Possibility of a power temperature dependence of the low-temperature penetration depth in an *S*-wave superconductor

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A number of experimental measurements of λ in HTSC have revealed a power temperature dependence of $[\lambda(T)-\lambda(0)]$ at $T \ll T_c$, which is often considered as an unambiguous evidence of the existence of gap nodes, and which is attributed to unconventional pairing. However, electron coupling to the low-lying excitations with an energy $\lesssim T$ *always* gives rise to a power dependence of $\lambda(T)$ even for S-wave superconductors without gap zeros. Negligible in conventional superconductors, this effect may be large enough to be observed experimentally in high-T_c compounds. Thus, power dependence of $[\lambda(T)-\lambda(0)]$ does not exclude S coupling. In particular, linear and quadratic temperature dependences of λ may occur. A calculational illustration is presented, in addition to the analytical results.

The temperature dependence of λ is usually associated with that of the superfluid density n_S: $\lambda^{-2} \sim n_S/m^*$, where m^* is an effective mass of the carriers. On the other hand, in the case of electron coupling with low-energy excitations (say, with phonons) m^* also becomes temperature-dependent. This may result in a power dependence of $\lambda(T)$ even in an S-wave superconductor (see, e.g., Ref. 1), where n_S changes only exponentially at low temperatures. To make this article complete, we reiterate some results of Ref. 1. Pure electron–phonon interaction results in a rather weak, $\sim T^5$, dependence[2] which is difficult to observe experimentally even in strong-coupling superconductors:

$$\frac{\lambda(T)-\lambda(0)}{\lambda(0)} \sim \frac{T}{\Delta}\left(\frac{T}{\omega_D}\right)^4, \tag{1}$$

where ω_D is the Debye frequency.

One reason for the appearance of such a high power of T is that in this particular case thermal excitations—acoustic phonons—transfer small momenta $q \lesssim T/c_s$, and so their effect on the transport properties, including the London penetration depth, almost cancels out. To obtain a more pronounced dependence, for example, quadratic or linear, we phenomenologically assume that electrons interact with low energy modes which are quasilocal, so that the momentum transfer in the scattering on such modes remains large ($\sim p_F$) as the energy transfer approaches zero. Given the effective spectral density of the modes $g(E)$ [in the case of electron–phonon interaction $g(E)$ is usually called the Eliashberg function $\alpha^2F(E)$], we obtain

$$\frac{\lambda(T)-\lambda(0)}{\lambda(0)} \sim \frac{\int_0 g(E)N(E)dE}{\Delta}, \quad N(E)=\frac{1}{e^{E/T}-1}. \tag{2}$$

For a simple form of g we therefore have

$$\frac{\lambda(T)-\lambda(0)}{\lambda(0)} \sim \frac{g(E\sim T)T}{\Delta}. \tag{3}$$

Such excitations might be related, for example, to disorder.[3] Depending on low-energy behavior of $g(E)$, different types of $\Delta\lambda(T)=\lambda(T)-\lambda(0)$ functions can be obtained.

Origin of the power term in λ(T)

Within the scope of the strong-coupling theory of superconductivity the normal and anomalous parts of the electron self-energy are obtained from the well-known Eliashberg equations.[4] For our purpose we can limit the analysis to the case of clean, isotropic, one-band, S-wave superconductor with the same coupling to the normal and anomalous electron self-energies:

$$[1-Z(i\epsilon_n)]i\epsilon_n=-i\pi T\sum_{\epsilon_m} A(\epsilon_n-\epsilon_m)\frac{\epsilon_m}{\sqrt{\epsilon_m^2+\Delta^2(i\epsilon_m)}}, \tag{4}$$

$$Z(i\epsilon_n)\Delta(i\epsilon_n)=\pi T\sum_{\epsilon_m} A(\epsilon_n-\epsilon_m)\frac{\Delta(i\epsilon_m)}{\sqrt{\epsilon_m^2+\Delta^2(i\epsilon_m)}}, \tag{5}$$

