Crystalline and liquid phases of a pion condensate

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The critical phenomena in a π-condensate phase transition of neutron matter are analyzed in the Thomas–Fermi approximation. The transition line $n = n_c(T)$ and the discontinuity in the pion field are determined. The pion lattice has a low melting point.

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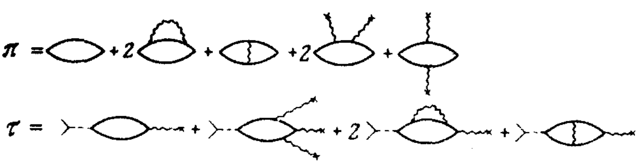
1. The proximity of nuclear matter to a π-condensate phase transition is manifested in a softening of the spin–acoustic excitations with the quantum numbers of a pion.[1] Because of the large phase volume of the fluctuations of the pion field, the transition changes from second order to first order.[2] The pronounced heating of nuclear matter which occurs in collisions of heavy ions[3] has attracted interest to the problem of thermal fluctuations.[4,5] In this letter we will examine the critical phenomena during π condensation by the Thomas-Fermi method, which has been used by Migdal[1] to study the properties of a well-developed condensate in the long-wave approximation, $k_0^2 \ll 4p_F^2$, where k_0 is the wave vector of the field. This approximation is used as a zeroth approximation, on which fluctuations will be superimposed. It is possible to derive an exact solution, i.e., to find the discontinuities in the basic thermodynamic properties on the $n = n_c(T)$ melting curve. Heating of the matter to $T \sim 1$ MeV destroys even a well-developed pion lattice. At high T, no important changes occur on the melting curve; the discontinuities in all the thermodynamic quantities are small. As the lattice melts, only the long-range order is disrupted; the short-range order remains and is manifested by a pronounced increase in the occupation numbers of the pion excitations at $k \approx k_0$, $n_k \gg 1$. In the liquid phase, the sphere $k = k_0$ is occupied uniformly by excitations, and if $\Delta \ll T$, where Δ is the characteristic pion frequency, the excitations can be treated classically. The neutrons are scattered by the field fluctuations as if they were being scattered by magnetic impurities, so that there is a pronounced decrease in the mean free path of the neutron quasiparticles. In the crystalline phase the function $n_{\mathbf{k}}$ is anisotropic, since the pions condense at six points (six in the case of a three-dimensional lattice) on the $k = k_0$ sphere. The energies of all the phases of the condensate are approximately the same, so that the observation of a phase transition in such crude experiments as the collisions of heavy nuclei seems improbable. Even if there is a pronounced increase in the density of the matter, its heating makes the liquid phase preferable.

2. We will restrict the present discussion to a neutron medium with a static interaction

$$V_{1,2} = -f^2 \frac{(\vec{\sigma}_1 \mathbf{k})(\vec{\sigma}_2 \mathbf{k})}{k^2 + m^2}.$$

For the pion distribution function $D = V/1 - V\pi$ and for the anomalous spin vertex $\tau_{\mathbf{k}} = (\vec{\sigma}\mathbf{k})\varphi_{\mathbf{k}}$ we can write the system of equations



(1)

A solid line in (1) represents the neutron Green's function G; a dashed line represents V; a wavy line represents D; and a line with an asterisk represents the field φ. Assuming that the field φ is a long-wave field, we can break up the integration into separate integrations over the nucleon and pion variables. The fermion loops in (1) then contract to a point, and their magnitude determines the effective $\pi\pi$ interaction through excitation of the neutron medium. We can therefore parametrize the D function[2]:

$$D(\mathbf{k}, \omega) = -f^2 \frac{\left(\vec{\sigma}_1 \frac{\mathbf{k}}{k_0}\right)\left(\vec{\sigma}_2 \frac{\mathbf{k}}{k_0}\right)}{\xi^2 + \gamma^2\left(\frac{k^2}{k_0^2} - 1\right)^2 - i\,\frac{|\omega|}{\epsilon_0}} \qquad \epsilon_0 = \frac{2k_0 V}{\pi} \quad \gamma^2 = \frac{k_0^2}{12 p_F^2} \quad k_0^2 = 2\sqrt{3}\,p_F\, m \, . \qquad (2)$$

