Hot-hole electrooptic effect

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Two new electrooptic effects have been observed in semiconductors: a change in the refractive index and an anisotropy of this index upon the heating and drift of holes in a strong electric field. These effects are shown to result from a heating and drift of holes in the electric field and from the complex structure of the valence band. Experiments were carried out on p-type germanium. The experimental results agree well with theoretical predictions.

INTRODUCTION

The contribution of the free holes to the dielectric constant of p-type semiconductor crystals is known to be governed by both intraband and interband transitions:[1] $n_h = n_{\text{inter}} + n_{\text{intra}}$. We show below how this contribution varies during the heating and drift of holes in an electric field. We examine the anisotropy $(n^{\parallel} - n^{\perp})$ of the refractive index for light of two polarizations, parallel and perpendicular to the field. In weak fields we have $(n^{\parallel} - n^{\perp}) \propto E^2$, so this effect might be called the "hot-hole Kerr electrooptic effect."

THEORY

1. Intraband transitions of holes. The contribution of free carriers to the dielectric constant ϵ in the course of intraband transitions is

$$(\epsilon_{\alpha\alpha})_{\text{intra}} = \frac{4\pi e^2}{\hbar^2 \omega^2} \int \frac{\partial^2 \mathscr{E}(\mathbf{k})}{\partial k_\alpha^2} f(\mathbf{k}) \frac{2d\mathbf{k}}{(2\pi)^3}, \tag{1}$$

where $\alpha = x,y,z$, and $f(\mathbf{k})$ is the distribution function of the hot charge carriers in the strong electric field. If the carriers have an anisotropic dispersion relation (e.g., if the constant-energy surfaces are corrugated spheres) and/or if this dispersion relation is non-parabolic, and the carrier momentum distribution in the electric field varies because of the heating and drift of the carriers, then ϵ_{intra} also varies because of the heating and drift.

The dispersion relation for heavy holes in semiconductors with the diamond structure is described by[2]

$$\mathscr{E}_1(\mathbf{k}) = \frac{\hbar^2}{2m_0} \{Ak^2 - [B^2 k^4 + C^2 (k_x^2 k_y^2 + k_y^2 k_z^2 + k_z^2 k_x^2)]^{1/2}\}. \tag{2}$$

The dispersion relation for heavy holes can be assumed to be parabolic, but the constant-energy surfaces are corrugated spheres.

The corrugation is only slight for light holes, but the dispersion relation is nonparabolic.[2] We approximate this relation by

$$\mathscr{E}_2(k)=\frac{\mathscr{E}_0}{2}\left[\left(1+\frac{2\hbar^2k^2}{m_2\mathscr{E}_0}\right)^{1/2}-1\right], \tag{3}$$

where m_2 is the mass of a light hole near the top of the valence band, and the parameter $\mathscr{E}_0$ has the value 0.5 eV for germanium, for example.

We assume that the distribution functions of the hot and light holes with respect to the momentum $\hbar k$ can be written as shifted Boltzmann distributions

$$f_i(\mathbf{k})=A_i\exp\left(-\frac{\mathscr{E}_i(\mathbf{k})-\hbar\mathbf{k}\mathbf{v}_{dr_i}}{k_BT_i}\right), \tag{4}$$

where A_i are normalization factors, and i takes on the values 1 and 2 for the heavy and light holes.

We first consider the *contribution of heavy holes*. We substitute (2) and (4) into (1). We expand the distribution function in powers of v_{dr_1}, and we retain terms up to $v_{dr_1}^2$. Assuming the corrugation to be slight, we then find the following expression for the refractive indices $n^{\perp}$and $n^{\parallel}$ for the polarizations $\mathbf{e}_\omega\|\mathbf{v}_{dr_1}$ and $\mathbf{e}_\omega\perp\mathbf{v}_{dr_1}$:

$$n_{\text{intra}}^{\perp,\parallel}=-\frac{4\pi e^2N_1}{2nm_1\omega^2}+\frac{4\pi e^2N_1C^2\mathscr{E}_{dr_1}}{2nm_1\omega^2Bk_BT_1}\beta^{\perp,\parallel}, \tag{5}$$

