 2 m/s. The Reynolds number Re found from the parameters of the nozzle is 4×10^4.

The experiments were carried out in the following way. A flat bottom was first installed, and a regime with a stable vortex filament was established by adjusting the nozzle directions for a given diameter of the exit aperture ($d_{\text{ex}}=40-70$ mm). Small air bubbles were injected into the chamber to visualize the flow. Since there is a pronounced pressure decrease at the axis of the vortex filament, bubbles collect into a continuous thin air filament, which effectively visualizes the physical axis of the vortex. To determine the parameters of the vortex and to prove that a vortex filament exists, we measured the velocity profile by a contactless optical method: a method of stroboscopic visualization of particles. These particles (or tracers) were air bubbles. Figure 2 shows typical profiles of the tangential and axial components of the velocity. The experimental data are generalized by the empirical expressions proposed in Ref. 9:

$$V(r)=A[1-\exp(-\alpha_1 r^2)]/r, \tag{1}$$

$$W(r)=W_1+W_2\exp(-\alpha_2 r^2), \tag{2}$$

FIG. 3. Visualization of the flow in a vertical plane passing through the center of the channel.

where A, α_1, α_2, W_1, and W_2 are empirical constants. It follows from this figure that the radius of the vortex determined from the maximum tangential velocity is 1.7 mm. This radius is much less than the width of the chamber and the length of the vortex, which is the same as the height of the chamber, $H=430$ mm.

When the flat bottom was replaced by a two-inclined-plane bottom (Fig. 1a), we observed a quasilinear two-vortex structure. A photograph is shown in Fig. 1b; a corresponding kinematic diagram is shown in Fig. 1a. On the photograph, the bright lines are thin air filaments which visualize the axes of two vortex filaments. That these are indeed axes follows from the visualization of the flow in the vertical plane by means of a light "knife" (Fig. 3). It is clear that the trajectories of the individual bubbles twist around the air filaments. Because of slight deviations from axial symmetry, only one of the spirals is predominant; the second is not seen.

The kinematics of the flow can be described as follows. On the whole, the flow in the chamber is twisted to the right. The tangential velocity component is predominant at the walls (Fig. 1a). In each vortex filament, the twisting of the flow is again rightward, but with a significant axial component of the velocity (Fig. 2). There is thus an interaction of two vortex filaments of a common sign. The result of the interaction is a double spiral, whose spiral axes are also twisted to the right. Exclusively because of this topology, there can be a kinematic coupling of two filaments of a common sign. Specifically, the velocity vectors of the two vortex tubes couple at the geometric axis of the chamber and are directed vertically upward.

The inclination (β) of the sloping bottoms of the chamber strongly influences the parameters of the double spiral (the distance between filaments, S; the wavelength λ; and the number of half-waves, j). At $\beta=50°$, for example, we have $j=3$, $\lambda=250$ mm, and $S=60$ mm (Fig. 1b), while at $\beta=30°$ we have $j=6$, $\lambda=115$ mm, and $S=25$ mm (Fig. 1c).

The mechanism for the interaction of the vortex filaments is evidently based on the Biot–Savart law, which also operates because of a self-induced motion of the vortices. However, viscous effects must also be taken into account in a theoretical analysis.

The literature reveals many theoretical studies[5,6] of the motion of two vortices (vortex filaments) differing in sign. On the other hand, there has been only a single theoretical study[10] of the interaction of identically twisted vortex filaments, in the case of an ideal liquid. A steady-state, slightly nonlinear solution was constructed on the basis of the Biot–Savart law and the "localized induction equation" which follows from it. The solution consists of a double spiral of vortex filaments which moves in a translational manner along the longitudinal axis. The conditions for the applicability of the solution are $r \ll S \ll L$, where r is a characteristic radius of the vortex, S is the distance between filaments, and L is a truncation length, which is on the order of the wavelength λ. For the experiment we are discussing here, these conditions hold approximately. Accordingly, the picture observed can be assumed to correspond completely to the model of Ref. 10, at least at a qualitative level. The circumstance that the double spiral remained immobile in this experiment, on the other hand, can be attributed to a cancellation of the phase velocity by the average motion of the flow and by a tying of the vortex axes to the sloping bottoms.

Despite the difficulty in observing two-spiral structures, there are some fragmentary reports in the literature of similar effects, but all have been time-varying. Chandrsuda *et al.*[11] detected a spiral twisting of two long filaments in a boundary layer. Boubnov and Golitsyn[12] observed a time-varying spiral pairing of two vortex filaments during natural convection in a rotating volume. The final stage was the coalescence of the two filaments into a single stronger filament. The system of spindle-shaped vortices in the wake behind a body in a boundary layer is clearly of a two-spiral nature.[4] A double spiral also forms during the decay of a vortex.[13]

In summary, these experiments have yielded the first observation and description of a steady-state, two-spiral structure of interacting vortex filaments of a common sign. A study of this structure makes it possible to construct a corresponding theory for the interaction of 3D vortices in real liquids and to make a contribution to research on coherent structures in turbulence.

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