Influence of localization effects in a normal metal on the properties of SNS junction

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It is shown that quantum corrections to the conductivity of a normal metal, located between two superconductors, depend on the difference in the phases of the order parameter in the superconductors. If a voltage is applied to the SNS junction, then the current through the junction oscillates at twice the Josephson frequency. A boundary condition is proposed for the "cooperon" at the boundary between the normal metal and the superconductor.

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1. It is well known that in a normal metal with randomly situated impurities the phase coherence of the electronic wave function is conserved over distances L_φ, much greater than the mean free path l. This phenomenon does not occur in classical kinetics ($p_F l \gg 1$). However, it has recently been discovered that quantum corrections in the kinetic coefficients are entirely determined by this phase memory.[1,2] The sensitivity of the phase of the wave function to weak magnetic fields leads to anomalous magnetoresistance,[3,4] as well as to the Aaronov-Bohm effect in disordered conductors.[5,6]

In this paper, it is shown that the phase of the electronic wave function in a normal metal adjacent to a superconductor is sensitive to the phase χ of the order parameter in the superconductor $\Delta = |\Delta| e^{i\chi}$. This leads to the fact that the conductivity of an *SNS* junction is an oscillating function of the difference between the phases of the superconductors $\varphi = \chi_1 - \chi_2$ with period π. If a voltage V is applied to the *SNS* junction, then the current through the junction, in addition to a constant component, also contains a component oscillating with frequency $\omega = 4\ \mathrm{eV}/\hbar$, which is twice the Josephson frequency.

2. We shall examine a junction made from a superconductor and a normal metal at a temperature $T \ll |\Delta|$. An electron, incident from the normal metal on the boundary with the superconductor, can undergo both the usual and Andreev reflection, with which the electron is transformed into a hole with oppositely directed momentum. The amplitude of Andreev reflection is equal to $re^{i\chi}$ ($r \leqslant 1$).

The mechanism for the appearance of quantum corrections to the conductivity is qualitatively illustrated in Fig. 1. The probability for a transition of an electron from point 1 to point 2 is determined by the square of the sum of probability amplitudes, corresponding to different diffusion trajectories connecting points 1 and 2 (trajectories I, II, III in Fig. 1). For $p_F l \gg 1$, different trajectories correspond to strongly differing phases of probability amplitudes. For this reason, interference with amplitudes, corresponding to different trajectories, can be neglected. Trajectories for which points 1 and 2 coincide to within the wavelength of an electron $\lambdabar = h/p_F$ (trajectories III and IV in

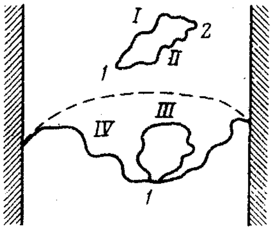

FIG. 1.

Fig. 1), are an exception. Each such trajectory can be traversed in two ways, corresponding to the different directions in which a closed loop can be circumscribed. Taking into account interference in amplitudes corresponding to these two trajectories leads to the quantum corrections to conductivity.[1,2] The return trajectories include trajectories in which the electron (continuous line in Fig. 1) undergoes Andreev reflection at the boundary with the superconductor and is transformed into a hole (dashed line in Fig. 1). Interference terms, corresponding to such trajectories, are proportional to $\cos 2\varphi (\varphi = \chi_1 - \chi_2)$. This occurs for the following reason: when the electron is transformed into a hole, the wave function acquires an additional phase $\chi_{1,2}$; when a hole is converted into an electron, it acquires the phase $-\chi_{2,1}$. The interference examined decays exponentially, when the thickness of the normal metal exceeds the diffusion phase interruption length $L_\varphi = \sqrt{D\tau_\varphi}$, where $D = \frac{1}{3} v_F l$ is the coefficient of diffusion of electrons, and τ_φ is the phase interruption time. At low temperatures, $L_\varphi \gg \xi_T = \sqrt{D\hbar/T}$ (ξ_T is the coherence length of the normal metal, at which proximity effects disappear).