where $\mathscr{E}_{dr_1}=m_1v_{dr_1}^2/2$, $m_1=m_0/(A-B-C^2/10B)$, n is the refractive index of the lattice, and the coefficients $\beta^{\perp,\parallel}$ depend on the direction of $\mathbf{v}_{dr_1}$. In the case $\mathbf{v}_{dr_1}\|[100]$ (for example) we have $\beta^{\parallel}=1/15$, and $\beta^{\perp}=4/15$. For $\mathbf{v}_{dr_1}\|[111]$ we have $\beta^{\parallel}=4/15$ and $\beta^{\perp}=-2/135$. For $\mathbf{v}_{dr_1}\|[110]$ we have $\beta^{\parallel}=7/30$, and $\beta^{\perp}$ depends on the polarization of the light in the (110) plane. For example, in the case $\mathbf{e}_\omega\|[\bar{1}10]$ we have $\beta^{\perp}=0.1$, while in the case $\mathbf{e}_\omega\|[010]$ we have $\beta^{\perp}=4/15$. The first term in (5) is the well-known contribution of free carriers to the refractive index of a crystal in the absence of an electric field. According to (5), the anisotropy of the refractive index is

$$(n^{\perp}-n^{\parallel})_{\text{intra}}=\frac{4\pi e^2N_1C^2\mathscr{E}_{dr_1}}{2nm_1\omega^2Bk_BT_1}(\beta^{\perp}-\beta^{\parallel}). \tag{6}$$

It can be seen that in weak fields, with $T_1=T_0$ (T_0 is the lattice temperature), we have $|n^{\perp}-n^{\parallel}|_{\text{intra}}\propto E^2$, and this difference depends on the direction of E with respect to the crystallographic axes through the dependence of $\beta^{\perp}$ and $\beta^{\parallel}$ on this direction. In the absence of a corrugation ($C=0$) there is no anisotropy.

2. We now consider the *contribution of light holes*. Substituting (3) into (1), and assuming $k_BT_2/\mathscr{E}_0$ and $\mathscr{E}_{dr_2}/\mathscr{E}_0\ll 1$, we find

$$n_{\text{intra}}^{\perp,\parallel}=-\frac{4\pi e^2N_2}{2nm_2\omega^2}\left\{1-\frac{21}{4}\frac{k_BT_2}{\mathscr{E}_0}-10\frac{\mathscr{E}_{dr_2}}{k_BT_2}\beta^{\perp,\parallel}\right\}, \tag{7}$$

where $m_2=m_0/(A+B+C^2/10B)$, $\beta^{\parallel}=1.1$, and $\beta^{\perp}=-0.7$. We immediately note that in the case of p-Ge the contribution of light holes to the electrooptic effects is small in comparison with that of the heavy holes. Nevertheless, this contribution was taken into account in the comparison of the results of calculation with experiment.

3. We now consider the *contribution of interband transitions.* To calculate this contribution we can use the standard expression for the dielectric constant (Ref. 1, for example). However, we find the following method more graphic. The relationship between the refractive index and the absorption coefficient for light is determined from the Kramers–Kronig relation:

$$n_{\text{inter}}=\frac{C}{\pi}\sum_{ij}\int\frac{\alpha_{ij}(\omega')d\omega'}{\omega'^2-\omega^2}, \tag{8}$$

where α_{ij} is the absorption coefficient for light in the transitions of holes from band i to band j. In the wavelength region of interest here, that of a CO_2 laser, the contribution of transitions of heavy holes into the light-hole band in α_{12} is the governing contribution, according to the calculations. This contribution is[2]

$$\alpha_{12}=\frac{2e^2k^3}{cn\hbar^2\omega}\left(\frac{d^2\mathcal{E}_1}{dk^2}-\frac{d^2\mathcal{E}_2}{dk^2}\right)^{-1}\int_{\Omega}W_{12}(\mathbf{k})[f_1(\mathbf{k})-f_2(\mathbf{k})]d\Omega, \tag{9}$$

where

$$W_{12}(\mathbf{k})=\frac{\hbar^2|\mathbf{e}_\omega\times\mathbf{P}_{12}|^2}{m_0k^2}; \quad W_{12}(\mathbf{k})=\langle W_{12}(k)\rangle_{\Omega}\frac{3}{8\pi}\sin^2(\mathbf{e}_\omega,\mathbf{k}).$$

The integration is over the solid angle Ω. According to (6), the sign and magnitude of n_{inter} depend on ω and $T_{1,2}(E)$. In the absence of a field, under the condition $\hbar\omega \gg k_BT_0(m_1/m_2)$, expression (6) for n_{inter} becomes the expression derived in Ref. 1. It can be seen from (6) and (7) that at small values of E, under the condition $(\mathcal{E}_{dr_1}/k_BT_1)^2 \ll 1$, the anisotropy of the refractive index of the light for the two light polarizations $\mathbf{e}_\omega \| \mathbf{v}_{dr}$ and $\mathbf{e}_\omega \perp \mathbf{v}_{dr}$ is $(n^\perp - n^\|)_{\text{inter}} \sim \mathcal{E}_{dr_1}/k_BT_1$.

