Description of optical echo without the given-field approximation

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(Submitted 8 June 1994)
Pis'ma Zh. Eksp. Teor. Fiz. **60**, No. 2, 85–88 (25 July 1994)

The occurrence of optical-echo phenomena in two-level media is analyzed without the approximation of a given field. The polarization of the medium and the intensity of the echo signal depend in a complex way on both the areas under the pulses and on the area under the "material" field and the relation between (on the one hand) the photon densities of each pulse and (on the other) the density of two-level atoms (excitons) of the medium.

The two-pulse optical echo in an inhomogeneously broadened system of two-level atoms has usually been analyzed in the approximation of a given field (see Ref. 1 and the papers cited there). The effect can be summarized as follows: Two pulses of resonant coherent laser light are applied to a medium of two-level atoms, at different times. The lengths of these pulses are shorter than the relaxation time of the medium. These pulses give rise to a polarization, which in turn induces a pulse of electromagnetic radiation, the "echo signal." The intensity of this echo signal is proportional to the square of the number of atoms in the medium. The two-pulse optical echo is seen most clearly when the area under the first pulse is $\pi/2$, and that under the second is π. The echo signal arises a time T after the end of the second pulse. This time is the time of the free evolution of the two-level medium between the two successive pulses. These features prevail in the approximation of a given field, in which the photon density of each pulse is much greater than the density of two-level atoms of the medium. There is, on the other hand, the question of just how this process would proceed under conditions such that the density of photons is on the order of or less than the density of atoms of the medium, i.e., conditions such that the depth of the temporal modulation of the pulse is equal to the amplitude of the pulse itself, and the given-field approximation breaks down.

In this letter we analyze the optical echo without using the given-field approximation, for either a medium of two-level atoms or a system of coherent excitons and biexcitons in the M-band region[2] in semiconductors with quantum wells. In such semiconductors or superlattices, an inhomogeneously broadened transition line arises because of fluctuations in the sizes of the quantum wells. Below we assume that the inhomogeneous-broadening function is a Gaussian function with a half-width Δ_0. In the approximation of envelopes which vary slowly with the time, we construct a system of nonlinear differential equations for the field amplitude $\mathcal{E}$, for the densities of excitons (n) and biexcitons (N; or for the difference between the populations for two-level atoms, $\rho = N - n$), and for the polarization components of the medium, u and v:

 0021-3640/94/020089-05$10.00

$$\dot{u}=\Delta v, \quad \dot{v}=-\Delta u-\frac{1}{2}\sigma\rho\mathscr{E},$$
$$\dot{n}=-\sigma v\mathscr{E}, \quad \dot{N}=\sigma v\mathscr{E}, \quad \dot{\rho}=2\sigma v\mathscr{E}, \quad \dot{\mathscr{E}}=-\alpha\sigma v. \tag{1}$$

Here $\alpha=2\pi\hbar\omega$, $\Delta=\omega-\omega_0$ is the detuning from the resonance between the frequency (ω) of the electromagnetic wave and the transition frequency ω_0 in the M band, which stems from optical exciton–biexciton conversion, and σ is the conversion constant.[2,3] System of equations (1) is analogous to that for two-level atoms,[1] differing only in the last equation, which determines the self-consistent time evolution of the pulse envelope $\mathscr{E}$. There is no equation for the field amplitude $\mathscr{E}$ in the given-field approximation, and the shape of the pulse envelope is given.

We assume that the incident pulses, arriving at different times, are square with amplitudes $\mathscr{E}_{01}$ and $\mathscr{E}_{02}$ and lengths τ_1 and τ_2, respectively. We denote by τ the time scale of the free evolution of the system between the first and second pulses. We assume that the times τ_1, τ_2, and τ are all much smaller than the relaxation time of the excitons and biexcitons. Depending on the relation between the photon densities f_{01} and f_{02} $\{f_{01}=\mathscr{E}_{01}^2/4\pi\hbar\omega$, $f_{02}=\mathscr{E}_{02}^2/4\pi\hbar\omega)$ of the two pulses and the density of excitons (two-level atoms), n_0, the optical-echo phenomenon can take different paths, which differ radically from the behavior that has been established for two-level atoms in the given-field approximation. We skip over the details of the calculations to the final results. In the approximation which assumes that the free-evolution time is short ($\Delta_0\tau\ll 1$) in comparison with the half-width of the inhomogeneous-broadening function, there are four types of solutions for the average polarization of the medium, $\mathscr{P}$, which is formed after the application of the two pulses.