3. The quantum correction to the conductivity is determined by the fanlike diagram,[2] which is illustrated in Fig. 2 taking into account the possibility of Andreev reflection. An important element of this diagram is the cooperon $C(\mathbf{r}, \mathbf{r}', t_1, t_2, t_3, t_4) = \langle \psi(\mathbf{r}, t_1)\, \psi(\mathbf{r}_2, t_2)\, \psi^+(\mathbf{r}', t_3)\, \psi^+(\mathbf{r}', t_4) \rangle$. The quantum correction to the conductivity is expressed in terms of the cooperon with the help of equation

$$\Delta\sigma = -\frac{2\sigma_0}{\pi\nu} \int_{-\infty}^{\infty} d\eta\, C^{\,t}_{-\eta\eta}(\mathbf{r}, \mathbf{r}). \tag{1}$$

Here ν is the density of states at the Fermi surface, and σ_0 is the conductivity of the metal, determined by Drude's equation. The cooperon $C^t_{\eta,\eta'}(\mathbf{r}, \mathbf{r}')\,\delta_{t,t'}$ satisfies the equation[2,3,7,8]

$$\left\{ \frac{\partial}{\partial\eta} + D\left[-i\vec{\nabla} - \frac{e}{c\hbar}\mathbf{A}\left(\mathbf{r}, t+\frac{\eta}{2}\right) - \frac{e}{c\hbar}\mathbf{A}\left(\mathbf{r}, t-\frac{\eta}{2}\right)\right]^2 + \frac{1}{\tau_\varphi} \right\} C^t_{\eta,\eta'}(\mathbf{r}, \mathbf{r}') = \frac{1}{\tau}\,\delta(\eta-\eta')\,\delta(\mathbf{r}-\mathbf{r}'). \tag{2}$$

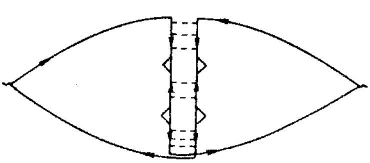

FIG. 2.

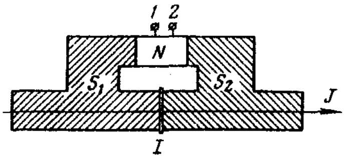

FIG. 3.

Here $\tau = l/v_F$ is the free flight time, $\eta = t_1 - t_2$, $t = \frac{1}{2}(t_1 + t_2)$, $\eta' = t_3 - t_4$, $t' = \frac{1}{2}(t_3 + t_4)$, and $\mathbf{A}(\mathbf{r}, t)$ is the vector potential of the electromagnetic field.

In the normal metal, adjacent to a superconductor, as is well known, it is necessary to use a two-component wave function $C^{\pm}$, which includes the wave functions of the electron and hole. For this reason, Eq. (2) for C^+ must be supplemented by a complex conjugate equation for the function C^-. At the boundary with the superconductor, these equations must be supplemented by the following boundary conditions:

$$\mathbf{n}\vec{\partial}\, C^+_{\eta,\eta'} = \mathbf{n}\vec{\partial}\, C^-_{\eta,\eta'} \exp\left\{i\chi\left(t + \frac{\eta}{2}\right) + i\chi\left(t - \frac{\eta}{2}\right)\right\}, \tag{3}$$

$$C^+_{\eta,\eta'} - C^-_{\eta,\eta'} \exp\left\{i\left[\chi\left(t + \frac{\eta}{2}\right) + \chi\left(t - \frac{\eta}{2}\right)\right]\right\} = \begin{cases} 0, & \text{for } r \lesssim 1 \\ \dfrac{D}{\pi v_F r^2}\, \mathbf{n}\vec{\partial}\, C^+_{\eta,\eta'}, & \text{for } r \ll 1, \end{cases} \tag{4}$$

