The sign of the intrinsic Hall effect in clean superconductors

N. B. Kopnin

L. D. Landau Institute for Theoretical Physics, Russian Academy of Sciences, Moscow, Russia

(Submitted 21 June 1994)
Pis'ma Zh. Eksp. Teor. Fiz. **60**, No. 2, 123–127 (25 July 1994)

The ohmic and Hall flux flow conductivities are calculated at low temperatures for a general electronic spectrum in the normal state. The sign of the Hall effect is predicted to depend on the shape of the Fermi surface.

1. Introduction

The electric field in the mixed state is associated with a flux flow, $\mathbf{E}=[\mathbf{B}\times\mathbf{u}]/c$. If the vortex velocity $\mathbf{u}$ is directed not at right angles to the transport supercurrent, a Hall voltage appears, in addition to the dissipative component. The Hall effect shows a very unusual behavior: In some cases its sign reverses as a result of the transition from the normal state to the superconducting state. There are reasons to believe that this behavior is peculiar to the flux motion in type-II superconductors (see Ref. 1 and the references cited there). This assumption is corroborated by the fact that the sign reversal was observed for ordinary V and Nb superconductors.[2,3]

The quasiclassical kinetic equations[4] for clean superconductors, $l \gg \xi$, which can describe the Hall effect in the mixed state, were recently derived. Interest in clean superconductors is based on the recent experiments in the superclean regime,[5,6] where a large Hall angle and a high vortex viscosity at low temperatures were observed.

In the present paper we use the new kinetic equations to calculate the ohmic and Hall flux flow conductivities at low temperatures, $\Delta^2/E_F \ll T \ll T_c$, employing the same physical ideas as those in Refs. 7–9. Being interested in the intrinsic aspects of the flux flow, we ignore the pinning effects and assume a free flow of vortices. We consider a general electronic spectrum and investigate the sign of the Hall effect, depending on the shape of the Fermi surface of the normal metal. The only assumption on the Fermi surface is that it is isotropic in the crystal (ab) plane, which is relevant to the high-T_c materials with a uniaxial anisotropy. We assume the most symmetric alignment of the magnetic field along the c axis with the current flowing in the (ab) plane. We consider isolated vortices in the limit of low fields $H \ll H_{c2}$.

2. Kinetic equations

The vortex velocity $\mathbf{u}$ and the average electric field are determined by the relaxation of perturbations produced by the moving vortices. One of the kinetic equations for clean superconductors derived in Ref. 4 is

$$\left(e\mathbf{v}_F\mathbf{E}g_- + \frac{1}{2}\left[f_- \frac{\hat{\partial}\Delta^*}{\partial t} + f_-^+ \frac{\hat{\partial}\Delta}{\partial t}\right]\right) \frac{\partial f^{(0)}}{\partial \epsilon} + \mathbf{v}_F\nabla(f_2 g_-)$$

$$-\left(\frac{1}{2}\left[(\hat{\nabla}\Delta)f_-^+ + (\hat{\nabla}\Delta^*)f_-\right] - \frac{e}{c}[\mathbf{v}_F\times\mathbf{H}]g_-\right) \frac{\partial f_1}{\partial p} = J_1. \quad (1)$$

The nonequilibrium corrections to the distribution functions f_1 and f_2 are, respectively, odd and even in ϵ and $\mathbf{p}$. The operators $\hat{\partial}/\partial t = \partial/\partial t \pm 2ie\varphi$ and $\hat{\nabla} = \nabla \mp (2ie/c)\mathbf{A}$, when they act on Δ (upper sign) or on Δ^* (lower sign). The collision integral is $J_1 = J_1^{(1)}\{f_1\} + J_1^{(2)}\{f_2\}$; for simplicity, we assume an isotropic scattering by impurities, which is characterized by the mean free time τ. We set $g_\pm = \frac{1}{2}(g_\epsilon^R \pm g_\epsilon^A)$, etc., where $g^{R(A)}$ are the regular retarded (advanced) quasiclassical Green's functions.

The new kinetic equations differ from the usual quasiclassical equations[10] in the terms with $\partial f/\partial \mathbf{p}$ multiplied by "forces" that include the Lorentz force $(e/c)[\mathbf{v}_F \times \mathbf{H}]$ and the forces due to the spatial variations of the order parameter. The force terms are generalizations of the result of Ref. 11, which was derived for slow spatial variations of the order parameter.