1. Under the conditions $f_{01}>n_0$ and $f_{02}>n_0\mathrm{cn}^2(\theta_1/2)$ we find

$$\mathscr{P}=\hbar\sigma n_0\mathrm{sn}\,\frac{\theta_1}{2}\,\mathrm{cn}\,\frac{\theta_1}{2}\left\{1+\mathrm{dn}\Psi_2\left(\frac{(f_{02}/n_0)\mathrm{sn}^2\Psi_2}{(f_{02}/n_0)+\mathrm{sn}^2(\theta_1/2)}-\mathrm{cn}^2\Psi_2\right)\right.$$
$$\left.\times\left(1-\frac{\mathrm{sn}^2(\theta_1/2)\mathrm{sn}^2\Psi_2}{(f_{02}/n_0)+\mathrm{sn}^2(\theta_1/2)}\right)^{-2}\right\}e^{-1/4\Delta_0^2(T-\tau)^2}\sin\omega t, \tag{2}$$

where

$$k_1^2=n_0/f_{01}, \quad k_2^2=\left(\frac{f_{02}}{n_0}+\mathrm{sn}^2\frac{\theta_1}{2}\right)^{-1}, \quad \theta_1=\sigma\mathscr{E}_{01}\tau_1,$$
$$\theta_2=\sigma\mathscr{E}_{02}\tau_2, \quad \Psi_2=\frac{1}{2}\theta_2\left(1+\frac{n_0}{f_{02}}\mathrm{sn}^2\frac{\theta_1}{2}\right)^{1/2}.$$

Here k_1 and k_2 are the moduli of elliptic functions with the variables θ_1 and Ψ_2, respectively; θ_1 and θ_2 are the areas under the pulses; and T is the time at which the echo signal is observed after the end of the second pulse. In the limit $f_{01}, f_{02}\gg n_0$, we obtain the expression derived for the polarization in the given-field approximation.[1] It can be seen from (2) that the polarization of the medium (and the intensity of the echo signal) are determined not only by the areas under the pulses, θ_1 and θ_2, but also by the pulse amplitudes and the density of excitons (two-level atoms). Furthermore, the maximum polarization does not correspond to the areas $\theta_1=\pi/2$ and $\theta_2=\pi$. The factor that determines the effect of the second pulse on the polarization of the medium contains elliptic functions of argument Ψ_2, which is expressed not only in terms of the area under the second pulse (as in the given-field approximation) but also in terms of the area under the first pulse and the photon density f_{02} and the exciton density n_0. The reason is that by the time at which the second pulse arrives, the parameters of the medium are determined by the parameters of the first pulse, because of a "memory." These first-pulse parameters in turn determine the subsequent evolution of the polarization of the medium as the next laser pulse is applied to it.

2. Under the conditions $f_{01}>n_0$ and $f_{02}<n_0\mathrm{cn}^2(\theta_1/2)$ we find

$$\mathscr{P}=\hbar\sigma n_0 \mathrm{sn}\frac{\theta_1}{2}\,\mathrm{cn}\frac{\theta_1}{2}$$

$$\times\left\{1+\mathrm{cn}\varphi_2\frac{[(2f_{02}/n_0+\mathrm{sn}^2(\theta_1/2)]\ \mathrm{sn}^2\varphi_2-1}{[1-\mathrm{sn}^2(\theta_1/2)\mathrm{sn}^2\varphi_2]^2}\right\}e^{-1/4\Delta_0^2(T-\tau)^2}\sin\omega t, \qquad (3)$$

where

$$k_1^2=n_0/f_{01},\quad k_2^2=\frac{f_{02}}{n_0}+\mathrm{sn}^2\frac{\theta_1}{2}2,\quad \theta_1=\sigma\mathscr{E}_{01}\tau_1,\quad \varphi_2=\sqrt{\alpha n_0/2}\sigma\tau_2.$$

In this case the polarization of the medium is expressed in terms of the area under the first pulse, θ_1, and is independent of the area θ_2. Instead of the area under the second pulse in terms of the amplitude of the wave electric field, $\mathscr{E}_{02}$, the polarization is expressed as a function of a certain "area" φ_2 of a "material field," which is proportional to the square root of the exciton density n_0. Introducing the area of the exciton-wave field (or the area of the material field, in the case of a system of two-level atoms) by means of the expression

$$\theta_{\mathrm{ex}}=\sigma\int_0^{\tau_2}\mathscr{E}_{\mathrm{ex}}(t)dt,\quad \mathscr{E}_{\mathrm{ex}}^2=4\pi\hbar\omega n_0,$$

we find $\varphi_2=1/2\sigma\mathscr{E}_{\mathrm{ex}}\tau_2=1/2\theta_{\mathrm{ex}}$. Here, as in case 1, the polarization is determined not only by the areas under the first pulse and the material field but also by the field amplitudes $\mathscr{E}_{01}$ and $\mathscr{E}_{02}$ and the exciton density n_0 . An important point here is that in this case the polarization of the medium and the intensity of the echo signal become independent of the area under the second pulse, but it becomes dependent on a certain area which is proportional to the product of the amplitude of the exciton-wave field and the length of the second laser pulse. Under the condition $f_{02}=n_0\mathrm{cn}^2(\theta_1/2)$, we find from (2) and (3) that $\mathscr{P}$ depends on the area under the second pulse through hyperbolic functions.