where the contribution of the Coulomb interaction is omitted; $\epsilon_n=(2n+1)\pi T$, and

$$A(\omega)=2\int_0^{+\infty} dE\,\frac{Eg(E)}{E^2+\omega^2}.$$

If the penetration depth $\lambda(T)$ is calculated *without including corrections to the electromagnetic vertex*, then

$$\lambda^{-2}(T)=\frac{8\pi}{3}\frac{e^2}{c^2}\nu_F v_F'^2\pi T\sum_{\epsilon_m}\frac{\Delta^2(i\epsilon_m)}{Z(i\epsilon_m)[\epsilon_m^2+\Delta^2(i\epsilon_m)]^{3/2}}, \tag{6}$$

where ν_F is the spectral density of electrons with a given spin projection in the normal state. We must introduce the somewhat artificial parameter v_F'—the Fermi velocity of the noninteracting electrons; see the discussion of the vertex corrections in the following section.

It can be shown that at $T \ll T_c$ Eqs. (4)–(6) can be rewritten with exponential accuracy $(e^{-\Delta/T})$ in the following way:

$$[1-Z(i\epsilon_n)]\epsilon_n=-\frac{1}{2}\int_{-\infty}^{+\infty} d\epsilon' A(\epsilon_n-\epsilon')\frac{\epsilon'}{\sqrt{\epsilon'^2+\Delta^2(i\epsilon')}}$$
$$-2\pi\int_0^{+\infty} dE g(E)N(E)\,\mathrm{Re}\left\{\frac{\epsilon_n+iE}{\sqrt{(\epsilon_n+iE)^2+\Delta^2(i\epsilon_n-E)}}\right\}, \tag{7}$$

$$\Delta(i\epsilon_n)Z(i\epsilon_n)=\frac{1}{2}\int_{-\infty}^{+\infty} d\epsilon' A(\epsilon_n-\epsilon')\frac{\Delta(i\epsilon')}{\sqrt{\epsilon'^2+\Delta^2(i\epsilon')}}$$

$$+2\pi\int_0^{+\infty} dE g(E)N(E)\mathrm{Re}\left\{\frac{\Delta(i\epsilon_n-E)}{\sqrt{(\epsilon_n+iE)^2+\Delta^2(i\epsilon_n-E)}}\right\}, \tag{8}$$

$$\lambda^{-2}(T)=\frac{8}{3}\frac{\nu_F v_F'^2}{c^2}\int_0^{+\infty}\frac{\Delta^2(i\epsilon)}{Z(i\epsilon)[\epsilon^2+\Delta^2(i\epsilon)]^{3/2}}d\epsilon, \tag{9}$$

where $N(E)=[e^{E/T}-1]^{-1}$, and $\epsilon_n>0$, $\Delta=\Delta^R$, and $Z=Z^R$ are obtained by analytical continuation of $Z(i\epsilon_n)$, $\Delta(i\epsilon_n)$. The idea of derivation of Eqs. (7)–(9) was suggested in Ref. 1. It uses the exponential smallness within the gap of the differences between retarded and advanced $Z(\epsilon)$ and $\Delta(\epsilon)$:[5] $(\Delta^R-\Delta^A)$, $(Z^R-Z^A)\sim e^{(|\epsilon|-\Delta)/T}$ at $|\epsilon|<\Delta$.

From (7) and (8) we obtain

$$Z(i\varepsilon,T)=Z(i\varepsilon,0)+2\pi\frac{\int_0^{+\infty}dEg(E)N(E)}{\sqrt{\varepsilon^2+\Delta^2(i\varepsilon,0)}}+O[T^3g(T)]\sim g(T)\frac{T}{\Delta},$$
$$[\Delta(i\epsilon,T)-\Delta(i\epsilon,0)]\sim\frac{\int_0^{+\infty}dEg(E)N(E)E^2}{\Delta^2}\sim\frac{O[g(T)T^3]}{\Delta^2}. \tag{10}$$

Additional smallness ($\sim T^2/\Delta^2$) of change in Δ is a consequence of the Anderson theorem, since the power terms arise from the interaction with low-lying excitations.[6] In general, such a cancellation in the temperature dependence of Δ is model-dependent. It is due to identical coupling of normal and anomalous electron self-energies to the low-energy modes, which is not the case if, for example, $g(E\lesssim T)$ contains a contribution from the spin fluctuations or if Δ is anisotropic[6] and $g(E\lesssim T)$ corresponds to large momenta, $\sim p_F$.