Equation (1) can be solved if the regular functions are given. For low temperatures, $T \ll \Delta_\infty$, we can use the solution obtained in Ref. 12. Here we used that result. We choose the coordinate z axis along the magnetic field and assume that the vortex has a positive circulation around the z axis, which corresponds to the positive charge of the carriers. Let $\mathbf{v}_\perp$ be the projection of the quasiparticle velocity $\mathbf{v}_F$ onto the (x,y) plane. The vector $\mathbf{v}_\perp$ makes an angle α with the x axis. If ρ and ϕ are the distance and the azimuthal angle in the cylindrical frame, then the impact parameter of a quasiparticle moving with $\mathbf{v}_\perp$ through the point (ρ,ϕ) will be $b = \rho\sin(\phi - \alpha)$. The distance along the trajectory is $l = \rho\cos(\phi - \alpha)$, so that $\rho^2 = b^2 + l^2$. In the leading approximation in ϵ/Δ_∞ we have

$$g_- = \frac{\pi v_\perp}{2C(v_\perp)} \exp[-K(\rho)]\delta[\epsilon - \epsilon(b)], \quad (2)$$

and

$$f_- e^{-i\phi} = -f_-^+ e^{i\phi} = \frac{i\pi v_\perp}{2C(v_\perp)} \cos(\phi - \alpha)e^{-K(\rho)}\delta[\epsilon - \epsilon(b)]. \quad (3)$$

Here

$$K(\rho) = \frac{2}{v_\perp}\int_0^\rho \Delta_0 d\rho', \quad C(v_\perp) = \int_0^\infty \exp[-K(\rho)]d\rho. \quad (4)$$

The bound state energy is[13]

$$\epsilon(b) = bC^{-1}(v_\perp)\int_0^\infty \frac{\Delta_0(\rho)}{\rho} \exp[-K(\rho)]d\rho. \quad (5)$$

The impact parameter $b = -n/p_\perp$, where n is the angular momentum of quasiparticles, and $p_\perp$ is the momentum projection onto the (xy) plane. Therefore, the distance between the energy levels with angular momenta n and $n \pm 1$ is $\omega_0 = (1/p_\perp)(\partial\epsilon/\partial b)$.

Using the second kinetic equation (omitted for brevity), we can show that the function f_1 is constant on the quasiparticle trajectory.[14] Integrating Eq. (1) along the trajectory, we obtain

$$([\mathbf{v}_\perp \times \mathbf{u}]\hat{z}) \frac{\partial f^{(0)}}{\partial \epsilon} \frac{\partial \epsilon(b)}{\partial b} \pi\delta[\epsilon-\epsilon(b)] + \left(\left[\mathbf{v}_\perp \times \frac{\partial f_1}{\partial \mathbf{p}_\perp}\right]\hat{\mathbf{z}}\right)$$

$$\times \frac{\partial \epsilon(b)}{\partial b} \pi\delta[\epsilon-\epsilon(b)] + \int_{-\infty}^{\infty} J_1^{(1)} dl = 0. \tag{6}$$

The collision integral is

$$J_1^{(1)} = -\frac{1}{\tau}\left(f_1\langle g_-\rangle g_- + \frac{1}{2}\langle f_1 f_-^+\rangle f_- + \frac{1}{2}\langle f_1 f_-\rangle f_-^+\right), \tag{7}$$

where

$$\langle g\rangle = \frac{1}{\nu(0)} \int \frac{dS_F}{(2\pi)^3 v_F} g(\mathbf{p}_F, \mathbf{r}). \tag{8}$$

The integration is carried out over the Fermi surface, and $\nu(0)$ is the average density of states.

Equation (6) can be solved only after certain simplifications of the collision integral. In Refs. 7 and 14, the model proposed in Ref. 12 was used. According to this model, the vortex core shrinks at low temperatures to $\xi_0 T/\Delta_\infty$. As a result, the dependence of $\epsilon(b)$ on $v_\perp$ reduces to a simple form, and one can calculate the averages in the collision integral for a spherical Fermi surface. However, this model is an oversimplification of the real order parameter behavior. Furthermore, it does not help much if the Fermi surface is not spherical.

To avoid integrals of unknown functions, we use here another approximation which is equivalent to a renormalization of τ: Instead of the last two equal terms in the true collision integral [Eq. (7)], we use the first term twice. In the moderately clean regime, both models give the same ohmic conductivity. We now have

$$\int_{-\infty}^{\infty} J_1^{(1)} dl = -\frac{\pi v_\perp \ln(\Delta_\infty/T)}{\tau C(v_\perp)} \left\langle \frac{v_\perp}{C(v_F)(\partial\epsilon/\partial b)} \right\rangle \delta[\epsilon-\epsilon(b)] f_1. \tag{9}$$

The logarithmic divergence in the integral for small ρ is cut off at distances $\rho \sim b \sim \xi(T/\Delta_\infty)$.

