Magnetoelectric effects in a 2D strip with a Hall current

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Possible reasons for the onset of a coordinate nonlinearity in the distribution of electrical properties for a 2D strip with a Hall current at nearly integer values of the filling factor are discussed.

Measurements of the linear electrooptic effect in a 2D strip with a Hall current show that, on the Hall plateaus, the potential distribution along the cross section of the strip is a nonlinear function of the coordinate x, which runs normal to the axis of the strip.[1,2] The reason for the nonlinearity is believed to be a screening mechanism which is characteristic of 2D electron systems with an integer filling factor, as described by MacDonald *et al.*[3] Corresponding ideas were invoked in Refs. 4 and 5 to explain some size effects in the behavior of the critical Hall current.

In this letter we wish to call attention to an alternative possibility for the onset of nonlinearities in this problem. Specifically, we will discuss magnetoelectric effects, which are well known in the theory of the magnetocapacitance of 2D electron systems, although there is no theory as such. Each investigator (see, for example, Refs. 6–8) independently calculates the details of the magnetocapacitance corresponding to the given experiment, citing an unpublished paper by Stern.[9]

1. The equations of Ref. 3, which contain a coordinate nonlinearity for the electrochemical potential $\varphi(x)$, take the following form in the problem of current flow along a strip of width $2w$ under the conditions $\sigma_{yy}=0$:

$$\delta n(x)=\frac{e\nu}{h\omega_c}\varphi''(x), \quad \varphi(x)=\begin{cases}0, & x=-w,\\ V, & x=+w,\end{cases} \tag{1}$$

$$\omega_c=eH/m_*c,$$

$$\varphi'(x)=\frac{2e}{\kappa}\int_{-w}^{+w} ds\,\frac{\delta n(s)}{x-s}, \quad \delta n=n-n_d, \tag{2}$$

$$j_y(x)=\frac{e\nu}{h}\varphi'(x). \tag{3}$$

Here V is the Hall potential difference, $e\,\delta n$ is the local charge density, $j_y(x)$ is the local current density, κ is the dielectric constant, m_* is the effective mass of an electron, ν is the filling factor, which is assumed in the theory of Ref. 3 to be an integer, and h is Planck's constant. MacDonald *et al.*[3] believe that the boundary conditions in (1) can be imposed on the electric potential. As a result, the first two equations are closed, determining $\varphi(x)$ in terms of ν, V, and the geometry of the strip. The current distribution

$j_y(x)$ in (3) is consistent with the behavior $\varphi'(x)$. A similar situation is possible because of the degeneracy of the 1D problem, in which the condition div$j=0$ automatically holds. As a result, scheme (1)–(3) is self-consistent without the involvement of the electrochemical potential μ in the problem.

2. We now note that Eq. (1) of Ref. 3 is approximate. A more comprehensive form of this equation is

$$\pi l_H^2\left(\delta n-\frac{e\nu}{\hbar\omega_c}\varphi''\right)=\left\{\sum_l f(\epsilon_l+e\varphi-\mu)-\pi l_H^2 n_d\right\},$$

$$\epsilon_l=\hbar\omega_c(l+1/2),\quad l_H^2=c\hbar/eH,\quad f(x)=[e^{x/T}+1]^{-1}. \tag{4}$$

In (4), l_H is the magnetic length, $\mu(x)$ is the current value of the electrochemical potential, and n_d is the density of donors. In writing (4) we omitted the contribution of edge states of magnetic origin (this simplification was allowed in Ref. 3). According to (4), the electron density deviates from its equilibrium value $n=n_d$ for two reasons. First, there is a shift of the centers of the oscillator functions due to the field $\varphi'(x)$ [this effect was discussed in Ref. 3; it leads to the term $\sim\varphi''$ on the right side of (4)]. The second reason is the x dependence of the combination $e\varphi-\mu$ in the arguments of the Fermi functions on the right side of (4). Although omitted in Ref. 3, this effect is nonetheless responsible for magnetoelectric effects in the magnetocapacitance problem. Assuming below that the electric potential $\varphi(x)$ is a linear function of the coordinates for a current-carrying strip, i.e., intentionally ignoring the nonlinearity effect of Ref. 3, and restricting the discussion to conditions such that the filling factor $\nu(x)=\pi l_H^2 n(x)$ does not exceed 2, we can put (4) in a form convenient for the discussion below:

$$\mu(x)=\tfrac{1}{2}\hbar\omega_c+e\varphi(x)-T\ln S(x), \tag{5}$$

$$S(x)=\frac{1}{2}\left(\frac{1}{\nu}-1\right)+\sqrt{\frac{1}{4}\left(\frac{1}{\nu}-1\right)^2+\epsilon\left(\frac{2}{\nu}-1\right)}, \tag{5a}$$

$$\nu\equiv\nu(x)=\pi l_H^2 n(x),\quad n(x)=n_d+\delta n(x),\quad \epsilon=\exp\left(-\frac{\hbar\omega_c}{T}\right)\ll 1. \tag{5b}$$

