Reductions of a Lax pair for self-duality equations of the Yang–Mills model

M. Legaré
Department of Mathematics, University of Alberta, Edmonton, T6G 2G1, Canada

A. D. Popov
Theoretical Physics Laboratory, Joint Institute for Nuclear Research (Dubna), Box 79, 101100 Moscow, Russia

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Reductions of the self-duality equation of the Yang–Mills model in $d=4$ in terms of the action of continuous symmetry groups lead to systems of differential equations in a lower dimensionality. An algorithm is written for reducing a Lax pair for self-duality equations with respect to an arbitrary subgroup of the conformal transformation group of R^4 space. The compatibility condition for the reduced Lax pair is shown to be the same as the self-duality equations reduced in terms of the action of the same symmetry group. The general scheme is illustrated with three examples.

1. The self-duality equations of the Yang–Mills model in Euclidean space R^4 with the metric $\delta_{\mu\nu}$ were introduced in a landmark paper by Belavin, Polyakov, Schwarz, and Tyupkin.[1] A Lax pair for the self-duality equations was written by Belavin and Zakharov.[2] The self-duality equations are thus integrable both by the method of the inverse scattering problem[2] and by methods of twistor theory.[3] Also integrable are the self-duality equations in $R^{2,2}$ space with the pseudo-Euclidean metric $(g_{\mu\nu})=\mathrm{diag}(1,1,-1,-1,)$ (Ref. 4). A reduction of these equations to the equations of modified chiral model in $R^{2,1}$ and their integrability by the method of the inverse scattering problem were studied by Zakharov and Manakov.[5]

It has recently been shown that numerous integrable equations in (1+0), (1+1), (0+2), and (1+2) dimensions (the equations of a generalized Kovalevskaya top, the P_{I}–P_{VI} Painlevé equations, the Korteweg–de Vries equation, the Boussinesq equation, the N-wave equation, the Ernst equation, the Kadomtsev–Petviashvili equation, and others) can be derived through a reduction of the self-duality equations.[6–9] The self-duality equations in $d=4$ thus play the role of a universal integrable system, from which many other equations can be derived through a reduction in terms of symmetry groups and through the imposition of algebraic constraints on the Yang–Mills potentials. The literature reveals no general method for reducing a Lax pair for self-duality equations. In most cases, the reduction is carried out with respect to a subgroup of the translation group. The Lax pairs corresponding to the reduced equations are found through a trivial reduction of the Lax pair for the self-duality equations.

In this letter we describe a reduction of the Lax pair of the self-duality equations with respect to an arbitrary subgroup of the conformal transformation group of R^4 space, which is isomorphic to the $SO(5,1)$ group. The condition for compatibility of the reduced

Lax pair leads in turn to self-duality equations which are reduced with respect to the same subgroup. Although the reduction algorithm is written in this letter exclusively for Euclidean space R^4, it can also be applied, after certain modifications, to the reduction of the Lax pair of the self-duality equations in $R^{2,2}$.

The Lax pair introduced for the Ernst equation by Belinski and Zakharov[10] contains a derivative with respect to a spectral parameter. We will show that derivatives with respect to a spectral parameter arise in a reduced Lax pair if and only if the symmetry group contains one generator Y of the "anti-self-dual" subgroup $SO(3)$ of the $SO(4)$ rotation group of R^4 space (an explicit expression for Y is given in Sec. 3).

2. We denote by A_μ the Yang–Mills potentials in R^4 with values in the Lie algebra $gl(n,C)$. The self-duality equations are[1]

$$\frac{1}{2}\,\epsilon_{\mu\nu\rho\sigma}F_{\rho\sigma}=F_{\mu\nu}, \tag{1}$$

where $\mu,\nu,...=1,...,4$; $F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu+[A_\mu, A_\nu]$ are gauge fields of the Yang–Mills model; $\partial_\mu\equiv\partial/\partial x^\mu$; and $\epsilon_{\mu\nu\rho\sigma}$ is the Levi–Civita density in R^4 ($\epsilon_{1234}=1$). A Lax pair[2] for Eqs. (1) can be written

