Nonlinear structures and their collective excitations in a coherent state of excitons during resonant pumping

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This letter examines nonlinear structures of a coherent state of Wannier–Mott excitons created and sustained by optical pumping. The spectrum of collective excitations and the stability of the structures are analyzed.

1. Equations describing a system consisting of coherent excitons and an electromagnetic field were first derived in Ref. 1. A concentration bistability, which exists in a coherent exciton condensate in a certain interval of pump intensities, was studied in Ref. 2. The properties of a uniform exciton condensate were described in detail in Refs. 3 and 4. Instabilities with respect to spatial stratification were studied in Refs. 5 and 6. Those papers also examined the spectrum of linear excitations of a uniform coherent state of Wannier–Mott excitons produced and sustained by optical pumping in a semiconductor or a semiconducting system with quantum wells in situations with and without resonators. Experiments carried out to probe an exciton condensate by means of picosecond light pulses in quantum-well structures were described in Refs. 7 and 8.

In this letter we examine nonlinear structures of an exciton condensate. We examine the spectrum of their collective excitations and the stability of the structures.

2. In the given field of a uniform monochromatic pump wave $E=E_0\exp(i\omega t)$ we use, in accordance with Refs. 1 and 2, the following equation for the wave function Φ, which describes the macroscopic quantum state of the excitons (recombination processes are being taken into account):

$$i\,\frac{\partial\Phi}{\partial t}+i\,\frac{\gamma}{2}\,\Phi-[(\omega_0-\omega)-\theta|\Phi|^2]\Phi+\frac{\hbar}{2m}\,\nabla^2\Phi=\frac{1}{\hbar}\,dE_0. \tag{1}$$

Here $\omega_0-\omega\equiv\Delta$ is the detuning of the pump frequency ω from the frequency of the exciton transition, ω_0; γ is the reciprocal exciton lifetime; θ is the nonlinear interaction coefficient of the excitons; m is the effective mass of an exciton; d is the dipole matrix element for an exciton transition; E_0 is the amplitude of the pump field (Φ and E_0 are normalized in such a way that $|\Phi|^2$ and $|E_0|^2$ are the densities of coherent excitons and photons in the pump wave, respectively); and $\hbar$ is Planck's constant. Equation (1) is valid under the condition $Nr_0^3\ll 1$ (r_0 is the radius of an exciton, and N is the concentration of excitons) over times scales $\delta t\gg\omega^{-1}$ and over spatial scales $\delta r\gg N^{-1/3}$. From the parameters in Eq. (1) we construct a length scale $L_{ex}=(\hbar/m\gamma)^{1/2}$. With $m\approx 0.1m_0$ (m is the mass of a free electron) and with $\gamma\sim 10^9$ s^{-1} we would have $L_{ex}\sim 10^{-4}$ cm. It is legitimate to discuss nonuniform structures with a length scale L_{ex} under the condition $N^{-1/3} \ll L_{ex}$. For $r_0 \approx 0.5 \times 10^{-6}$ cm, this condition holds at $N \sim 10^{17}$ cm^{-3}.

Let us assume that a thin film of a semiconductor of thickness $l < L_{ex}$ is pumped by a wave which is uniform in the plane of the film (the X,Y plane) and which is propagating along the normal (along the Z axis) to the surface of the film. We assume that the distribution of excitons along the Z axis is uniform; we ignore absorption of the wave over the thickness of the film. The nonlinearity coefficient can have either sign.[1]

Equation (1) is supplemented by the boundary conditions (for the 1D case)

$$\frac{\partial \Phi}{\partial x}\Big|_{0,L} = 0, \tag{2}$$

which correspond to boundaries ($x=0$ and $x=L$) that are impenetrable to excitons.

