Anomalously high 1/*f* noise in a nonuniform semiconductor

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A system with an exponentially broad distribution of resistances is analyzed for the case in which the distributions of fields and currents within each of the resistances play an important role. This situation is inherent in a continuum problem. Allowance for these distributions further increases the relative spectral density of the $1/f$ noise in comparison with that of the discrete problem. A critical exponent of the relative spectral density is derived. It contains a term which is attributable to the nonuniformity of the current and field distributions within each resistance.

The relative spectral density of the $1/f$ noise in a uniform conductor is described well by[1] $C=\omega VS/R^2$, where $S_R=\{\delta R\,\delta R\}$ is the Fourier transform of the correlation function of the fluctuations in the resistance R, V is the volume of the sample, and ω is the frequency. The relative spectral density C is a constant of the material. According to Hooge's hypothesis,[2] we have $C=\alpha/n$, where n is the density of free carriers. The universal coefficient $\alpha\sim 10^{-3}$ is known as the "Hooge constant." In a nonuniform conductor, on the other hand, the total noise of the sample is characterized by C^e:

$$C^e=\frac{\langle C(\mathbf{r})[\mathbf{E}(\mathbf{r})\cdot\mathbf{j}(\mathbf{r})]^2\rangle}{[\langle\mathbf{E}(\mathbf{r})\rangle\cdot\langle\mathbf{j}(\mathbf{r})\rangle]^2}, \tag{1}$$

where $\langle ...\rangle$ means an average over volume, and $\mathbf{E}(\mathbf{r})$ and $\mathbf{j}(\mathbf{r})$ are the local electric field and the local current density.

It is clear from (1) that in order to determine C^e we need to know the detailed distributions of the fields and the currents. The nonuniformity of the distribution is more important for the $1/f$ noise (the fourth moment) than in a determination of the effective conductivity (the second moment, $\sigma^e=\langle\mathbf{E}\cdot\mathbf{j}\rangle/\langle\mathbf{E}\rangle^2$). Since the fourth moment generally cannot be expressed in terms of the second, various models of the nonuniform medium, which can be used to find $\mathbf{E}(\mathbf{r})$ and $\mathbf{j}(\mathbf{r})$, are employed in order to determine C^e (Refs. 3 and 4).

The most general case of nonuniform medium is a semiconductor with a large-scale potential well $V=V(\mathbf{r})$ which modulates the bottom of the conduction band.[5,6] If the potential varies sufficiently smoothly (if the length scale of the potential satisfies $b\gg l$, where the right-hand side is the mean free path), then one can introduce a local electrical conductivity $\sigma(\mathbf{r})=e\mu\cdot n(\mathbf{r})$, where μ and $n(\mathbf{r})$ are the mobility and density of the free carriers. For simplicity, we assume that the mobility is independent of the coordinates (for example, the only scattering is a scattering by phonons). In the nondegenerate case, the density is modulated by $V(\mathbf{r})$ in accordance with $n(\mathbf{r})=n_0\exp\{-V(\mathbf{r})/kT\}$. The problem of determining an effective conductivity of the system is thus analogous to the problem of finding σ^e for a medium with an exponentially broad distribution of resistances.[5] The effective conductivity of the system is governed by the carrier density near the percolation level V_c in the potential $V(\mathbf{r})$ only at an exponential accuracy.[5,7,8] As was shown in Refs. 9 and 10, in determining the preexponential factor we need to assume that the system is in the diffuse (smeared out) region Δ. This approach requires more detailed knowledge of the structural elements of the percolation conducting cluster, because the resistance of the system is governed not only by a saddle point of height V_c (the "key resistance" in the terminology of Ref. 11) but also by other saddle points, which are the analogs of the resistances of bridges and interlayers.[4,9] The number and heights of the saddle points are determined by the size of the diffuse region Δ. The size of Δ was found in Refs. 9 and 10.

In a calculation of the integral properties of continuum systems, the primary elements of the percolation structure are thus saddle points of the random potential which are near V_c. In the simplest case, $V(\mathbf{r})$ is characterized by a single energy scale, V_0, and a single spatial scale, b. Near any saddle point the form of $V(\mathbf{r})$ is then universal:

$$V(\mathbf{r})=V_s+\frac{V_0}{b}(-x^2+y^2+z^2), \tag{2}$$

where V_s is the height of the saddle point. To determine the resistance of each saddle point we need to know the distributions of fields and currents in the vicinity of the saddle point. According to Ref. 6, we have

$$n(\mathbf{r})=n_s\exp(-[y^2+z^2-x^2]/L^2), \quad n_s=n_0\exp(-V_s/kT), \tag{3}$$

where $L=b\sqrt{kT/V_0}$, and

$$E_Y=E_Z+0, \quad E_X=(\varphi_0/L\sqrt{\pi})\exp(-x^2/L^2), \quad j=e\mu n(\mathbf{r})E(\mathbf{r}), \tag{4}$$

where φ_0 is the potential difference across the saddle V_s. The resistance of the saddle point, R_s, is thus one of the actual resistances in the exponentially broad spectrum; it is equal to $R=1/(\sqrt{\pi}Len\mu)$. A distinctive feature of continuum problems has emerged in (3) and (4): The length scale b of the system (which is an analog of the minimum size in a lattice problem) has been joined by a new length scale $L\ll b$, which determines the microscopic geometry of the key resistances.