$$[D_1+iD_2-\lambda(D_3+iD_4)]\Psi(x,\lambda)=0,$$

$$[D_3-iD_4+\lambda(D_1-iD_2)]\Psi(x,\lambda)=0, \tag{2}$$

where $D_\mu=\partial_\mu+A_\mu$, and $\Psi\in C^n$ is a vector function which depends on the coordinates x_μ of R^4 space with the metric $\delta_{\mu\nu}$ and also on the complex parameter $\lambda\in CP^1$. For the mathematically oriented reader we note that Ψ is a section of the complex vector stratification $\tilde{E}\simeq Z\times C^n$ given on the space of twistors $Z=R^4\times CP^1$ for R^4 space.[3,11] Equations (2), along with the equation $\partial\Psi/\partial\bar{\lambda}=0$, mean that the stratification (fiber bundle) $\tilde{E}$, which is a lifting of the bundle $E\simeq R^4\times C^n$ with self-dual connection, is holomorphic.[3,11]

It is simple to verify that the vector parts $V_1=\partial_1+i\partial_2-\lambda(\partial_3+i\partial_4)$ and $V_2=\partial_3-i\partial_4+\lambda(\partial_1-i\partial_2)$ of the differential operators in (2) determine a basis of antiholomorphic vector fields with respect to the following complex structure J^μ_ν in R^4 (Ref. 11):

$$J^\mu_\nu=-\delta^{\mu\sigma}\bar{\eta}^a_{\sigma\nu}s_a,$$

where $\bar{\eta}^a_{\sigma\nu}=\{\epsilon^a_{bc},\ \sigma=b,\ \nu=c;\ \delta^a_\nu,\ \sigma=4;\ -\delta^a_\sigma,\ \nu=4\}$ are the 't Hooft anti-self-dual tensors;[12] $a,b,...=1,2,3$; the s_a parametrize $S^2\simeq CP^1(s_as_a=1)$; and $\lambda=(s_1+is_2)/(1+s_3)$. We find a definition of the 't Hooft self-dual tensor $\eta^a_{\sigma\nu}$ if we change the signs of δ^a_ν and δ^a_σ in the expression for $\bar{\eta}^a_{\sigma\nu}$. Using the identities for 't Hooft tensors,[12] we can easily show that we have $J^\mu_\sigma J^\sigma_\nu=-\delta^\mu_\nu$, $J^\mu_\sigma V^\sigma_1=-iV^\mu_1$, and $J^\mu_\sigma V^\sigma_2=-iV^\mu_2$.

3. Both the self-duality equations and their Lax pair are invariant under the conformal-transformation group $SO(5,1)$ of Euclidean space R^4. We will now formulate a general algorithm for reducing a Lax pair for self-duality equations with respect to an arbitrary subgroup G of the $SO(5,1)$ group.

A. First, we specify a homomorphism of the Lie algebra $so(5,1)$ of the $SO(5,1)$group into a Lie algebra of the vector fields X_ξ $[\xi\in so(5,1)]$ in R^4:

$$X^a=\eta^a_{\mu\nu}x_\mu\partial_\nu,\quad Y^a=\bar\eta^a_{\mu\nu}x_\mu\partial_\nu,\quad P_\mu=\partial_\mu,\quad K_\mu=\frac{1}{2}x_\sigma x_\sigma\partial_\mu-x_\mu D,\quad D=x_\sigma\partial_\sigma,\tag{3}$$

where $\{X^a\}$ and $\{Y^a\}$ are generators of two commuting $SO(3)$ subgroups in $SO(4)$, the P_μ are translation generators, the K_μ are generators of special conformal transformations, and D is a dilitation (or extension) generator.

B. Second, we must determine the action of $SO(5,1)$ on Z which conserves the holomorphic nature of the bundle $\tilde E\to Z$. This is possible if, after the elevation $X_\xi\to\tilde X_\xi$, the generators in (3) on Z, $\tilde X_\xi$ are infinitesimal automorphisms of the complex structure J^μ_ν in R^4 and of the canonical complex structure ϵ^i_j in $R^2\subset CP^1$ (Ref. 13):

$$\mathscr{L}_{\tilde X_\xi}J^\mu_\nu\equiv\tilde X_\xi J^\mu_\nu+J^\mu_\sigma\tilde X^\sigma_{\xi,\nu}-J^\sigma_\nu\tilde X^\mu_{\xi,\sigma}=0,\quad\forall\xi\in so(5,1),$$

