Nonlinear response of dirty conductors in an alternating electric field

F. T. Vas'ko
Institute of Semiconductors, Academy of Sciences of the Ukrainian SSR

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The nonlinear response of a dirty conductor to a low-frequency ($\Omega\tau < 1$) field **E** cos Ωt is calculated. A nonanalytic dependence on **E** is found. The temporal profile of the response is greatly distorted.

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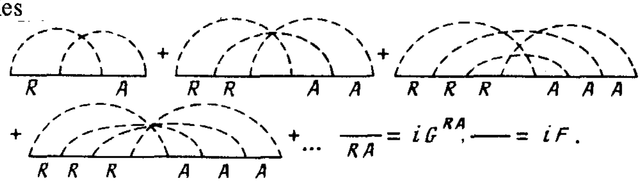
Study of the quantum corrections to the conductivity of disordered metals and degenerate semiconductors (see the bibliography in Refs. 1 and 2) has explained several aspects of the linear kinetic coefficients of these materials (the negative magnetoresistance and the logarithmic temperature dependence of the two-dimensional conductivity). Nonohmic voltage–current characteristics have also been measured for dirty metal films[3,4] and inversion layers of a semiconductor[5] in weak electric fields. It is therefore worthwhile to calculate the quantum corrections to the nonlinear response. The present calculations were carried out for the case of noninteracting electrons being scattered by a δ-correlated random potential. The results show that the current depends in a nonanalytic manner on a weak alternating electric field[1)] and that the temporal profile of the response is highly distorted.

The "weak" localization effects are described in terms of the Keldysh nonequilibrium diagrams[7] by the contribution to the eigenenergy function $\Omega(\mathbf{x}_1 t_1, \mathbf{x}_2 t_2)$ from the series



This contribution is expressed in terms of the average of two exact Green's functions $\mathcal{G}^{RA}$ by means of

$$\Delta\Omega(\mathbf{x}_1 t_1, \mathbf{x}_2 t_2) = w^2 \int_{-\infty}^{\infty} dt_1' \int_{\infty}^{\infty} dt_2' \left\langle \mathcal{G}^R(\mathbf{x}_1 t_1, \mathbf{x}_2 t_2') \mathcal{G}^A(\mathbf{x}_1 t_1', \mathbf{x}_2 t_2) \right\rangle F(\mathbf{x}_2 t_2', \mathbf{x}_1 t_1'), \quad (1)$$

where the coefficient w^2 [$w\delta(\mathbf{x}_1 - \mathbf{x}_2)$ is the correlator of the random field] arises from the outermost dashed curves. In a uniform alternating electric field $\mathbf{E}\cos\Omega t$ we can transform this expression to (here d is the dimensionality of the problem)

$$\Delta\Omega_{\mathbf{p}}(\omega t) = \frac{w\tau^{-1}}{(2\pi)^d} \int d\mathbf{k} \int_{-\infty}^{\infty} d\omega' \int_{-\infty}^{\infty} dt' C_{\mathbf{k}}(\omega t, \omega' t') F_{\mathbf{k}-\mathbf{p}}(\omega' t'), \quad (2)$$

by introducing the classical times $t = (t_1 + t_2)/2$, $t' = (t_1' + t_2')/2$ and taking Fourier transforms in $t_1 - t_2$, $t_1' - t_2'$. The function $C_k(\omega t, \omega' t')$ is large in the region $kl < 1$ (l is the mean free path, and τ is the momentum relaxation time; $\Omega\tau < 1$), and an equation for it is found from the Bethe-Salpeter equation for $\langle \mathcal{G}^R \mathcal{G}^A \rangle$. Expression (2) describes the contribution of $F_{-\mathbf{p}}$ to the equation for $F_{\mathbf{p}}$; i.e., it incorporates the oppositely directed flux of particles which results from the quantum interference.

We can calculate the correction to the current from (2), $\Delta\mathbf{j}_t$, by solving the equation for F through iterations in $\Delta\Omega$. Using the Green's functions

$$G_{\mathbf{p}}^R(\omega, t) = G_{\mathbf{p}}^A(\omega t)^* = \left[\omega - (\mathbf{p} + \frac{e}{\Omega}\mathbf{E}\sin\Omega t)^2/2m + i/2\tau\right]^{-1}, \quad (3)$$

and integrating over ω and ω' (an energy dependence on ϵ_F remains for the highly degenerate electrons), we find

$$\Delta\mathbf{j}_t = -\sigma_B \mathbf{E} \frac{2w\tau}{(2\pi)^d} \int d\mathbf{k} \int_{-\infty}^{\infty} dt' \cos\Omega t' \, C_{\mathbf{k}}(t, t'), \quad (4)$$

where σ_B is the Born-approximation conductivity, and the field is limited by the inequality $(e/\Omega)E < p_F l^{-1}$ (in this case the contribution from the classical nonlinearity is slight). In the hydrodynamic ranges of the variables, t, $t' > \tau$, $kl < 1$, the following equation is derived for a "cooperon"[2,6] (we are neglecting small quantities of order $1/\epsilon_F\tau$):

$$\left\{\frac{1}{2}\left(\frac{\partial}{\partial t} - \frac{\partial}{\partial t'}\right) + D\left[\mathbf{k} + \frac{e}{\Omega}\mathbf{E}(\sin\Omega t + \sin\Omega t')\right]^2\right\} C_{\mathbf{k}}(t, t') = \delta(t - t'), \quad (5)$$

where D is the diffusion coefficient.