In any case, if corrections to the electromagnetic vertex[1] can be ignored in the calculations of $\lambda(T)$, then from (9) and (10) we obtain estimate (3). Such a correction should be considered if the momentum of the thermal bosons, q_T, is small in comparison with p_F. We then have additional cancellations:

$$\frac{\lambda(T)-\lambda(0)}{\lambda(0)}\sim\frac{g(E\sim T)T}{\Delta}\left(\frac{q_T}{p_F}\right)^2 \tag{11}$$

(in much the same fashion as they occur in the normal state transport properties). For phonons $[g(E)\sim E^2/\omega_D^2\quad q_T/p_F\sim T/\omega_D]$ it is the vertex corrections that change the dependence of $\lambda(T)$ from $\sim T^3$ to $\sim T^5$.

Another peculiarity of long-wave excitations is that in calculations of *their* effect on the electron self-energy parts the Migdal's theorem no longer apply for modes with a momentum $q\lesssim\Delta/v_F$. Since long-wave soft modes slightly affect the penetration depth, we restrict the discussion below to the case of quasilocal thermal modes. Then for a qualitative estimate (2) holds true. Let us formulate some direct consequences of (2): First, if $g(E)\sim a(E/E_c)^n$, then

$$\frac{\lambda(T)-\lambda(0)}{\lambda(0)}\sim\frac{aT^{n+1}}{E_c^n\Delta}.$$

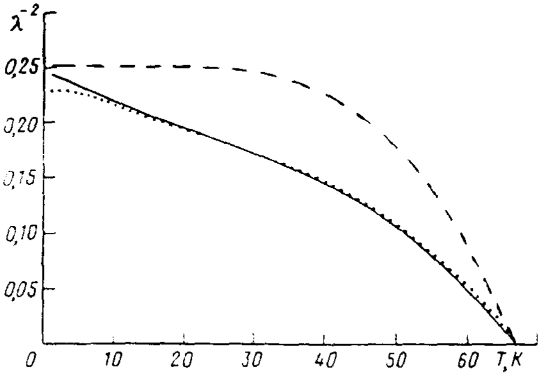

FIG. 1. How $\lambda(T)$ in an S-wave superconductor is affected by the electron interaction with low-lying bosonic excitations (solid and dotted curves). The dashed curve is $\lambda(T)$ computed in the absence of soft excitations. The gap in the electronic spectrum remains large: $\Delta_0 \sim 200$ K.

Secondly, if there is a gap E_0 in the spectrum of the coupling excitation, which is smaller than Δ_0, then the dependence of $[\lambda(T)-\lambda(0)]$ is exponential $e^{-E_0/T}$ only at $T \lesssim E_0$. In particular, given a low-energy peak in g: $g=aE_0\delta(E-E_0)$, the dependence of λ changes from exponential to linear aT/Δ at $T\sim E_0$:

$$\frac{\lambda(T)-\lambda(0)}{\lambda(0)} \sim \frac{aE_0}{\Delta} \frac{1}{e^{E_0/T}-1} .$$

Figure 1 shows that the low-energy part of $g(E)$ has a strong effect on $\lambda(T)$. The quantity λ^{-2} is given in dimensionless units:

$$\lambda^{-2}(T)=\pi T\sum_{\epsilon_m} \frac{\Delta^2(i\epsilon_m)}{Z(i\epsilon_m)[\epsilon_m^2+\Delta^2(i\epsilon_m)]^{3/2}} .$$