$$\mathscr{L}_{\tilde X_\xi}\epsilon^i_j\equiv\tilde X_\xi\epsilon^i_j+\epsilon^i_k\tilde X^k_{\xi,j}-\epsilon^k_j\tilde X^i_{\xi,k}=0,\quad\forall\xi\in so(5,1),\tag{4}$$

where $\mathscr{L}_{\tilde X_\xi}$ is a Lie derivative along the vector field $\tilde X_\xi$ on Z; $\epsilon^2_1=-\epsilon^1_2=1$; and $i,j,...=1,2$. We are using local coordinates y_i on the sphere $S^2\simeq CP^1$, which are related to the coordinates s_a by the equations $y_1=s_1/(1+s_3)$, $y_2=s_2/(1+s_3)$, $y_1+iy_2=\lambda$. The necessary realization of the Lie algebra $so(5,1)$ as the subalgebra $\{\tilde X_\xi,\xi\in so(5,1)\}$ in the algebra of vector fields on Z is given by

$$\tilde X^a=X^a,\quad\tilde Y^a=Y^a+2Z^a,\quad\tilde P_\mu=P_\mu,\quad\tilde K_\mu=K_\mu+\bar\eta^a_{\sigma\mu}x_\sigma Z^a,\quad\tilde D=D,\tag{5}$$

where the generators Z^a of the $SO(3)$ rotation group on S^2 are given by

$$Z_a=\epsilon^a_{bc}s_b\frac{\partial}{\partial s_c}.$$

It is simple to verify that Eqs. (4) do indeed hold for vector fields (5).

C. To reduce the linear system (2) with respect to the action of the subgroup G of the conformal group, we impose the conditions of $G-$invariance on the potentials A_μ and on the vector function $\Psi(x,\lambda)$ (Refs. 13 and 14):

$$\mathscr{L}_{X_\xi}A_\mu\equiv X_\xi A_\mu+A_\sigma X^\sigma_{\xi,\mu}=0,\quad\forall\xi\in\mathscr{G},\tag{6a}$$

$$\mathscr{L}_{\tilde X_\xi}\Psi\equiv\tilde X_\xi\Psi=0,\quad\forall\xi\in\mathscr{G},\tag{6b}$$

where $\mathscr{G}$ is the Lie algebra of the subgroup $G\subset SO(5,1)$. For simplicity we are restricting the discussion to strict-invariance conditions (6), although A_μ and Ψ in general, could be invariant within gauge transformations.[15]

D. In accordance with the general method of reduction with respect to symmetry groups (see Ref. 14 and the papers cited there), we choose as the "new" coordinates on Z the coordinates θ_ξ on orbits of the G group and also the invariant coordinates θ_A and ζ, which parametrize the orbit space and which satisfy the conditions

$$\frac{\partial}{\partial\bar\lambda}\zeta=0,\quad\mathscr{L}_{\tilde X_\xi}\theta_A\equiv\tilde X_\xi\theta_A=0,\quad\mathscr{L}_{\tilde X_\xi}\zeta\equiv\tilde X_\xi\zeta=0,\quad\forall\xi\in\mathscr{G}.\tag{7}$$

We call the invariant complex coordinate ζ the new "spectral parameter."

The most general form of G-invariant potentials A_μ, which are solutions of Eqs. (6a), can be written in terms of the $gl(n,C)$-valued functions of the invariant coordinates θ_A and the functions of the coordinates θ_ξ on the orbits. The solution of Eqs. (6b) on the other hand, is an arbitrary function of the invariant coordinates: $\Psi=\psi(\theta_A,\zeta)$.

E. We must substitute the functions A_μ and ψ into Lax pair (2). We then obtain a reduced Lax pair as a system of two linear differential equations in terms of functions of invariant coordinates. The compatibility condition for this system is the same as the self-duality equations reduced with respect to the action of the G group, as follows from the general theory of the reduction of differential equations with respect to symmetry groups.[14] If the generators of the group G contain one of the three generators Y_a introduced in (3), then the reduced Lax pair will contain derivatives with respect to the new spectral parameter ζ. The reason is that elevation $Y_a \to \tilde{Y}_a$ on $Z=R^4\times S^2$ is nontrivial, and the $\tilde{Y}_a$ rotations which are generated are combinations of rotations in R^4 and S^2.