Substituting the inhomogeneous solution of this equation in (4), and integrating over $\mathbf{k}$, we find the nonlinear correction to the current to be

$$\Delta\mathbf{j}_t = -\sigma_B \mathbf{E} A_d \,\mathrm{Re}\, e^{i\Omega t} \int_{\Omega\tau}^{\Omega\tau_\varphi} \frac{dx}{x^{d/2}} \exp[-i2x - 2af(x)\sin^2(\Omega t - x)],$$

$$A_d = a_d \frac{(\Omega\tau)^{d/2-1}}{(\epsilon_F\tau)^{d-1}}, \quad a_1 = \sqrt{\frac{2}{\pi}}, \quad a_2 = \frac{1}{2\pi}, \quad a_3 = \frac{3}{16}\sqrt{\frac{3}{2\pi}}, \quad (6)$$

$$a = \frac{2D}{\Omega}\left(\frac{e}{\Omega}E\right)^2, \quad f(x) \cong \begin{cases} x & x > 1 \\ 2x^5/45, & x < 1 \end{cases}.$$

The integral at the top is cut off after a time on the order of τ_φ, which determines the phase relaxation of the wave function.[2,6]

For low frequencies ($\Omega\tau_\varphi < 1$) the customary harmonic expansion of $\Delta\mathbf{j}_t$ [in which the $(2k+1)$th harmonic is proportional to a^k] is valid only in a weak field, where $2a/45 < (\Omega\tau_\varphi)^{-5}$. For $2a/45 > (\Omega\tau_\varphi)^{-5}$ we find a dependence which is nonanalytic in the field:

$$\Delta\mathbf{j}_t \cong -\sigma_B \mathbf{E} A_d \cos\Omega t \int_{\Omega\tau}^{x(t)} \frac{dx}{x^{d/2}}, \quad x(t) \cong \begin{cases} \left(\frac{4a}{45}\sin^2\Omega t\right)^{-1/5} & \left|t - k\frac{\pi}{\Omega}\right| > \tau_p \\ (4a/45)^{-1/7}, & \Omega t = k\pi \quad k = 0, \pm 1\ldots \end{cases}. \quad (7)$$

The temporal profile of the response is distorted substantially by the integral in (7); the half-width τ_p of the peaks which arise at the extrema of $\Delta\mathbf{j}_t$ is estimated to be $\Omega\tau_p \sim (\Omega\tau_\varphi)^{-5/2}(4a/45)^{-1/2}$. Since Ω is bounded from below only by the inequality $e/\Omega E < l^{-1}$, the shape of the distortion of the response can apparently be observed directly in the rf range. In a strong field, $2a/45 > (\Omega\tau)^{-5}$, the increment $\Delta\mathbf{j}_t$ is suppressed.

In the high-frequency region, with $\Omega > \tau_\varphi^{-1}$ (but $\Omega\tau < 1$), we carry out a Fourier expansion of $\Delta\mathbf{j}_t$; the coefficient of the $(2k+1)$th harmonic is proportional to a^k only for $a < 1/\Omega\tau_\varphi$. For $1/\Omega\tau_\varphi < a < 1$, the response to the $(2k+1)$th harmonic is proportional to $a^{d/2}$, and it oscillates rapidly with a period $2|k+1|/a$. With increasing field [for $1 < (2a/45)^{1/5} < 1/\Omega\tau$] we find the asymptotic behavior

$$\Delta\mathbf{j}_t \cong -\sigma_B \mathbf{E} A_d \sum_{k=0}^{k_{max}} B_k \cos(2k+1)\Omega t,$$

$$B_0 - B_0^{eq} = \begin{cases} \ln(2a/45)^{-1/5} + \text{const}, d = 2 \\ -1.6\,(2a/45)^{1/10} + \text{const}, d = 3 \end{cases}, \quad B_k = \begin{cases} b_k + 0[(2a/45)^{-1/5}], & d = 2 \\ c_k\,(2a/45)^{1/10} + 0[(2a/45)^{-1/10}], & d = 3 \end{cases}. \quad (8)$$

The coefficients $b_k c_k$ fall off slightly with increasing harmonic index ($0.44 > b_k > 0.32$; $0.4 > c_k > 0.29$) up to $k_{max} \sim 1/2\,(2a/45)^{1/5}$. In strong fields all the harmonics are suppressed by the field in a $1/\sqrt{a}$ fashion. These asymptotic expressions are extremely crude (since the dependence on a is smooth), and the basic result is the appearance of a large number of harmonics of nearly identical amplitude. This effect may be of interest for frequency conversion in the IR range.

A numerical estimate for $D \sim 10$ cm^2/s and $\tau_\varphi \sim 10^{-9}$ shows that a narrow peak (of half-width $\Omega\tau_p \sim 0.1$) in the response at the frequency 10^8 s^{-1} arises at $E > 0.2$ V/cm. For $\Omega \sim 2 \times 10^{11}$ s^{-1}, the condition $a \sim 1$, under which several harmonics are generated, is satisfied in a field of order 19 V/cm.

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[1)] This fact was pointed out by Altshuler *et al.*,[6] who calculated the photoconductivity which results from the suppression of localization effects by an alternating field.

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