The dashed curve $\lambda^{-2}(T)$ is computed for the spectral density without a low-energy part: $g(E)=2E_0\delta(E-E_0)$, where $E_0=200$ K. The other two curves show $\lambda^{-2}(T)$ when a low-energy spectral density is added: $g(E)=2E_0\delta(E-E_0)+2E_1\delta(E-E_1)$; $E_1=15, 5$ K for dotted and solid curves, respectively. As a result, the linear dependence of $\lambda^{-2}(T)$ begins at $T\sim E_1$. Our results can easily be generalized to another physically possible situation in which there is coupling with low-energy nonbosonic excitations. Some effects of disorder, for example, can be described in terms of the electron interaction with two-level systems.[3] The associated *effective* spectral density $g(E)$ is temperature dependent at thermal energies; in the simplest case[8]

$$g_T(E)=g_0(E)\tanh\left(\frac{E}{2T}\right). \tag{12}$$

Then

$$\frac{\lambda(T)-\lambda(0)}{\lambda(0)} \sim \frac{1}{\Delta}\int_0 \left[g_T(E)\coth\left(\frac{E}{2T}\right)-g_0(E)\right]dE+O\left(g(E\sim T)\,\frac{T^2}{\Delta^2}\right). \tag{13}$$

Thus the particular case (12) of the coupling with two-level centers is the exception as the major term in (13) vanishes. Specifically, given a constant low-energy density of centers $g_0(E)$, we obtain a quadratic temperature dependence of the London penetration depth λ

for the interaction with two-level centers given by (12), and a linear dependence in a more general model [see also, Ref. 7]. In any event, if the power term in $\lambda(T)$ is attributable principally to the effects of disorder, it should strongly depend on the sample, which may be one of the reasons for the discrepancy in the experimental data (see, e.g., Refs. 11–15).

The effect of vertex corrections on the penetration depth in the *whole* temperature range (0, T_c)

Near T_c the correction in the substitution of the real Fermi velocity of the interacting system, v_F, for v_F' in (6) is

$$\lambda^{-2}(T)=\frac{8\pi}{3}\frac{e^2}{c^2}\nu_F v_F^2 \pi T\sum_{\epsilon_m}\frac{\Delta^2(i\epsilon_m)}{Z(i\epsilon_m)[\epsilon_m^2+\Delta^2(i\epsilon_m)]^{3/2}},$$

$$v_F=-\frac{dG^{-1}}{dp}(\epsilon=0,\ p=p_F)=\frac{v_F'}{1-\gamma_1}.$$

Here γ_1 is the (dimensionless) first spherical harmonic of the vertex function[9] Γ^k. At lower temperatures the renormalization of λ changes.[10] In particular, there appears a "ladder contribution"[1] of the electron–boson interaction to the electromagnetic vertex, which cannot be found analytically in general. Our discussion is therefore restricted to two illustrative examples: If the gap function is small as compared with the characteristic boson frequency ω_0, then the BCS-like approximation is valid: $\Delta(i\epsilon)\approx\text{const}$, $Z(i\epsilon)\approx 1+\lambda_{\text{ph}}$, $\epsilon\ll\omega_0$. The zero-temperature penetration depth is[10]

$$\lambda^{-2}(0)=\frac{2}{3}\frac{\nu_F v_F^2}{c^2}\frac{1-\gamma_1}{1+(\lambda_{\text{ph}}-\lambda_{\text{ph}}^1)(1-\gamma_1)},$$

where λ_{ph} and λ_{ph}^1 are the (dimensionless) zeroth and first harmonics of the electron–boson interaction.