Consequently, the electrooptic effect is proportional to E^2 even in the case of interband transitions at small values of E.

EXPERIMENTAL PROCEDURE AND RESULTS; COMPARISON WITH THE THEORY

As a model sample we used a crystal of p-type germanium cut in such a manner that the electric field and the hole drift were approximately along the [110] direction. We studied a modulation of laser light with a wavelength $\lambda=10.6$ μm by a p-Ge crystal in a pulsed electric field. The length of the electric field pulse was 0.3 μs. To study the changes in Δn with E for the two polarizations of the light, we used a crystal with plane-parallel faces. This crystal served as a Fabry–Perot interferometer. The light intensity transmitted through this interferometer, $\mathcal{J}_\nu$, depends on n (see the inset in Fig. 1). If we choose the working point near the maximum of the derivative $d\mathcal{J}_\nu/dn$, we find that the modulation of the light upon a change in n is at a maximum. If we now choose the point with $d\mathcal{J}_\nu/dn=0$, we find that the modulation is associated with only a change in the light absorption in the field. It was thus possible to separately measure the two contributions to the modulation of the light, i.e., those caused by changes in Δn_h and

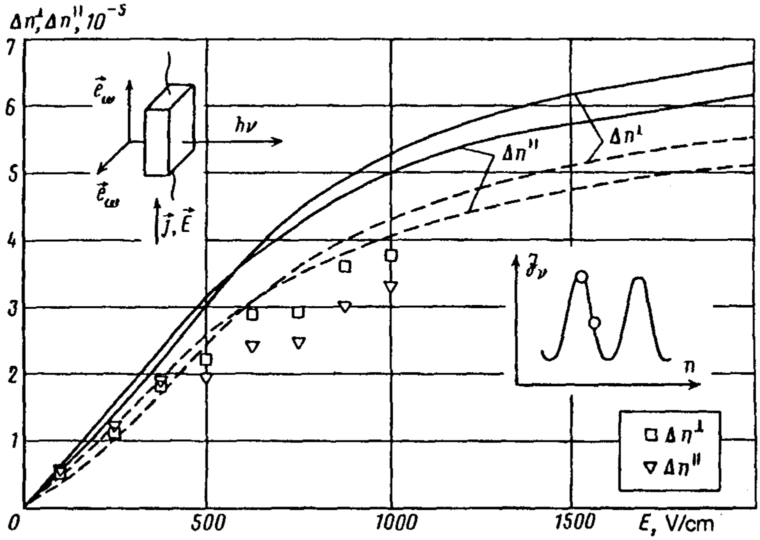

FIG. 1. Change in the refractive index of the p-Ge for light of two polarizations, $\mathbf{e}_\omega \perp \mathbf{E}$ and $\mathbf{e}_\omega \| \mathbf{E}$, as a function of the applied electric field. The points are experimental. □—$\Delta n^{\perp}$; ▽—$\Delta n^{\parallel}$. The dashed curves are theoretical. They incorporate the contribution to the refractive index from only intersubband transitions of hot holes. At $E=0$ this contribution is $n_{\text{intra}} = -5.4 \times 10^{-5}$. The solid curves are theoretical, incorporating both intersubband transitions and intraband transitions of hot heavy and light holes for $\mathbf{E}\|[110]$, $\mathbf{e}_\omega\|[110](\mathbf{e}_\omega\|\mathbf{E})$ and $\mathbf{e}_\omega\|[001](\mathbf{e}_\omega \perp \mathbf{E})$. The experimental conditions are $T=80$ K, $N_h = 5\times10^{14}$ cm^{-3}, and $\lambda = 10.6$ μm. The inset shows the experimental layout.

$\Delta\alpha_{12}$. The working point was shifted by changing the crystal temperature, since the n of the crystal depends strongly on T: A change in T of a few degrees was sufficient to span the entire $\mathcal{J}_\nu(T)$ dependence from $\mathcal{J}_{\nu\min}$ to $\mathcal{J}_{\nu\max}$.