Steady-state homogeneous solutions Φ_0 of Eq. (1) are

$$\Phi_0 = \frac{dE_0}{\hbar(i\gamma/2 - \Delta + \theta|\Phi_0|^2)}, \quad |\Phi_0|^2 = \frac{|dE_0|^2}{\hbar^2[(\gamma/2)^2 + (\Delta - \theta|\Phi_0|^2)^2]}. \tag{3}$$

The equation for $|\Phi_0|^2$ can have from one to three solutions, depending on the parameters. In the three-solution case, which prevails under the conditions $\Delta\theta > 0$ and $|\Delta| > \gamma\sqrt{3}/2$, the quantity $|\Phi_0|^2$ is an S-shaped[2] function of $|E_0|^2$, and the central state is not realized. There are accordingly a concentration bistability and a corresponding hysteresis in the system.

Linearizing Eq. (1) near a steady, uniform, and otherwise arbitrary state, we find a dispersion relation for fluctuations, which are assumed to be proportional to $\exp(\Omega t + ikx)$:

$$\Omega = -\frac{\gamma}{2} \pm i\,\frac{\gamma}{2}\left[\left(\frac{2\Delta}{\gamma} - \frac{6\theta|\Phi_0|^2}{\gamma} + L_{ex}^2 k^2\right)\left(\frac{2\Delta}{\gamma} - \frac{2\theta|\Phi_0|^2}{\gamma} + L_{ex}^2 k^2\right)\right]^{1/2}.$$

Under the conditions $|\theta||\Phi_0|^2 > \gamma/2$ and $-\Delta + 2\theta|\Phi_0|^2 > 0$, Eq. (4) describes[5,6] an instability [$\mathrm{Re}\,\Omega(k) > 0$] of a uniform coherent state of excitons with respect to periodic spatial stratification with a length scale $\sim L_{ex}$. The stratification can be present in the original uniform monostable and bistable cases.

3. We would like to point out that there can be simultaneous instabilities with respect to stratification on the lower and upper branches of the S-shaped curve of $|\Phi_0|^2$ versus $|E_0|^2$. This situation prevails under the conditions $\theta > 0$ and $1 > \Delta/\gamma > \sqrt{3}/2$.

Figure 1a shows a structure which grows after a small local perturbation of the initial uniform state of the lower branch of the bistable curve. A similar perturbation of a uniform state of the upper branch of the bistable curve, for the same parameter values, leads to the structure shown in Fig. 1b. Under the conditions $\theta > 0$ and $\Delta/\gamma > 1$, an instability occurs on only the upper branch; under the conditions $\theta < 0$ and $\Delta/\gamma < -1$, it occurs on only the lower branch.

We wish to stress that in the system described by Eq. (1) there can also be some aperiodic structures which are excited in a "hard" manner. In particular, there can be

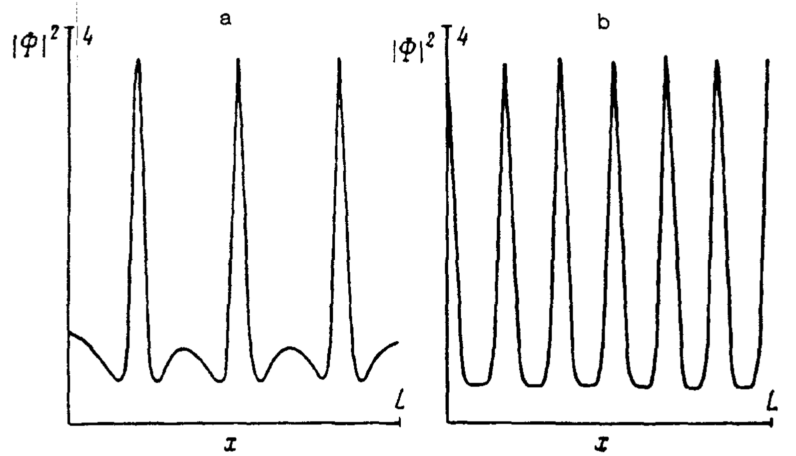

FIG. 1. Structures which grow from unstable, uniform steady states corresponding to the two branches of the bistable curve. The parameter values are $L=24.7L_{ex}$, $\Delta/\gamma=0.9$, and $2dE_0/\hbar\gamma=1.2807$. In this figure and in those that follow, $|\Phi_0|^2$ is expressed in units of $\gamma/(2\theta)$. a—Lower branch, $\theta\,|\Phi_0|^2=1.005\gamma/2$; b—upper branch, $\theta\,|\Phi_0|^2=1.524\gamma/2$.

solitary bunches of excitons which are nucleated by a perturbation of finite magnitude under conditions such that the instability with respect to periodic stratification has not yet been reached [$\mathrm{Re}\,\Omega(k)<0$].

