Deficiency of the Gross–Llewellyn Smith sum rule and QCD vacuum polarization effect

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From the analysis of the recent CCFR collaboration data for the structure function $xF_3(X,Q^2)$ ($0.015<x<0.65$ and $1.2\ \mathrm{GeV}^2<|Q^2|<501\ \mathrm{GeV}^2$) of the deep inelastic neutrino–nucleon scattering we conclude that probably a part of the nucleon baryon number is due to the vacuum polarization effects.

The deep inelastic lepton–nucleon scattering processes (DIS) occurring at small distances characterize the internal structure of the elementary particles. In the past few years, new experimental data with high precision and in large kinematic region have become available.

Recently the next to next-to-leading order QCD analysis of the most precise data for the neutrino–nucleon DIS structure function $xF_3(x,Q^2)$ measured by the CCFR collaboration at the FERMILAB collider[1] has been carried out.[2] This analysis gives an estimate of the Gross–Llewellyn Smith (GLS) sum rule[3] in the wide range of the squared momentum transfer Q^2, $2\ \mathrm{GeV}^2<Q^2<500\ \mathrm{GeV}^2$,

$$\mathrm{GLS}(Q^2)=\frac{1}{2}\int_0^1 \frac{xF_3^{\bar{\nu}p+\nu p}(x,Q^2)}{x}\,dx \tag{1}$$

and reveals at the level of the statistical experimental errors the effect of the deviation[4] from the perturbative QCD prediction:

$$\mathrm{GLS}_{\mathrm{QCD}}(Q^2)=3\left[1-\frac{\alpha_s(Q^2)}{\pi}+O[\alpha_s^2(Q^2)]+O(1/Q^2)\right]. \tag{2}$$

The deficiency $\Delta\mathrm{GLS}\equiv\mathrm{GLS}_{\mathrm{QCD}}-\mathrm{GLS}_{\mathrm{exp}}$ at $Q^2=10\ \mathrm{GeV}^2$ with four active flavors and the value of the QCD parameter $\Lambda_{\overline{MS}}^{(4)}=213\pm31$ MeV is

$$\Delta\mathrm{GLS}(Q^2=10\ \ \mathrm{GeV}^2)=0.180\pm0.107(\mathrm{stat}). \tag{3}$$

This value decreases only logarithmically with the squared momentum transfer over all experimentally accessible regions up to $500\ \mathrm{GeV}^2$. We choose the reference scale at $Q^2=10\ \mathrm{GeV}^2$, where the data are statistically most valuable,[1] and where the high twist effects and the target mass corrections are negligible.[2] For this scale a large helicity and flavor asymmetries of the proton sea have recently been observed in the EMC (Ref. 5)–SMC (Ref. 6) and NMC (Ref. 7) experiments.

In the present letter we propose a mechanism which explains the possible violation of the GLS sum rule based on the nonperturbative QCD dynamics. Keeping in mind the different experimental and theoretical uncertainties in extracting the value (3), we assume this number to be the upper bound of the effect. The mechanism proposed by us is closely related to the one violating the Ellis–Jaffe and Gottfried sum rules[8] which characterize the helicity and flavor distributions in the nucleon constituents.

In the framework of the parton model the GLS sum rule for the proton structure function $F_3(x,Q^2)$ corresponds to the conservation of the baryon number, B:[9]

$$\frac{1}{3}\int_0^1[(u(x,Q^2)-\bar{u}(x,Q^2))+(d(x,Q^2)-\bar{d}(x,Q^2))]dx=B\left(1-\frac{\alpha_s(Q^2)}{\pi}\right). \tag{4}$$

The baryon charge operator in the quark model is defined by

$$\hat{B}=\frac{1}{6}\int_0^1[\{u^+(\mathbf{x}),u(\mathbf{x})\}_+ +\{d^+(\mathbf{x}),d(\mathbf{x})\}_+]d\mathbf{x} \tag{5}$$

and the baryon number is related to the low-energy spin-averaged matrix element of the isoscalar vector current $J_\mu(\mathbf{x})=\bar{u}\gamma_\mu u+\bar{d}\gamma_\mu d$ through the proton state:

$$\langle p|J_\mu(0)|p\rangle=12p_\mu B. \tag{6}$$

If the proton state $|p_0\rangle$ contains only free quarks, then the baryon number would be equal to one, $B=1$. The index 0 in $|p_0\rangle$ means that a proton (and quarks) is considered in the perturbative QCD vacuum with zero contribution from the Dirac sea quarks to the baryon number: $\langle p_0|\hat{B}^{\text{sea}}|p_0\rangle=0$.

