# Electromagnetic radiation induced by a gravitational wave in a laser beam

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The conversion of a gravitational wave into an electromagnetic wave in a laser beam is about 25 orders of magnitude more efficient than in a static magnetic field. Two harmonics are generated: $\omega_0 \pm \omega_g$. Conditions for detecting these harmonics by ultrahigh-resolution laser spectroscopy are discussed.

The conversion of gravitational and electromagnetic waves into each other has been studied by several authors.[1–6] It has been shown in particular that a gravitational wave in a static magnetic field $H_0$ filling a volume of size $L$ induces an electromagnetic wave with an energy conversion coefficient[5]

$$\alpha = GH_0^2L^2/c^4. \tag{1}$$

Under the conditions prevailing in a laboratory on the earth we would have $\alpha \sim 10^{-35}$. This value is too low for an observation of the induced radiation. In this letter we show that the conversion of gravitational radiation is much more efficient in a laser beam.

We start from Maxwell's equations in vector form.[7] In the field of a weak gravitational wave, these equations take a form analogous to that written in Ref. 5:

$$\frac{\partial^2 \mathbf{E}}{\partial x^{02}} - \Delta \mathbf{E} = \mathbf{F}_E, \quad \frac{\partial^2 \mathbf{H}}{\partial x^{02}} - \Delta \mathbf{H} = \mathbf{F}_H, \tag{2}$$

where

$$\begin{aligned} \mathbf{F}_E &= -\frac{\partial^2(\hat{h}\mathbf{E})}{\partial x^{02}} - \frac{\partial\, \mathrm{curl}(\hat{h}\mathbf{H})}{\partial x^0} + \mathrm{grad}[\mathrm{div}(\hat{h}\mathbf{E})], \\ \mathbf{F}_H &= -\frac{\partial^2(\hat{h}\mathbf{H})}{\partial x^{02}} + \frac{\partial\, \mathrm{curl}(\hat{h}\mathbf{E})}{\partial x^0} + \mathrm{grad}[\mathrm{div}(\hat{h}\mathbf{H})]. \end{aligned} \tag{3}$$

Here $\hat{h} = (h_{\alpha\beta})$ is the metric of the transverse traceless gravitational wave.

We assume that the laser beam is propagating along the $x$ axis between mirrors 1 and 2, which are separated by a distance $L$. The gravitational radiation is propagating along the direction $\mathbf{n}_g$:

$$\mathbf{n}_g = (\sin\theta\cos\varphi,\ \sin\theta\sin\varphi,\ \cos\theta). \tag{4}$$

We seek a solution of Eqs. (3) as the sum of a reference wave and an induced wave:

$$\mathbf{E} = \mathbf{E}_0 + \tilde{\mathbf{E}}, \quad \mathbf{H} = \mathbf{H}_0 + \tilde{\mathbf{H}}. \tag{5}$$

Here we are assuming that the reference wave and the gravitational waves are both plane waves. In the frame of reference of the gravitational wave, the tensor $\hat{h}$ has two independent components: $h_{011}=-h_{022}=a$, and $h_{012}=h_{021}=b$. The gravitational radiation of all known astrophysical and man-made sources is very weak ($h\sim 10^{-19}$–$10^{-22}$) and low-frequency ($\omega_g/2\pi\sim 10^3$ Hz). The ratio of the wave vectors of the gravitational and reference waves is thus $k_g/k_0\sim 10^{-12}$. We can calculate the induced radiation by carrying out an expansion in the two small parameters $h$ and $k_g/k_0$, ignoring the terms on the order of $h^2$ and $k_g/k_0$ in $\mathbf{F}_E$ and $\mathbf{F}_H$ . As a result, for the case of a reference wave propagating along the $x$ axis and back, we find the following result for the vector $\mathbf{F}_E$:

$$\mathbf{F}_{E\pm}=\pm\mathbf{k}\frac{1}{4}k_0^2E_{0\pm}(h_{33}+h_{22})\{\exp i[(\omega_0+\omega_g)t-(\mathbf{k}_0+\mathbf{k}_g)\mathbf{r}]$$

$$+\exp i[(\omega_0-\omega_g)t-(\mathbf{k}_0-\mathbf{k}_g)\mathbf{r}]\}+\text{c.c.} \tag{6}$$

Here $(h_{33}+h_{22})=-h_{11}=-a(\cos^2\theta\cos^2\varphi-\sin^2\varphi)+b\sin 2\varphi\cos\theta$.