As the temperature varies from T_c to zero, the vertex contribution to $\lambda^{-2}(T)$ thus changes by the factor

$$\frac{(1-\gamma_1)(1+\lambda_{\text{ph}})}{1+(\lambda_{\text{ph}}-\lambda_{\text{ph}}^1)(1-\gamma_1)},$$

which amounts to $(1-\gamma_1)$ in the weak-coupling limit, $\lambda_{\text{ph}}\ll 1$. If the first harmonic of the electron–boson interaction can be ignored (i.e., no 'bosonic' ladder corrections to the electromagnetic vertex are considered), we obtain

$$\lambda^{-2}(T)=\frac{8\pi}{3}\frac{e^2}{c^2}\nu_F v_F'^2\frac{1-\gamma_1}{1-\gamma_1[1-\Pi(T)]},$$

$$\Pi(T)=\pi T\sum_{\epsilon_m}\frac{\Delta^2(i\epsilon_m)}{Z(i\epsilon_m)[\epsilon_m^2+\Delta^2(i\epsilon_m)]^{3/2}}.$$

The renormalization of λ due to the vertex corrections therefore changes by the amount

$$\frac{1-\gamma_1}{1-\gamma_1[1-\Pi(T)]}$$

as the temperature drops from T_c. These examples show again that in the quantitative calculations of $\lambda(T)$ the corrections to the electromagnetic vertex should be included, as they multiply the penetration depth by the *temperature-dependent* factor, causing them to change:

$$\left(\frac{\lambda(0)}{\lambda(T)}\right)^2 \text{ vs } \left(\frac{T}{T_c}\right).$$

It is shown that nodes in the gap are not the necessary prerequisite for a power low-temperature dependence of the penetration depth. Given the considerable coupling of the electrons to the soft ($E \ll T_c$) short-wave modes in an S-superconductor, the power term in $\lambda(T)$ is strong enough to be detected experimentally.

It is unclear whether such an S-wave scenario is realized in HTSC, but the question may in principle be resolved experimentally.

We have also demonstrated the importance of the vertex corrections in the calculations of the penetration depth.

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[1] G. M. Eliashberg, G. V. Klimovitch, and A. V. Rylyakov, J. Supercond. **4**, 393 (1991).

[2] At very low temperatures ($T \sim T_c \omega_D / E_F$) the dependence smoothly changes to $\sim T^6$.

[3] P. W. Anderson, B. I. Halperin, and C. Varma, Phil. Mag. **25**, 1 (1972).

[4] G. M. Eliashberg, Zh. Eksp. Teor. Fiz. **38**, 966 (1960) [Sov. Phys. JETP **11**, 696 (1960)].

[5] G. M. Eliashberg, Zh. Eksp. Teor. Phys. **39**, 1437 (1960) [Sov. Phys. JETP **12**, 1000 (1961)].

[6] A. J. Millis, Subir Sachdev, and C. M. Varma, Phys. Rev. B **37**, 4975 (1988).

[7] It is even possible to contrive *a certain* coupling with three-level systems which would lead to *nonmonotonic* $\lambda(T)$. But such a coupling seems unreasonable from the physical point of view.

[8] G. M. Eliashberg, JETP Lett. **45**, 35 (1987).

[9] A. A. Abrikosov, L. P. Gorkov, and I. E. Dzyaloshinsky, *Methods of Quantum Field Theory in Statistical Physics*, ed. by R. A. Silverman (Dover Publications, New York, 1975), p. 175.

[10] G. M. Eliashberg, private communications.

[11] W. N. Hardy, D. A. Bonn, D. C. Morgan *et al.*, Phys. Rev. Lett. **70**, 3999 (1993) and the references cited there.

[12] J. E. Sonier, R. F. Kielf, J. H. Brewer *et al.*, Phys. Rev. Lett. **72**, 744 (1994).

[13] N. Klein, N. Tellmann, H. Schulz *et al.*, Phys. Rev. Lett. **71**, 3355 (1993), and the references cited there.

[14] Zhengxiang Ma, R. C. Taber, L. W. Lombardo *et al.*, Phys. Rev. Lett. **71**, 781 (1993).

[15] Dong Ho Wu, Jian Mao, S. N. Mao *et al.*, Phys. Rev. Lett. **70**, 85 (1993).

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