Gauge theories without Faddeev–Popov ghosts

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Physical quantities can be calculated in gauge theories by using simply the "naive" Feynman rules in Lorentz-covariant gauges, without appealing to Faddeev–Popov ghosts.

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In this letter we will show that, in principle, physical quantities can be calculated in gauge theories by using the "naive" Feynman rules, without fictitious particles: the Faddeev-Popov "ghosts." For simplicity, we will discuss the theory of a pure Yang-Mills field. The standard action for this case is

$$S = -\frac{1}{4} G^2_{\mu\upsilon} - \frac{1}{2} (\partial_\mu \kappa^{\mu\upsilon} A^a_\upsilon)^2 + C^a \partial_\mu \kappa_{\mu\upsilon} \nabla^{ab}_\upsilon C^b \ .$$

We will show that in a certain calculation method the fictitious particles make a zero contribution to the Green's functions of the gauge fields and may therefore be omitted. Let us outline the guiding considerations. First, it is necessary to specify the regularization in addition to the action. The dimensional regularization of Ref. 1 is convenient and has the remarkable property that quantities of the type $\delta^{(n)}(0)$ turn out to be zero in this regularization.[2] Second, we know that in the Coulomb gauge ($\kappa_{0\mu} = 0, \kappa_{ij} = \delta_{ij}$) the contribution of fictitious particles to the Green's functions of the gauge fields is described effectively by the action $-i\delta^{(1)}(0)\mathrm{Spln}\partial_i\nabla_i$. If $\delta^{(1)}(0)$ were replaced by $\delta^{(n)}(0)$, this expression would vanish.

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We propose the following calculation rules: The dimensional regularization is used, and $\kappa_{\mu\nu}$ is continued into the n-dimensional space by the method

$$\kappa_{\mu\nu} \to \kappa^{(0)}_{\mu\nu} = \mathrm{diag}\,(\underbrace{a, \ldots, a}_{4+\epsilon},\ \underbrace{0, \ldots, 0}_{\epsilon}),$$

where $n = 4 + 2\epsilon$ (Lorentz covariance is explicitly retained in the first four dimensions). The contribution of the fictitious fields is then described by the action $-i\delta^{(\epsilon)}(0)\mathrm{Spln}\partial_{\hat{\mu}}\nabla_{\hat{\mu}}$ ($\hat{\mu}$ means that the index μ takes on the values $0, 1, \ldots, 4+\epsilon-1$) and is zero. Possibly more convenient is a slight modification of this approach in which $\kappa_{\mu\nu}$ is continued into the n-dimensional space in the following manner (with Lorentz covariance preserved in the first four dimensions):

$$\kappa_{\mu\nu} \to \kappa^{(\beta)}_{\mu\nu} = \mathrm{diag}(a, a, a, a,\ \underbrace{a\beta, \ldots, a\beta}_{2\epsilon}).$$

All the calculations are carried out for $\epsilon < 0$; then β is allowed to go to zero, and the limit $\epsilon \to 0$ is taken. In this case again the contribution of the fictitious particles to the Green's functions of the gauge fields vanishes. We can illustrate this approach for the particular case of the single-loop contribution of fictitious fields to the gauge field propagator. The corresponding expression is proportional to the integral

$$\int d^4k d^{2\epsilon}\mathbf{k} \frac{\widetilde{k}_\mu(\widetilde{k}_\nu - \widetilde{p}_\nu)}{(\mathrm{k}^2 + \beta\mathbf{k}^2)((\mathrm{k}-\mathrm{p})^2 + \beta(\mathbf{k}-\mathbf{p})^2)},$$

where $\widetilde{k}$ means that the 2ϵ components of k_μ are multiplied by β, k^2 is the square of the first four components, and $\mathbf{k}^2$ is the square of the last 2ϵ components. If we immediately set $\beta = 0$ within the integral, then we can perform a factorization $\int d^{2\epsilon}\mathbf{k} \sim \delta^{(2\epsilon)}(0)$, which should be assumed equal to zero. This technique is an illustration of the first method (the remaining integral over d^4k diverges; to keep this from happening, we set only ϵ components of $\kappa_{\mu\nu}$ equal to zero in the first method, while the other ϵ components regularize the integral over $d^{4+\epsilon}k$). With $\beta \neq 0$, we use the substitution of variables $\mathbf{k} \to \beta^{1/2}\mathbf{k}$. The Feynman integral then becomes

$$\sim \beta^{-\epsilon}\,\Pi(\mathrm{p}, \mathbf{p}, \beta, \epsilon); \tag{1}$$

Π exists and has a finite limit in the limit $\beta \to 0$ (and for ϵ not equal to an integer). Accordingly, if we take the limit $\beta \to 0$ with $\epsilon < 0$, this integral vanishes. It is not difficult to show that all the other fictitious-particle loops are of the form in (1), and their contributions can be suppressed by a similar approach.

We thus see that these proposed calculation methods make it possible to manage without using fictitious fields at all. The expressions derived for physical quantities (the S matrix and the matrix elements of the gauge-invariant operators) are the same as the results calculated by the ordinary rules. This conclusion follows from the circumstance that with $\epsilon \neq 0$ the entire modification reduces to simply a special choice of $\kappa_{\mu\nu}$, i.e., to a particular choice of gauge. The physical quantities, however, are independent of the gauge for all n.

This approach obviously also works in the general case of an arbitrary gauge theory (provided, of course, that there exists a gauge-invariant regularization, part of which is an analytic continuation in the dimensionality of the space). This approach may in fact prove more convenient for specific calculations, since the structure of the fictitious-particle Lagrangian becomes significantly more complex in gauge theories of the general type,[3-6] and in general this Lagrangian may be effectively impossible to construct. At the same time, results for physical quantities can be found by using the naive Feynman rules. This statement applies in particular to the expanded supergravity, in which the structure of the ghost Lagrangian has not yet been established.

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