We seek the induced wave in the form

$$E^{\pm}=\frac{1}{2}i\mathbf{A}^{\pm}(x)\exp[i(\omega t\mp kx)]+\text{c.c.} \tag{7}$$

We assume that the amplitude $\mathbf{A}^{\pm}(x)$ is a slowly varying function: $k|A_x|\gg|A_{xx}|$. Under the matching conditions $\omega=\omega_0\pm\omega_g$, $k=k_0\pm k_g$, we find the following equation for determining $\mathbf{A}^{\pm}(x)$:

$$-(k_0\pm k_g)\frac{d\mathbf{A}^{\pm}}{dx}=\mathbf{k}\frac{1}{4}k_0^2E_{01}(h_{33}+h_{22})\exp[\pm ik_g(1\mp\sin\theta\cos\varphi)x]+\text{c.c.} \tag{8}$$

Assuming that there have been $N$ reflections from the mirrors with homogeneous initial conditions, and taking into account the reflection conditions at the mirrors, we find the following equation for the wave amplitude:

$$\mathbf{A}^{-}(0)=-\mathbf{k}\frac{k_0}{2}E_{01}NLR^{N-\frac{1}{2}}\left[\frac{\sin Z^{+}}{Z^{+}}+\frac{\sin Z^{-}}{Z^{-}}\right](h_{33}+h_{22}), \tag{9}$$

where

$$Z^{\pm}=k_gL(1\pm\sin\theta\cos\varphi).$$

A corresponding result can be found for the magnetic component of the induced field. As a result, we find the energy flux of the induced harmonics $\omega_0\pm\omega_g$:

$$W_{\text{ind}}^{\pm}=\frac{A_N^2+B_N^2}{8\pi}c=W_0W_g\frac{4\pi G}{c^5}(LN)^2R^{2N-1}\left(\frac{\omega_0}{\omega_g}\right)^2 f(\theta,\varphi). \tag{10}$$

Here $W_0$ and $W_g$ are the energy fluxes of the laser light and of the gravitational waves, respectively, given by

$$W_0=\frac{E_{01}^2+H_{01}^2}{8\pi}c,\quad W_g=\frac{c^3\omega_g^2}{8\pi G}a^2,$$

and $f(\theta,\varphi)$ is the directional pattern. In the case $a=b$, this pattern is described by

$$f(\theta,\varphi)=\frac{1}{2}(\cos^2\theta\cos^2\varphi-\sin^2\varphi-\sin 2\varphi\cos\theta)^2.$$

According to (10), the energy conversion coefficient is

$$\alpha'=\frac{W_{\mathrm{ind}}}{W_g}=\frac{G(E_{01}^2+H_{01}^2)(L2N)^2}{c^4}R^{2N}\left(\frac{\omega_0}{\omega_g}\right)^2. \tag{11}$$

For a laser power $P=SW_0=1$ W, for a beam cross-sectional area $S=\pi r^2=\pi$ ($r=1$ cm), for $L=3$ km, for $\omega/2\pi=6\times10^{14}$ Hz ($\lambda=500$ nm), for $R=0.9999$, and for $N=N_{\mathrm{opt}}=1/(1-R)$, the conversion coefficient $\alpha'$ has the value $2\times10^{-10}$ because of the large factor $(\omega_0/\omega_g)^2$. This value is 25 orders of magnitude greater than the conversion coefficient in a static magnetic field. If the gravitational wave has an amplitude $a=10^{-20}$, and if the measurement time is $t=1$ s, the power of the induced radiation is a fraction $\delta I_g/I=3.84\times10^{-12}$ of the power of the laser beam. This figure corresponds to $4.84\times10^6$ photons in each of the harmonics $\omega_0\pm\omega_g$. These harmonics are seen as slight peaks on the laser spectral line, at distances $\pm\nu_g$ from the center of the line. If the lineshape is Lorentzian, the background $\delta I_c$ against which we observe the peaks, in the band of the natural linewidth $\delta\nu$, can be estimated from

$$\frac{\delta I_c}{I}=\frac{2}{\pi}\left(\frac{\delta\nu}{2\nu_g}\right)^2. \tag{12}$$

Here $I$ is the intensity of the laser spectral line. Assuming that the peaks lying at a distance $\nu_g=10^3$ Hz from the center of the line can be observed with adequate contrast, which is specified by the condition

$$\delta I_g/I_c=1, \tag{13}$$

we find $\delta\nu\sim6$ Hz from (12) and (13). According to Refs. 8 and 9, existing lasers can be stabilized in a band $\sim1$ ns. The requirements on the laser stabilization can be relaxed by carrying out the observations at a low contrast and by using mirrors with an even larger reflection coefficient. In contrast with the plans[10] for a laser-interferometric detector of gravitational radiation in the U.S., the arrangement we have been discussing here has a single arm, and the mirrors are rigidly mounted.

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