4. Some new collective excitations arise against the background of the periodic structure. We denote by Φ_p a complex periodic solution of Eq. (1) with wave vector k_p: $\Phi_p=\Phi(k_px)$. We seek a nearly periodic solution $\Phi=\Phi[k_px+\epsilon F(X,T)]$ which arises from a modulation of the structure by a collective excitation. Here X and T are slow variables ($X=\epsilon x, T=\epsilon t$). We introduce $\Psi=\partial\Phi_p/\partial(k_px)$. By analogy with Ref. 9, in which the instability of a periodic structure in a reaction–diffusion system was analyzed, we can then find an equation for the function F:

$$\frac{\partial F}{\partial T}=D\,\frac{\partial^2 F}{\partial X^2}, \tag{5}$$

where

$$D(k_p)=i\,\frac{\hbar}{2m}\,\frac{\int_0^{2\pi}d(k_px)\Psi^*\left(2k_p\frac{\partial}{\partial k_p}+1\right)\Psi}{\int_0^{2\pi}d(k_px)\Psi^*\Psi}. \tag{6}$$

The asterisk means complex conjugation. Even without being more specific about expression (6), we conclude that the coefficient D is complex. The real part of this coefficient determines the damping, while the imaginary part determines the frequency of long-wave collective excitations of the original, spatially periodic structure.

We have carried out a numerical study of the behavior of a periodic steady-state structure which grows from an initial monostable state after a perturbation consisting of

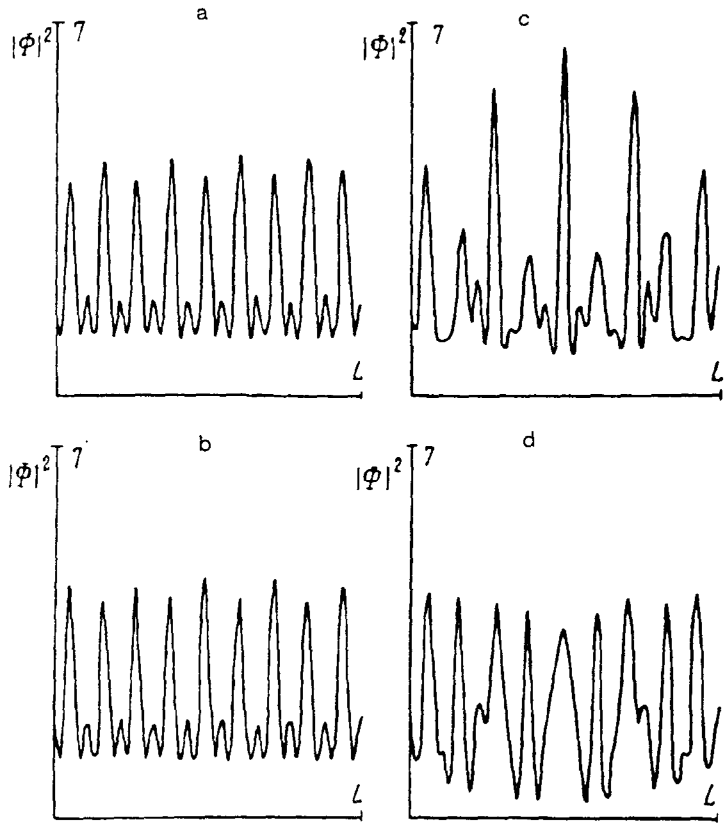

FIG. 2. Wave motion in a spatially periodic structure after local pulsed perturbations of various amplitudes. The parameter values are $L = 12.0$, $\Delta/\gamma = -4$, and $\theta|\Phi_0|^2 = 1.53\gamma/2$. a, b—Damped standing wave, with a time interval γ^{-1} between a and b; c, d—undamped standing wave, with a time interval $2\gamma^{-1}$ between c and d.