However, the physical proton is immersed in the strong interacting medium and we see that confinement and spontaneous chiral symmetry breaking occur. As was shown by Skyrme and Witten,[10,11] this highly nonlinear QCD vacuum can carry its own baryon number:

$$B^{\text{Skyrme}}=\frac{1}{24\pi^2}\epsilon_{0\mu\lambda\rho}\int \text{Tr}\{R_\mu R_\lambda R_\rho\}d\mathbf{x}, \tag{7}$$

where $R_\mu=(\partial_\mu U)U^+$ with $U^+U=1$ is constructed from bosonic fields. This is the effect of the fermion–boson transmutation. In the Skyrme model the chiral soliton baryon charge (7) is fully compensated for by the negative baryon charge induced by sea quarks.

Later, Rho, Goldhaber, and Brown[12] and Goldstone and Jaffe[13] have suggested that the baryon number (6) of the proton, which is surrounded by the nontrivial (Skyrme) vacuum, could be distributed between the normal (canonical) quark contribution, B^{valence}, and the part anomalously induced by the vacuum polarization, B^{sea}:

$$B=B^{\text{valence}}+B^{\text{sea}}. \tag{8}$$

The latter is related to the influence of the regularization procedure on the symmetry properties of the theory and is of pure quantum origin. The classical Skyrme field serves as a tool to define this procedure. Within the chiral bag model for the physical proton state $|p\rangle$ the valence part and the sea polarization part of the baryon number, respectively, are

$$\langle p|\hat{B}^{\text{valence}}|p\rangle=1,$$
$$\langle p|\hat{B}^{\text{sea}}|p\rangle=-B^{\text{Skyrme}}. \tag{9}$$

We can write the sum rule

$$B^{\text{valence}}+B^{\text{sea}}+B^{\text{Skyrme}}=1. \tag{10}$$

Here $B^{\text{sea}}=-B^{\text{Skyrme}}$ (by definition) and B^{Skyrme} is invisible in DIS since it characterizes the property of the background vacuum field.

This interpretation of the anomalous sea quark contribution to the baryon charge is in complete agreement with the interpretation of the total angular momentum sum rule for the proton.[14] The relative angular momentum, inactive in DIS, is produced in this case to compensate for the negative helicity of the sea quarks created in the field of a strong vacuum fluctuation, instanton.[8]

In the framework of the chiral bag model,[15] where a massless Dirac quark field is confined to a finite region of space by means of a chiral boundary condition which is parametrized by a chiral angle Θ that characterizes a leakage of the baryon charge, the anomalous baryon number of the vacuum is[13]

$$\begin{aligned} &B^{\text{Skyrme}}(\Theta)=-\frac{1}{\pi}\left(\Theta-\frac{1}{2}\sin 2\Theta\right), \quad -\frac{\pi}{2}<\Theta<\frac{\pi}{2}, \\ &B^{\text{Skyrme}}(\Theta+\pi)=B^{\text{Skyrme}}(\Theta), \qquad \text{outside the interval } \left[-\frac{\pi}{2},\frac{\pi}{2}\right]. \end{aligned} \tag{11}$$