Figure 1 shows experimental and theoretical results. The calculations were carried out using Eqs. (4)–(6). The drift velocities v_{dr_i}, the temperatures T_i, and the densities N_i of the hot, heavy holes and light holes were found from the momentum and power balance equations. We considered intraband and interband scattering of holes by optical phonons, acoustic phonons, and impurities. The Luttinger parameters for germanium, in terms of which A, B, and C are expressed in (2), were taken from Ref. 3. The calculation shows that the interband transitions to $\Delta n_h^{\perp,\parallel}$ are the governing contribution. The difference $\Delta n_h^{\perp} - \Delta n_h^{\parallel}$ turns out to be small in comparison with $\Delta n_h^{\perp}$ and $\Delta n_h^{\parallel}$ and an effort to determine the anisotropy in n_h by the experimental procedure described above results in a large error. We accordingly used a different method to study the optical anisotropy. Circularly polarized laser light was incident on a test sample. Because n_h for the case $\mathbf{e}_\omega \perp \mathbf{E}$ differs from n_h for $\mathbf{e}_\omega \| \mathbf{E}$, the circularly polarized light transmitted through the crystal was transformed into elliptically polarized light. The light leaving the crystal was analyzed by an analyzer oriented at an angle of 45° with respect to the direction of the electric field (see the inset in Fig. 2). The modulation depth of the light in this experiment was $M \simeq (2\pi L/\lambda)(n^{\perp} - n^{\parallel})$, where L is the length of the crystal. The results are shown

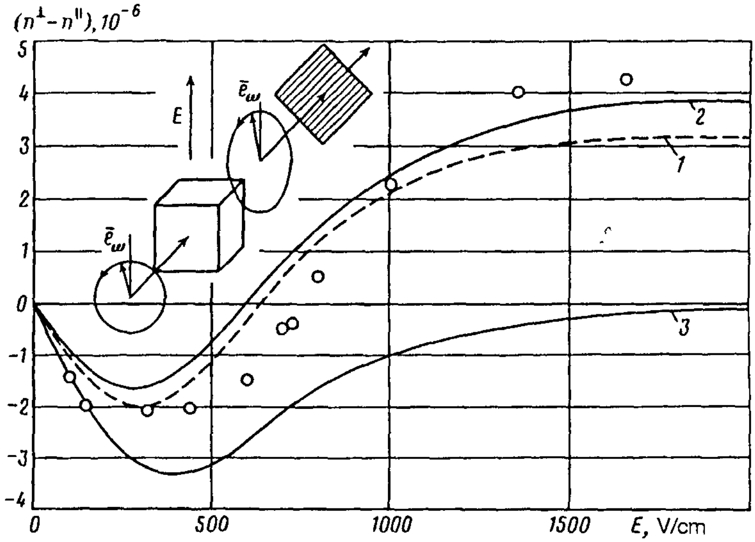

FIG. 2. Anisotropy of the refractive index of p-Ge versus the electric field (the Kerr electrooptic effect involving hot holes). The points are experimental. Dashed curve 1 shows the results of a calculation incorporating the contribution of only intersubband transitions of hot holes to the refractive index. The solid curves show the results of calculations incorporating both intersubband and intraband transitions of hot heavy and light holes. Curve 2—$\mathbf{E}\|[110]$, $\mathbf{e}_\omega\|[110]$ ($\mathbf{e}_\omega\|\mathbf{E}$) and $\mathbf{e}_\omega\|[001]$ ($\mathbf{e}_\omega\perp\mathbf{E}$); 3—$\mathbf{E}\|[111]$, $\mathbf{e}_\omega\|[111]$ ($\mathbf{e}_\omega\|\mathbf{E}$) and ($\mathbf{e}_\omega\perp\mathbf{E}$).

in Fig. 2. We see that the value of $(n^{\perp}-n^{\parallel})$ changes sign with increasing field. According to the calculated results, shown in the same figure, the interband contribution to the anisotropy is predominant for this particular geometry of the crystal. For the case $\mathbf{v}_{dr_1}\|[100]$, however, the intraband contribution, associated with corrugation, plays the major role according to the calculations. Although the study was carried out for p-type germanium, the mechanism for the change in n_h and for the onset of an anisotropy described above is shared by most p-type semiconductors.

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