At equilibrium we have $\mu=$const, and Eq. (5), along with Poisson equation (2), determines features of the magnetocapacitance (e.g., in the problem of a plane capacitor, one of whose plates is a 2D system). The effect of the magnetic field is determined by the combination $T\ln S$, which changes abruptly from zero to $\hbar\omega_c$ as the point $\nu=1$ is crossed.

When there is a steady-state current along the strip, Eq. (4) and thus (5) should remain unchanged [the range of applicability of (5) is the same as that of Eq. (1)]. However, the electrochemical potential μ no longer has to remain constant along the x axis. Furthermore, boundary conditions (1) should be imposed on μ, not φ. The reason is that it is specifically the deviation of μ from a constant value which may the reason for the onset of an average current $\langle j_y\rangle$along the strip.

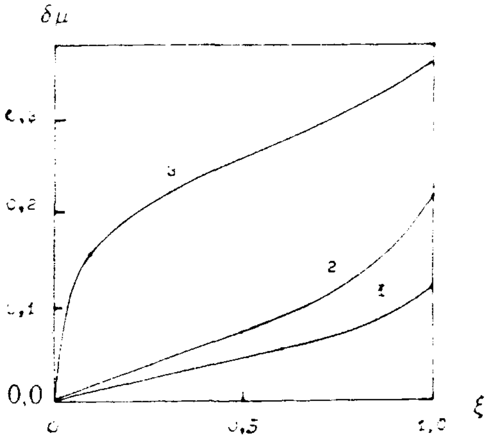

FIG. 1. Plot of $\delta\mu$ versus ξ at a fixed value of j, measured for three temperatures t. 1—$t=0.07$; 2—0.05; 3—0.03. The quantities $\delta\mu, t$, and ξ are defined in (11).

The proposed set of definitions thus takes the following form in our case. First, definition (3) makes it possible to relate the distribution of the current j_y to the field $\varphi'(x)$. If n_d is constant, then so is j_y. Since

$$\varphi'(x)=\frac{h}{e\nu} j_y \tag{6}$$

is known, we find, using (2), the distribution of the electron density:

$$\delta n(x)=\frac{\kappa h j_y}{2\pi e^2\nu}\frac{\xi}{\sqrt{1-\xi^2}},$$

$$\xi=x/w, \quad \int_{-w}^{+w}\delta n dx=0. \tag{7}$$

The last step is to determine the electrochemical potential $\mu(x)$. For this purpose we use Eq. (5) with $\delta n(x)$ from (7) and with $\varphi'(x)$ from (6). The constant φ_0 in the definition of $\varphi(x)$ in (6) can be found from the requirement

$$\mu(-w)=\mu_0. \tag{8}$$

At the other end we set

$$\mu(+w)=\mu_0+eV. \tag{9}$$

Combining (8) and (9), we find the current–voltage characteristic of a 2D strip with an approximately integer filling factor:

$$eV=2w\varphi'+T\ln\frac{S(+w)}{S(-w)}. \tag{10}$$

Let us demonstrate the degree of nonlinearity of the coordinate dependence $\mu(x)$ for the case of a monotonic variation of the temperature and at a fixed value of the total current $I=2wj_y$. Choosing some appropriate dimensionless variables

$$\delta\mu(\xi)=\frac{\mu(\xi)-\mu(0)}{\hbar\omega_c}, \quad t=\frac{T}{\hbar\omega_c}, \quad \xi=\frac{x}{w}, \tag{11}$$

and assuming that in the absence of a current we have

$$\pi l_H^2 n_d = 1,$$

we find the behavior $\delta\mu(\xi)$ from (5)–(7) for a fixed, small value of j_y and for three values of t: 0.07, 0.05, and 0.03 (Fig. 1).

The results found above do not apply directly to the experiments of Refs. 1 and 2, since there was a screening electrode in those experiments. However, the nonlinearity in the behavior of $\delta\mu(\xi)$ is obvious, and this is the point that was to be demonstrated.

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