3. Under the conditions $f_{01}<n_0$ and $f_{02}>n_0\mathrm{dn}^2\varphi_1$ we find

$$\mathscr{P}=\hbar\sigma\sqrt{n_0f_{01}}\mathrm{sn}\varphi_1\mathrm{dn}\varphi_1\left\{1+\mathrm{dn}\Psi_2\left(\frac{(f_{02}/n_0)\mathrm{sn}^2\Psi_2}{(f_{02}/n_0)+k_1^2\mathrm{sn}^2\varphi_1}-\mathrm{cn}^2\Psi_2\right)\right.$$

$$\left.\times\left(1-\frac{k_1^2\mathrm{sn}^2\varphi_1\mathrm{sn}^2\Psi_2}{(f_{02}/n_0)+k_1^2\mathrm{sn}^2\varphi_1}\right)^{-2}\right\}e^{-1/4\Delta_0^2(T-\tau)^2}\sin\omega t, \qquad (4)$$

where

$$k_1^2=\frac{f_{01}}{n_0}, \quad k_2^2=\left(\frac{f_{02}}{n_0}+k_1^2\mathrm{sn}^2\varphi_1\right)^{-1}, \quad \varphi_1=\sqrt{\alpha n_0/2}\sigma\tau_1=\frac{1}{2}\,\sigma\mathcal{E}_{ex}\tau_1=\frac{1}{2}\,\theta_{ex},$$

$$\Psi_2=\frac{1}{2}\,\theta_2\left(1+\frac{f_{01}}{f_{02}}\mathrm{sn}^2\varphi_1\right)^{1/2}, \quad \theta_2=\sigma\mathcal{E}_{02}\tau_2.$$

In this case the polarization does not depend on the area under the first pulse. In its place we find the area of the material field, φ_1 (which is proportional to the product of the amplitude of the exciton-wave field and the length of the second pulse). The effect of the second pulse is expressed through its area θ_2 and through the ratio of the photon densities of the two pulses.

4. Under the conditions $f_{01}<n_0$ and $f_{02}<n_0\mathrm{dn}^2\varphi_1$ we find

$$\mathcal{P}=\hbar\sigma\sqrt{n_0f_{01}}\mathrm{sn}\varphi_1\mathrm{dn}\varphi_1\left\{1+\mathrm{dn}\varphi_2\left(\frac{(f_{02}/n_0)\mathrm{sn}^2\varphi_2}{f_{02}/n_0+k_1^2\mathrm{sn}^2\varphi_1}-\mathrm{cn}^2\varphi_2\right)\right.$$
$$\left.\times\left(1-\frac{k_1^2\mathrm{sn}^2\varphi_1\mathrm{sn}^2\varphi_2}{f_{02}/n_0+k_1^2\mathrm{sn}^2\varphi_1}\right)^{-2}\right\}e^{-1/4\Delta_0^2(T-\tau)^2}\sin\omega t, \tag{5}$$

where

$$k_1^2=f_{01}/n_0, \quad k_2^2=\frac{f_{02}}{n_0}+k_1^2\mathrm{sn}^2\varphi_1,$$

$$\varphi_1=\sqrt{\alpha n_0/2}\sigma\tau_1=\frac{1}{2}\,\sigma\mathcal{E}_{ex}\,\tau_1, \quad \varphi_2=\sqrt{\alpha n_0/2}\sigma\tau_2=\frac{1}{2}\,\sigma\mathcal{E}_{ex}\tau_2.$$

It follows from (5) that the polarization of the medium does not depend on the areas under the two pulses. It is determined exclusively by the areas of the material field, φ_1 and φ_2, which are proportional to the product of the exciton-wave amplitude and the length of the corresponding pulse of the field of the incident wave. These areas are also proportional to the photon and exciton densities.

In cases 1 and 2, the polarization of the medium is proportional to the density of excitons (or of two-levels atoms), n_0, while the intensity of the echo signal is proportional to n_0^2. In cases 3 and 4, the polarization is proportional to $\sqrt{n_0f_{01}}\sim\mathcal{E}_{01}\sqrt{n_0}$, and the intensity of the echo signal is proportional to $n_0f_{01}\sim n_0\mathcal{E}_{01}^2$. This is an extremely important circumstance for an experimental study of optical echoes in cases in which the density of two-level atoms is higher than the density of photons of the first pulse.

In summary, if we abandon the given-field approximation, the polarization of the medium and the intensity of the echo signal are governed not only by the areas under the pulses but also by the field amplitudes $\mathcal{E}_{01}$ and $\mathcal{E}_{02}$ and the exciton density. Alternatively, they may be completely independent of the areas under the pulses but determined by the areas of the material field. There is also a change in the behavior of the intensity of the echo signal. With regard to the time evolution, we note that in the limit $\Delta_0\tau\ll 1$ the echo pulse arises at a time T after the end of the second pulse. The time T is equal to the time scale of the free evolution of the system between the two pulses.

[1] É. A. Manykin and V. V. Samartsev, *Optical Echo Spectroscopy* [in Russian] (Nauka, Moscow, 1984).
[2] P. I. Khadzhi, *Kinetics of the Recombination Radiation of Excitons and Biexcitons in Semiconductors* [in Russian] (Shtiintsa, Kishinev, 1977).
[3] P. I. Khadzhi, *Nonlinear Optical Processes in a System of Excitons and Biexcitons in a Semiconductor* [in Russian] (Shtiintsa, Kishinev, 1985).

Translated by D. Parsons