where $\vec{\partial} = \vec{\nabla} - i(e/c\hbar)[\mathbf{A}(\mathbf{r}, t + \frac{1}{2}\eta) + \mathbf{A}(\mathbf{r}, t - \frac{1}{2}\eta)]$ and $\mathbf{n}$ is the normal to the boundary. Conditions (3) and (4) are written in analogy with the boundary conditions for diffusion equations for a two-component mixture, in which there is a finite probability for mutual transformation at the boundary. If the factor $\exp\{i[\chi(t_1) + \chi(t_2)]\}$, is dropped, then Eq. (3) describes the conservation of the total number of particles, while Eq. (4) relates the difference in the concentrations of components at the boundary to the flow of one of the components. As usual, a difference in the concentrations arises only in the case $r \ll 1$; otherwise, the concentrations at the boundary are equal. The phase factor $\exp\{i[\chi(t_1) + \chi(t_2)]\}$ is related to the change in phase of the wave function of a quasiparticle under Andreev reflection.

Equation (2) and boundary conditions (3) and (4) allow solving the problem of the effect of a junction with a superconductor on the quantum corrections to the kinetic coefficients in a normal metal. The quantum correction to the resistance of a normal interlayer with thickness L in an SNS junction is

$$\Delta R = \frac{2e^2}{\pi^2\hbar} R^2 \frac{1}{L} \sum_{Q_\parallel} \int \frac{d\mathbf{Q}_\perp}{(2\pi)^2} \left(Q_\parallel^2 + Q_\perp^2 + \frac{1}{L_\varphi^2}\right)^{-1}. \tag{5}$$

Here $Q_\parallel$ is the wave vector, which is the characteristic value of the boundary value problem for the diffusion equation with boundary conditions (3) and (4). If $Ql \sim l/L \ll r^2$, then the right side of Eq. (4) can be set equal to 0. In this case, $Q_\parallel = (1/L)(\pi n - \varphi)$, where n is an integer.

Equation (5) coincides with the expression for the quantum correction to the conductivity of a hollow cylinder depending on the magnetic flux that penetrates this cylinder[5] (the Aaronov-Bohm effect in a disordered system), if in (5) φ is replaced by the ratio of the magnetic flux to the quantum magnetic flux, while L is replaced by one-half the circumference of the cylinder. Equation (5) is especially simple when the transverse dimensions of the junction are less than L_φ:

$$\Delta R = \frac{e^2}{\pi \hbar} R^2 \frac{L_\varphi}{L} \frac{\operatorname{sh} L/L_\varphi}{\operatorname{ch} \frac{L}{L_\varphi} - \cos 2\varphi} . \tag{6}$$

The remaining limiting cases are examined in Ref. 5.

4. The effects examined above can be observed in two regimes: a) in a regime when with the help of a Josephson junction, connected in parallel to the *SNS* junction, the phase is fixed (Fig. 3) and b) in the nonstationary regime. The last regime is most simply realized when a voltage V is applied to an *SNS* junction. In this case, the phase difference φ satisfies Josephson's equation

$$\frac{d\varphi}{dt} = \frac{2eV}{\hbar} . \tag{7}$$

The current through the *SNS* junction I, which is related to V by Ohm's law $I = \sigma(\varphi)V$, oscillates with frequency $\Omega = 4\text{eV}/\hbar$, which is twice the Josephson frequency:

$$I = \left(\sigma_0 + \sigma_1 \sin \frac{4e}{\hbar} V t\right) V. \tag{8}$$

Expression (8) is valid in the adiabatic approximation $\omega = 4\text{eV}/\hbar \ll DL^{-2}$, when the electrons have time to diffuse along a trajectory, connecting the two superconductors (trajectory IV in Fig. 1), within a period of the oscillations. For large ω, the amplitude of the oscillations decreases exponentially.

5. The phenomenon examined simulates the nonstationary Josephson effect in an *SNS* junction. The difference consists of the fact that the frequency of the oscillations exceeds by a factor of 2 the Josephson frequency $2\text{eV}/\hbar$. We emphasize that the stationary Josephson effect is missing in the system that we examined.

In order to observe the effects examined above, the condition $L_\varphi \gtrsim L \gg \xi_T = \sqrt{D\hbar/T}$ must be satisfied. The phase interruption time τ_φ can be related both to inelastic processes (electron-phonon and electron-electron)[1,8] and to elastic scattering by paramagnetic impurities.[9]

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