4. We will illustrate the scheme described above by means of three examples of the reduction of Lax pair (2) in terms of Abelian symmetry groups, one of whose generators is Y^3. A nontrivial reduction in terms of a non-Abelian subgroup $SO(3)$ of the $SO(5,1)$ group was described in Ref. 16.

Example 1. We consider a 1D Abelian group $SO(2)$, which is generated by the vector field Y^3. On $R^4\times CP^1$ we introduce the variable φ (a coordinate on an orbit) and the invariant variables $\{r,R,\chi,\zeta=a\ \exp(-i\eta)\}$ $(r,R,a>0,\ 0\leqslant\varphi,\ \chi,\eta<2\pi)$ which satisfy (7) and which are related to the coordinates $\{x_\mu,\lambda=a\ \exp(-i\xi)\}$ on $R^4\times CP^1$ $(0\leqslant\xi<2\pi)$ by

$$x_1=r\cos\left(\chi-\varphi-\frac{\eta}{4}\right),\quad x_2=-r\sin\left(\chi-\varphi-\frac{\eta}{4}\right),\quad x_3=R\cos\left(\chi+\varphi+\frac{\eta}{4}\right),$$

$$x_4=-R\sin\left(\chi+\varphi+\frac{\eta}{4}\right),\quad \xi=\frac{\eta}{2}-2\varphi\Rightarrow\lambda=\zeta\exp\left[i\left(\frac{\eta}{2}+2\varphi\right)\right].$$

For the vector field $\tilde{Y}^3$ elevated on Z we find

$$\tilde{Y}^3=x_1\partial_2-x_2\partial_1-x_3\partial_4+x_4\partial_3+2i(\lambda\partial_\lambda-\bar{\lambda}\partial_{\bar{\lambda}})=\partial_\varphi.$$

A solution of Eqs. (6a) is

$$A_1=a_1\cos\left(\chi-\varphi-\frac{\eta}{4}\right)+a_2\sin\left(\chi-\varphi-\frac{\eta}{4}\right),$$

$$A_2=-a_1\sin\left(\chi-\varphi-\frac{\eta}{4}\right)+a_2\cos\left(\chi-\varphi-\frac{\eta}{4}\right),$$

$$A_3=a_3\cos\left(\chi+\varphi+\frac{\eta}{4}\right)+a_4\sin\left(\chi+\varphi+\frac{\eta}{4}\right),$$

$$A_4=-a_3\sin\left(\chi+\varphi+\frac{\eta}{4}\right)+a_4\cos\left(\chi+\varphi+\frac{\eta}{4}\right), \tag{9}$$

where $a_\mu = a_\mu(r,R,\chi)$. A solution of Eqs. (6b) is $\Psi=\psi(r,R,\chi,\zeta)$. Substituting (8) and ψ into Lax pair (2), we find the reduced system

$$\nabla_X\psi \equiv (X+A_X)\psi = \left[\partial_r - \zeta\partial_R + \left(\frac{\zeta}{R} - \frac{1}{r}\right) i\partial_\chi + \left(\frac{1}{r} + \frac{\zeta}{R}\right)\zeta\partial_\zeta + a_1 + ia_2 - \zeta(a_3+ia_4)\right]\psi$$
$$=0, \tag{9a}$$

$$\nabla_Y\psi \equiv (Y+A_Y)\psi = \left[\zeta\partial_r + \partial_R + \left(\frac{1}{R} + \frac{\zeta}{r}\right) i\partial_\chi - \left(\frac{\zeta}{r} - \frac{1}{R}\right)\zeta\partial_\zeta + a_3 - ia_4 + \zeta(a_1-ia_2)\right]\psi$$
$$=0, \tag{9b}$$

where the vector parts in (9a) and (9b) are denoted by X and Y, respectively. Self-duality equations (1) reduce to[8]

$$\partial_r a_3 - \partial_R a_1 - \frac{1}{r}\partial_\chi a_4 + \frac{1}{R}\partial_\chi a_2 + [a_2,a_4] + [a_1,a_3] = 0,$$

$$\partial_r a_4 + \partial_R a_2 + \frac{1}{r}\partial_\chi a_3 + \frac{1}{R}\partial_\chi a_1 + [a_1,a_4] + [a_3,a_2] = 0,$$

$$\partial_R a_4 - \partial_r a_2 + \frac{1}{R}a_4 - \frac{1}{r}a_2 + \frac{1}{R}\partial_\chi a_3 - \frac{1}{r}\partial_\chi a_1 - [a_1,a_2] + [a_3,a_4] = 0 \tag{10}$$

and are the same as the compatibility condition $[\nabla_X, \nabla_Y] - \nabla_{[X,Y]} = 0$ of Lax pair (9).