The quantity ξ^2 in (2) is sensitive to changes in the density and the temperature; this quantity has the meaning of a dimensionless roton gap in the pion spectrum,

$$\xi^2 = 1 + \frac{m^2}{k_0^2} + f^2 \pi(k_0, 0)\,; \quad \xi_0^2 = \frac{n - n_c^0(T)}{3 n_c^0(0)}\,; \quad n_c^0(T) = n_c^0(0)\left(1 + \frac{\pi^2 T^2}{4\epsilon_F^2}\right), \qquad (3)$$

where ξ_0^2 is the nucleating roton gap and $n = n_c^0(T)$ is the transition line in the Thomas-Fermi approximation. In this approximation, the optimum lattice is a simple cubic lattice of the pion field:

$$\varphi(\mathbf{r}) = 2a\left(\frac{6}{7}\right)^{1/2}\frac{\epsilon_F}{k_0}\left\{\cos k_0 x + \cos k_0 y + \cos k_0 z\right\}. \tag{4}$$

The quantity a in (4) is the field amplitude; for weak fields we would have $a^2 \ll 1$. After this parametrization of D and φ, system (1) becomes a system of equations for the two quantities a^2 and ξ^2. A solution of these equations is

$$\xi^2 = \xi_0^2 + a^2 + 2\beta\xi(J(\xi^2) - 1) \qquad J(\xi^2) = \int_0^\infty \frac{\mathrm{sh}\frac{z}{2}\,dz}{\exp\left\{\frac{\xi^2\epsilon_0}{T}\,\mathrm{sh}z\right\} - 1}. \tag{5}$$
$$\xi^2 = a^2\delta^2, \quad \delta^2 = \frac{3}{28}, \quad \beta = \frac{7\epsilon_0 m}{6\epsilon_F p_F}$$

The parameter β in (5) is a small quantity of order $(k_0/2p_F)^3$; only δ^2 depends on the type of lattice. For a square lattice we would have $\delta_2^2 = \frac{5}{56}$, and for a one-dimensional lattice we would have $\delta_1^2 = \frac{1}{28}$. The free energy F can be found from (5), since the equation for a^2 determines the extremum of F as a function of a^2:

$$\frac{F - F_0}{E_0} = \xi_0^2 a^2 + \frac{(1-\delta^2)a^4}{2} - 2\beta^2\xi^2(J(\xi^2) - 1)^2 - \frac{4}{3}\beta\xi^3 - 2\beta\int_{\xi^2}^\infty J(\xi^2)\,\xi\,d\xi^2, \tag{6}$$

where F_0 is the energy of an ideal gas of neutrons, and $E_0 = (27/7)\epsilon_F n$. From (5) and (6) we can find all the characteristics of the system; the derivatives of F with respect to n and T give us the chemical potential μ, the compressibility, and the entropy S.

3. For an analysis of the limit $T = 0$, we introduce the variables $a^2 = \beta^2\varphi_0^2$, $\xi_0^2 = \beta^2\Delta_0$. In terms of these variables we find from (6)

$$\frac{F - F_0}{E_0\beta^4} = (2 + \Delta_0)\varphi_0^2 + \frac{(1-\delta^2)}{2}\varphi_0^4 - \frac{4}{3}(1 + \Delta_0 + \varphi_0^2)^{3/2} + 2\Delta_0 + \frac{4}{3}. \tag{7}$$

The expansion of (7) in powers of the small quantities Δ_0 and φ_0^2 is

$$4\frac{F - F_0}{E_0\beta^4} = \Delta_0^2\varphi_0^2 + \varphi_0^4(\Delta_0 - 2\delta^2) + \frac{1}{3}\varphi_0^6 - 2\Delta_0^2 + \frac{1}{3}\Delta_0^3. \tag{8}$$

The system is stable with respect to small variations of φ_0; since the term $\sim\varphi_0^2$ is always positive, the phase transition results from a change in the sign of the term $\sim\varphi_0^4$. This is a general property of both quantum[2] and classical[6] systems. The optimum value of φ_0 is found by taking a variation of (8) with respect to φ_0^2: $\varphi_0 = \delta + (\delta^2 - \Delta_0)^{1/2}$. The pion field arises abruptly: $a_c^2 = \delta^2\beta^2\ 6/2 + \sqrt{3}$. The discontinuity in the field is a small quantity on the order of $(k_0/2p_F)^6$ analytically and δ^2 numerically. From (5) we can find the roton gap in the liquid and crystalline phases:

$$\frac{\xi_1}{\beta} = (1 + \Delta_0)^{1/2} - 1, \quad \frac{\xi_2}{\beta} = \delta^2 + \delta(\delta^2 - \Delta_0)^{1/2}, \quad \Delta_0 = \frac{n_c^0 - n}{3\beta^2 n_c^0}.$$