This expression is given for the boundary separating the region of intermediate and large distances, where there are soft vacuum effects, which are topologically equivalent to a sphere. The chiral boundary condition of general form

$$-i(\hat{n}rs)\ q_{L|S}=M(\Theta)q_{R|S}, \tag{12}$$

where $\hat{n}$ is the outer normal to the surface, is due to the specific condition of the confinement of quarks in the closed region. The matrix M is such that the axial vector isotriplet current conservation should be satisfied and simultaneously the flavor singlet axial current should have the anomaly. These requirements fix the form of the chiral boundary condition as an effective surface interaction of the quark fields which are confined to the hadron with the external fields from the vacuum condensate due to the instanton exchange:[16]

$$-i\boldsymbol{\gamma}\cdot\hat{n}\ q|_s=\exp[i\gamma_5\Theta(\boldsymbol{\tau}\cdot\hat{n}+1)]\ q|. \tag{13}$$

As was shown in Ref. 13, we have the following picture of the baryon charge leakage induced by the background field. The chiral angle Θ varies from zero at a very large value of the bag radius, $R\gg f_x^{-1}$, to $-\pi$ as the bag radius goes to zero. It corresponds to the change in the baryon charge, carried by the Dirac sea quarks, from zero at the chiral angle $\Theta=0$ (large R) to -1 at $\Theta=-\pi$ (small R). When Θ passes $-\pi/2$, the occupied positive quark mode undergoes an abrupt transition to a negative charge level and the baryon charge of the Dirac sea changes by -1, (11).

Now we can relate the deficiency of the GLS sum rule (3) to the anomalous vacuum baryon number

$$-\frac{1}{\pi}\left(\Theta-\frac{1}{2}\sin 2\Theta\right)=0.060\pm0.036 \tag{14}$$

and then estimate the value of the chiral angle

$$\Theta = -\frac{\pi}{4}\left(0.86\,^{+0.16}_{-0.23}\right). \tag{15}$$

The numbers (14) and (15) correspond to the upper bound of the effect.

The origin of the anomaly in the singlet vector current results from the low-energy (QCD) box anomaly:[17] $\omega \rightarrow \pi^+ \pi^0 \pi^-$. At the same time, the isovector chiral flow along the surface, which is controlled by the boundary condition (13), is zero due to the equal number of left- and right-handed chiral quarks. The pseudoscalar isosinglet coupling (13) at the surface affects the description of the flavor singlet current of the proton (proton spin)[16] and leads to the color anomaly.[18]

Finally, does the explanation given above have a particular signal in the deep inelastic scattering, e.g., x or Q^2 dependence? The answer is positive,[19] because the structure function $F_3^{\nu p}(x,Q^2)$ is defined by the vector—the axial vector correlator is specific. It is well known[9] that in this channel the Regge singularities have a negative C parity, $C=-1$. They are the ω meson exchange, with the intercept $\alpha_\omega \approx 1/2$, and the odderon, the C-odd partner of the pomeron, with the high intercept $\alpha_O \geqslant 1$: $F_3(x) = \alpha_\omega x^{-\alpha_\omega} + \alpha_O x^{-\alpha_O}$. The first one is an exchange which is related to the momentum distribution of the valence quarks. The second singularity is due to the C-odd vacuum exchange and determines the x dependence of the sea quarks. In the high-energy, elastic, hadron–hadron interactions the cross sections of the particle and antiparticle are different if the odderon trajectory exists.[20] We hope to clarify the connection between the Dirac sea and the odderon singularity contributions to the GLS sum rule in the future publications.

Thus we can interpret the possible violation of the Gross–Llewellyn Smith sum rule observed by the CCFR collaboration in the neutrino–nucleon DIS in a wide Q^2 interval as an indication of a large polarization effect in the nonperturbative QCD vacuum that surrounds the hadron. We also point out that the peculiar interaction of the constituents induced by instantons is also responsible for the large helicity and flavor asymmetry of the sea quarks in the proton wave function and the sea quark distribution functions. These and related questions are currently under investigation.

In addition, the experimental study (and theoretical understanding) of the behavior of the structure function $F_3(x,Q^2)$ in the region of small x and the calculation of the different QCD corrections at large Q^2 are necessary. The nuclear effects must also be taken into account in order to draw a conclusion about the violation of the GLS sum rule.

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