Example 2. We now consider the 2D Abelian subgroup $SO(2)\times SO(2)$ in $SO(5,1)$, which is generated by the vector fields X^3 and Y^3. The orbits are parametrized by φ and χ, and the space of orbits is parametrized by the invariant coordinates r, R, and $\zeta = \lambda \exp[-i(\frac{1}{2}\eta + 2\varphi)]$. After an elevation on Z, the vector fields $\tilde{X}^3$ and $\tilde{Y}^3$ take the form $\tilde{X}^3 - \partial_\chi$, and $\tilde{Y}^3 = \partial_\varphi$.

A solution of Eqs. (6a) has the form of (8) with $a_\mu = a_\mu(r,R)$, while a solution of Eqs. (6b) is of the form $\Psi=\psi(r,R,\zeta)$. A reduced Lax pair can be found from (9) by setting $\partial_\chi\psi=0$. The reduced self-duality equations in turn follow from (10) in the case $\partial_\chi a_\mu = 0$.

Example 3. We now consider a 3D Abelian subgroup of the $SO(5,1)$ group with the generators Y^3, P_3, and P_4. Here we have

$$\tilde{Y}^3 = x_1\partial_2 - x_2\partial_1 - x_3\partial_4 + x_4\partial_3 + 2i(\lambda\partial_\lambda - \bar{\lambda}\partial_{\bar{\lambda}}) = \partial_\varphi, \quad \tilde{P}_3 = \partial_3, \quad \tilde{P}_4 = \partial_4,$$

where $x_1 = r\cos(\chi+\eta)$, $x_2 = -r\sin(\chi+\eta)$, $\lambda = a\exp[i2(\eta-\chi)]$, $r,a>0$, $0\leqslant\chi$, and $\eta<2\pi$. As the coordinates on the orbits we have selected χ, x_3, and x_4; as the invariant coordinates we have selected $r=\sqrt{x_1^2+x_2^2}$, $\zeta=\lambda\exp(i2\chi - i3\eta) = a\exp(-i\eta)$. The invariant Yang–Mills fields are

$$A_1 = a_1(r)\cos(\chi+\eta) + a_2(r)\sin(\chi+\eta), \quad A_2 = -a_1(r)\sin(\chi+\eta) + a_2(r)\cos(\chi+\eta),$$

$$A_3 = a_3(r)\cos(\chi+\eta) - a_4(r)\sin(\chi+\eta), \quad A_4 = a_3(r)\sin(\chi+\eta) + a_4(r)\cos(\chi+\eta), \tag{11}$$

and $\Psi=\psi(r,\zeta)$. After substitution of (11) and ψ, Lax pair (2) reduces to the system

$$\left[\partial_r+\frac{2}{r}\,\zeta\partial_\zeta+a_1+ia_2-\zeta(a_3+ia_4)\right]\psi=0,$$

$$\left[\zeta\partial_r-\frac{2}{r}\,\zeta^2\partial_\zeta+a_3-ia_4+\zeta(a_1-ia_2)\right]\psi=0. \quad (12)$$

It is simple to verify that the compatibility condition for system (12) is the same as the self-duality equations reduced with respect to the same subgroup, $SO(2)\times SO(2)$:

$$\dot{a}_2+\frac{1}{r}\,a_2+[a_1,a_2]+[a_4,a_3]=0,$$

$$\dot{a}_3-\frac{1}{r}\,a_3+[a_1,a_3]+[a_2,a_4]=0,$$

$$\dot{a}_4-\frac{1}{r}\,a_4+[a_1,a_4]+[a_3,a_2]=0, \quad (13)$$

where $\dot{a}_\mu\equiv da_\mu/dr$. After the change of variables $t=\ln r$, $u_1=ra_2$, $u_2=r^{-1}a_3$, $u_3=r^{-1}a_4$, and after the choice of gauge $a_1=0$, Eqs. (13) become the modified Nahm equations which we discussed in Ref. 8.

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