Hydrodynamic analogy of the Gunn effect

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It is shown that the thermal instability of fluid flow with increasing viscosity in a restricted temperature range leads to self-oscillations of the flow rate, owing to the motion of thermal domains. In He II, the corresponding effect is related to the appearance of He I regions.

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1. We shall examine a flow of fluid in a pipe, whose viscosity η first increases with temperature, and then saturates (an example of such a fluid is sulfur; the increase in η is related to reversible structural changes at a temperature ~450 K). For simplicity, we will assume a dependence

$$\eta(T< T_*) = \eta_1, \qquad \eta(T> T_*) = \eta_2, \qquad \eta_1 < \eta_2 . \tag{1}$$

We shall write the average (over the transverse coordinates) equations for the temperature $T(x)$ and flow velocity v in the form

$$\rho c \frac{\partial T}{\partial t} = \kappa \frac{\partial^2 T}{\partial x^2} - \rho c \mathrm{v} \frac{\partial T}{\partial x} + 12\eta \frac{\mathrm{v}^2}{d^2} - \frac{2\alpha}{d}(T - T_0), \tag{2}$$

$$\frac{dP}{dx} = - 12\eta \frac{\mathrm{v}}{d^2} . \tag{3}$$

Here, ρ, c, κ, and P are the density, heat capacity, thermal conductivity, and fluid pressure; the numerical coefficients correspond to a two-dimensional channel; d is the width of the channel; α is the coefficient of heat transfer to the external medium, whose temperature $T_0 < T_*$. In writing down (3), we took into account the fact that the values of v being examined are assumed to be small and the thermal processes are assumed to be slow (compared to hydrodynamic processes). It is also assumed that $\alpha d \ll \kappa$.

For small values of the pressure differential ΔP, according to (1)-(3), a uniform, with respect to x, flow regime is possible and the fluid velocity and temperature are

$$\mathrm{v} = d^2 (12\eta_1 l)^{-1} \Delta P, \qquad T = T_0 + 6\eta_1 \mathrm{v}^2 (\alpha d)^{-1}, \tag{4}$$

where l is the length of the channel. For large ΔP, a different uniform regime is realized:

$$\mathrm{v} = d^2 (12\eta_2 l)^{-1} \Delta P, \qquad T = T_0 + 6\eta_2 \mathrm{v}^2 (\alpha d)^{-1}, \tag{5}$$

and the temperature in this case exceeds T_*. The velocity intervals for both regimes partially overlap; for a fixed flow rate, T and ΔP reach the values (4) or (5), depending on the initial conditions.

If the pressure differential is given, then in the interval

$$\sqrt{\eta_1} < \Delta P (2\sqrt{6} l)^{-1} d^{3/2} [\alpha(T_f - T_0)]^{-1/2} < \sqrt{\eta_2} \tag{6}$$

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there are no uniform stationary solutions. [In the case of a continuous dependence $\eta(T)$, the smooth curve (1), a uniform solution exists for all ΔP, but in the interval (6), it is unstable. The unstable solution corresponds to $\mathbf{v}$ as a decreasing function of ΔP.] The instability (or its absence) of uniform solutions leads to the formation of nonuniform flow regimes, in particular, stationary waves (thermal domains), moving together with the fluid with velocity $\mathbf{v}$. This velocity is approximately determined by the equality

$$3(\eta_1 + \eta_2)\, v_{cr}^2 = \alpha d(T_* - T_0)\,, \qquad (7)$$

while the width of the thermal domain s is determined by the condition

$$\Delta P = 12 v_{cr} d^{-2}[\eta_1(l - s) + \eta_2 s]\,. \qquad (8)$$

Equations (7) and (8) assume that the width of the transition layer, within which the temperature varies from the value (4) to (5) with $\mathbf{v} = \mathbf{v}_{cr}$, is small compared to s. The width of the transition layer is of the order of $\sqrt{\kappa d/\alpha}$.

When the thermal domain is ejected through the output section of the pipe, the drag decreases and the flow rate of the fluid increases. A new domain is then formed as a result of thermal instability. The frequency of the pulsations in the flow rate is[1)]

$$\omega \sim v_{cr}\, l^{-1}. \qquad (9)$$

Self-oscillations will be observed, if this quantity is less than the growth increment for the thermal instability.

The product αd and the velocity $\mathbf{v}_{cr}$ are nearly independent of the diameter. For $\alpha d = 10^{-4}$ W cm^{-1} K^{-1}, an estimate using Eqs. (7) and (9) for sulfur at $T_0 = 150°$ C gives the frequency ~ 1 s^{-1} for $l = 10$ cm.

2. Let us now turn to liquid helium.

A sufficiently large pressure differential will maintain a He I flow in a capillary, submerged in a medium with temperature $T_0 < T_\lambda$ and, at the same time, the temperature of the fluid in the capillary is greater than T_λ due to the frictional heat. For

$$\Delta P < 2\sqrt{6}\, l d^{-3/2}\, [\alpha\eta(T_\lambda - T_0)]^{1/2}\,, \qquad (10)$$

the He I flow is unstable and part of the fluid is transformed into He II. The width of the He I region, the "thermal domain" is determined by the magnitude of the pressure differential. In order to find this width and the fluid velocity, it is necessary to examine the boundary separating the He I and He II phases in the flow.[2] In Ref. 2, it is shown that the interphase boundary always moves relative to the fluid toward the downstream phase. For this reason, the transformation He I → He II occurs at one boundary of the He I region, while the transformation He II → He I occurs at the other. If the flow rate $\mathbf{v} < \mathbf{v}_{cr}$, then the He I region disappears and for $\mathbf{v} > \mathbf{v}_{cr}$, this region increases in size. Using the relations in Sec. 4 of Ref. 2, we find

$$v_{cr}^2 \approx \left[1 + \frac{c(T_0)}{c_I}\right] \frac{\alpha d}{6\eta} (T_\lambda - T_0), \qquad (11)$$

where c_I is the heat capacity of He I and it is assumed that $(T_\lambda - T_0) \ll T_\lambda$.

It is easy to see that this velocity will be established with ΔP fixed in a capillary with a "domain." Indeed, for $\mathbf{v} < \mathbf{v}_{cr}$, the He I region will contract, drag will decrease, which will increase the velocity, and so on. Of course, we have in mind a region of parameters, for which the estimate (11) yields a quantity that is smaller than the "usual" critical velocity, related to the formation of quantum vortex rings.

For $T_0 = 2$ K, $d = 10^{-5}$ cm, $\alpha = 10^{-5}$ W cm$^{-1}\cdot$K^{-1}, the velocity $\mathbf{v}_{cr} \approx 7.5$ cm/s. In order to estimate the frequency of pulsations, it is necessary to substitute in (9) the velocity u of the He I region in the laboratory frame; because of the phase transition, u differs from the flow velocity $\mathbf{v}_{cr}$. For values of parameters presented above, $u \approx 15$ cm/s.

The problem of the time of formation of thermal domains must be examined separately.

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1)The analogy with current pulsations with an N-shaped current-voltage characteristic, the Gunn effect,[1] is obvious.

1. J. B. Gunn, Solid State Commun. 1, 88 (1963).
2. É. N. Rumanov, Zh. Éksp. Teor. Fiz. 74, 1422 (1978) [Sov. Phys. JETP 47, 744 (1978)].

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