The discontinuity in the roton gap is large: $\xi_2^2(n_c) = \xi_1^2(n_c)(1 + \sqrt{3})^2$. Since the parameter δ^2 is small, the pions remain soft even against the background of a well-developed lattice.

4. Analyzing the expression for F in (6), we can find the function $n_c(T)$:

$$n_c(T) = n_c(0)\left(1+\frac{T^2}{T_1^2}\right), \qquad n_c(T) = n_c(0)\left(1+\frac{T^{2/3}}{T_2^{2/3}}\right) \qquad T_1 \cong 0{,}1\,\epsilon_F\,\frac{m^2}{p_F^2}.$$

$$T < T_3 \sim \epsilon_0 \beta^2 \delta^4, \qquad T > T_3, \qquad T_2 \cong 0{,}01\,\epsilon_F\,\frac{p_F}{m}.$$

For real nuclei we would have $T_1 \sim T_2 \sim 1$ MeV and $T_3 \sim 0.1$ MeV. To see why the T_{1-3} are small, we introduce an expression for the energy of the system at $T=0$ in the supercritical region: $n > n_c$, where the liquid phase is metastable:

$$F = F_0 - \frac{3\epsilon_F (n - n_c^0)^2}{14 n_c^0 (1 - \delta^2)}. \tag{9}$$

For the liquid phase we should set $\delta^2 = 0$ in (9). Since the parameter δ^2 is small for all types of lattices, the energies of all the phases are approximately equal, and a slight heating of the matter is sufficient to stabilize the liquid phase. All the phenomena associated with the phase transition occur at values of T which are so low that we may ignore the thermal diffusion of the momentum distribution of the neutron quasiparticles and consider only the temperature of the gas of pion excitations. In the classical region, $T > T_3$, the discontinuities in all quantities depend on T in a power-law manner:

$$\Delta S \sim T^{1/3}; \quad \Delta\mu \sim T^{2/3}; \quad a_c^2 \sim T^{2/3}; \quad \Delta\xi^2 \sim T^{2/3} \quad n = n_c(T).$$

5. The properties of the liquid phase are determined by the relationship between the roton gap $\Delta = \epsilon_0 \xi^2$ and T. The value of ξ^2 is found from (5) with $a^2 = 0$. There are three regions on the (n, T) plane. In the quantum region, $\Delta \gg T$, the system is a Fermi liquid with a soft pion mode. In the classical region, $\Delta \ll T$, the pion momentum distribution has a δ-shaped peak at

$$n(k) \sim \frac{T}{\epsilon_0}\,\frac{1}{\xi^2 + \gamma^2\left(\frac{k^2}{k_0^2} - 1\right)^2}, \qquad n_\pi = n\,\frac{9\pi T m}{\epsilon_0 p_F \xi}.$$

The pion excitation density n_π increases with decreasing size of the roton gap. The quantity $C_\pi = n_\pi / n$ is the concentration of these excitations (the number of pions per nucleon). The damping of the neutron quasiparticles is expressed in terms of this quantity:

$$G^{-1}(p, \epsilon) = \epsilon - (p - p_F)V + i\gamma\frac{\epsilon}{|\epsilon|}; \quad \epsilon > \Delta \qquad \gamma = \frac{2}{3} C_\pi \epsilon_F.$$

The quantity γ^{-1} is the scale time between collisions with pion excitations. The third region on the (n, T) plane results from the cancellation of the contributions of the quantum and classical fluctuations. In this semiquantum region, $\Delta = \lambda_0 T$, we have a number $\lambda_0 \sim 1$, and this number is independent of n and T in the limit $T \to 0$, $\xi_0^4 \ll T/\epsilon_0$. In this case there are neither real nor virtual excitations in the system, and the $k = k_0$ sphere is empty.

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