We can now formulate a standard grid problem concerning the effective conductivity. We partition the whole medium into cells with a size on the order of b, which is the analog of the minimum size in the standard percolation problem. With each cell we associate a resistance R_s, which can be written in terms of a random variable x:

$$R_s=\frac{1}{\sqrt{2\pi}e\mu b n_0\lambda^{1/2}}\exp\left(-\frac{V_c-V_0}{kT}\right)e^{-\lambda x}, \tag{5}$$

where $\lambda=2V_0/kT\gg 1$, and x is a random variable which lies on the interval [0, 1] (there are no correlations over distances greater than b). As usual, we assume that the distribution of this random variable is uniform.

According to Refs. 7–10, the effective conductivity of the medium is

$$\sigma^e=\sigma(x_c)\lambda^{-y}, \quad y=\frac{\alpha_1-\alpha_2}{2}+\nu(d-2)+\frac{1}{2}, \tag{6}$$

where we would have $\alpha_1=\alpha_2=1$ according to the NLB model. According to the weak-link model, we would instead have $\alpha_1=t-\nu(d-2)$ and $\alpha_2=q+\nu(d-2)$, where ν is the critical exponent of the correlation length, and t and q are critical exponents of the conductivity above and below the percolation threshold of two-phase systems. In the 3D case under consideration here we have $\nu_3=0.9$, $t_3=1.7$ and $q_3=0.7$. The extra 1/2 in the critical-like exponent y is associated with a redistribution of the fields and currents in the key resistances. A corresponding effect in the so-called Swiss-cheese models[12] leads to a change in the critical exponent for the conductivity of two-phase systems, t.

Here, in contrast with Ref. 10, we need to determine C_s, i.e., the noise in each resistance s associated with the microscopic geometry of this resistance, before we calculate C^e. Using Hooge's hypothesis $C=\alpha/n$ and (1) and (4), where $\langle ...\rangle$ is now an average over a volume on the order of the length scale of the potential, b, we find

$$C_s=\frac{\alpha}{n_s}\left(\frac{b}{L}\right)^3\frac{erf^3(2b/L)}{erf^6(b/L)}. \tag{7}$$

Since we have $b/L\gg 1$, expression (7) becomes the following expression, within terms on the order of $\exp(-b/L)$:

$$C_s=\frac{\alpha}{n_c}\left(\frac{b}{L}\right)^3. \tag{8}$$

Allowance for the strong variations in the distributions of currents and fields near the saddle point causes the amplitude of the $1/f$ noise to increase significantly, by a factor of $(b/L)^2=(V_0/kT)^{3/2}$, at each saddle point.

Once we have determined the noise amplitude in each resistance, (8), the problem reduces to the solved problem of determining the effective properties of a medium with an exponentially broad spectrum of resistances.[10] In our notation we find

$$C^e=\frac{\alpha}{n_c}\left(\frac{V_0}{kT}\right)^{\gamma}, \quad \gamma>\frac{3}{2}+m, \tag{9}$$

where $m=(t-q)/2+2\nu$. It can be seen from (6) and (9) that the effective spectral density of the $1/f$ noise in a nonuniform semiconductor with a large-scale potential well is higher by a factor of $\lambda^{2\nu+3/2}$ $(\gg 1)$ than in a uniform semiconductor with the same resistivity. The factor 2ν arises in the ordinary system of an exponentially broad spectrum of resistances, while the power of 3/2 is attributable to the microscopic geometry: the geometric shape of the regions which determine the resistance of the whole system. This additional increase in the noise is yet another example[13–15] of an effect of the microscopic geometry on the size of the critical exponents of the relative spectral density of the $1/f$ noise. We have thus clearly demonstrated a relationship between a deviation of the critical exponents from universal applicability, on the one hand, and the microscopic geometry, on the other.

As can be seen from (9), Hooge's hypothesis is not valid for describing the noise in the overall nonuniform system. The reason is that the relation $C(\mathbf{r})=\alpha/n(\mathbf{r})$ is not valid for the effective values $C^e = \alpha / n_m$, where n_m is an average density found experimentally. It is important to note that n_m depends on the measurement method. In a strong magnetic field, for example, we would find $n_M = \langle n \rangle$ from Hall measurements, while the density found from measurements of the magnetoresistance of the same sample would be[16,17] $n_m = n_c$ $[n_c / \langle n \rangle \sim \exp(-V/kT)]$. We are therefore not surprised by the large scatter in values which was mentioned in Ref. 1 (Table II).

Expressions (8) and (9) also give us a "rule" for the decrease in the relative spectral density of the $1/f$ noise: a decrease in the nonuniformity of the system (V/kT). There are two ways to achieve this goal. First, we could increase the density of mobile carriers in the band, by means of photoexcitation, for example. The effect would be a screening of the fluctuation potential and a decrease in V_0. The second possibility (a slightly paradoxical method) is to raise the temperature of the electron gas. The Nyquist noise will of course increase in the process. We do not know of any experiments designed expressly for determining the temperature dependence of the amplitude of the $1/f$ noise in percolation systems with a microscopic geometry. Published results (e.g., Figs. 18 and 19 in Ref. 1 and Refs. 17 and 18) describe both increases and decreases in the amplitude of the $1/f$ noise with increasing temperature.

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