a local pulsed increase in intensity. If the initial perturbation is not too large, damped wave motions propagate along the structure. The longest-lived of these wave motions is one in which neighboring peaks oscillate out of phase (Fig. 2, a and b). We thus see a demonstration of the existence of "phonon" modes in a "crystalline" structure which has grown. If the local pulsed excitation is large, an undamped standing envelope wave with a period much longer than the period of the original structure is set up in the structure (Fig. 2, c and d).

5. An increase in the pump level and a change in the detuning may cause these collective excitations to become unstable (the real part of D may go negative). A large number of spatial harmonics are excited spontaneously and successively (in time), and the system loses its symmetry. This effect ultimately leads to a random wave motion. Figure 3 shows several times during the onset of the instability.

We note in conclusion that an equation like (1) also describes the field excited by a

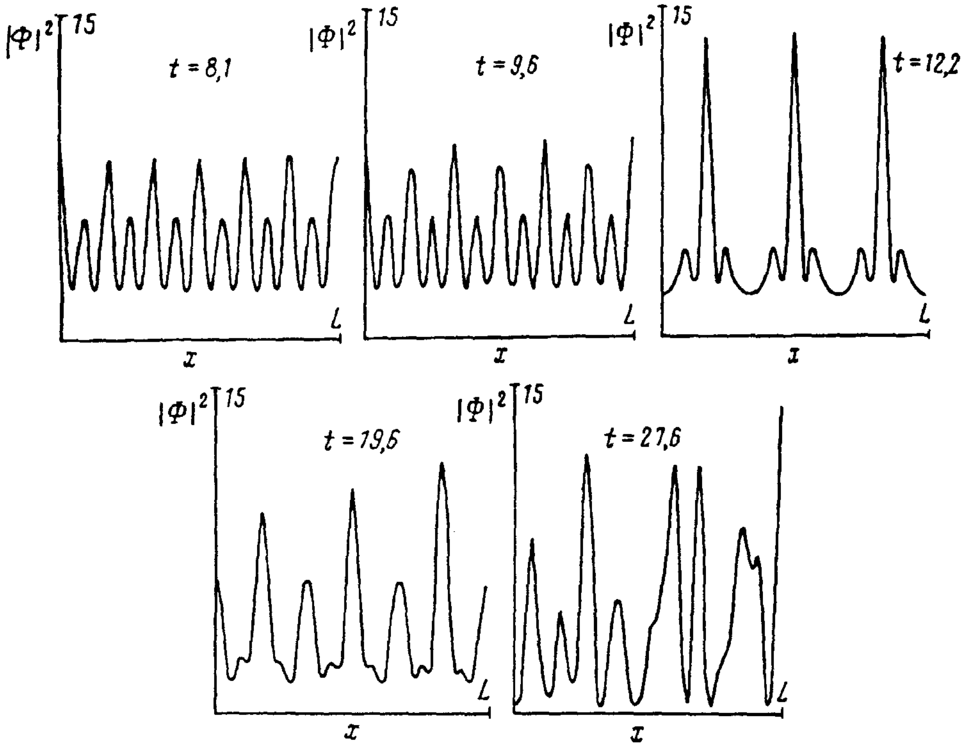

FIG. 3. Onset of an instability of a periodic structure. The parameter values are $L = 6.36L_{ex}$, $\Delta/\gamma = -5.75$, and $\theta\,|\Phi_0|^2 = 3.08\gamma/2$. The time is expressed in units of $2\gamma^{-1}$.

uniform pump in a wide-aperture optical cavity resonator with a Kerr nonlinear medium (in the approximation of a single longitudinal mode). An instability of the field with respect to stratification in a resonator of this sort was analyzed in Ref. 10. The results derived in the present letter also apply to that system.

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