For the Fermi surface isotropic in the (xy) plane we have

$$\left(\left[\mathbf{v}_\perp \times \frac{\partial f_1}{\partial \mathbf{p}_\perp}\right]\hat{z}\right) = \pm \frac{v_\perp}{p_\perp} \frac{\partial f_1}{\partial \alpha}. \tag{10}$$

Here the plus sign is for quasielectrons and the minus sign is for quasiholes, since the directions of $\mathbf{v}_\perp$ and $\mathbf{p}_\perp$ are either the same or opposite for these two types of quasiparticles, respectively.

The kinetic equation (6) gives

$$f_1 = -\frac{\partial f^{(0)}}{\partial \epsilon}\left[\frac{p_\perp \gamma H}{v_\perp}(\mathbf{u}\mathbf{v}_\perp) + \frac{p_\perp \gamma_0}{v_\perp}([\mathbf{u}\times\mathbf{v}_\perp]\hat{z})\right], \quad (11)$$

where

$$\gamma_H = \pm \frac{(\omega_0 \tau_{\rm eff})^2}{1+(\omega_0\tau_{\rm eff})^2}, \quad \gamma_0 = \frac{\omega_0 \tau_{\rm eff}}{1+(\omega_0\tau_{\rm eff})^2}. \quad (12)$$

The "effective scattering time" is $\tau_{\rm eff} = \tau\beta(v_\perp)$, where

$$\beta(v_\perp) = \frac{C(v_\perp)}{\ln(\Delta_\infty/T)\langle v_\perp [C(v_\perp)(\partial\epsilon/\partial b)]^{-1}\rangle} \quad (13)$$

is on the order of unity (aside from being a possible logarithm).

The transport current can be calculated using the general scheme described, for example, in Ref. 10. Using the distribution function of Eq. (11), we have

$$\mathbf{j}_{\rm tr} = e\int \frac{dS_F}{(2\pi)^3 v_F} v_\perp p_\perp(\gamma_H \mathbf{u} + \gamma_0[\hat{\mathbf{z}}\times\mathbf{u}]). \quad (14)$$

Since $\mathbf{u} = c[\mathbf{E}\times\hat{\mathbf{z}}]/B$, the flux flow conductivities are

$$\sigma_f^{(0)} = \frac{2|e|}{(2\pi)^3}\left[\int_{\rm q.e.} S(p_z)|\gamma_0|dp_z + \int_{\rm q.h.} S(p_z)|\gamma_0|dp_z\right]\frac{c}{B}, \quad (15)$$

$$\sigma_f^{(H)} = \frac{2e}{(2\pi)^3}\left[\int_{\rm q.e.} S(p_z)|\gamma_H|dp_z - \int_{\rm q.h.} S(p_z)|\gamma_H|dp_z\right]\frac{c}{B}. \quad (16)$$

The modulus of the charge appears in Eq. (15), since the circulation of the vortex was chosen along the positive z axis (along the magnetic field) for the positive charge of the carriers. In the opposite case, both γ_0 and $\partial\epsilon/\partial b$ change their signs, together with the vortex circulation. Equations (15) and (16) are generalizations of the result of Ref. 7 to the case of a nonparabolic spectrum of quasiparticles.

3. Discussion

In the superclean limit $l \gg \xi_0 E_F/\Delta_\infty$ or $\omega_0\tau \gg 1$ the Hall coefficient is $|\gamma_H| = 1$, and the dissipative part γ_0 vanishes as $(\omega_0\tau_{\rm eff})^{-1}$. The Hall angle, which is defined by $\tan\theta_H = \sigma_f^{(H)}/\sigma_f^{(0)} \sim \omega_0\tau$, is equal to $\pi/2$. The Hall conductivity is $\sigma_f^{(H)} = (N_e - N_h)ec/B$, just as for the normal metal in high magnetic fields. The transport current is $\mathbf{j}_{\rm tr} = (N_e - N_h)e\mathbf{u}$, where $N = 2V/(2\pi)^3$, and V_e and V_h are the volumes of the electron-like and hole-like parts of the Fermi surface, respectively.

The electron-phonon relaxation can also play a role if the impurity content is low enough. This mechanism can become more effective in the superclean regime, since for high-T_c superconductors the electron–phonon relaxation is substantial. It is possible that the increase in the Hall angle toward the superclean limit observed in Ref. 6 is due to the increase in $\tau_{\rm ph}$ with decreasing temperature.

In the moderately clean case we have $\gamma_0=\omega_0\tau_{\rm eff}\ll 1$. In the model of Ref. 12 for the vortex core structure, the ohmic conductivity is $\sigma_f^{(0)}=\nu(0)|e|\cdot\tau\Delta_\infty^2\ln(\Delta_\infty/T)c/B$, and it reproduces the results of Refs. 7 and 14.

In the limit $\omega_0\tau\ll 1$ we have $|\gamma_H|=(\omega_0\tau_{\rm eff})^2$. The Hall angle is small $\theta_H\sim\omega_0\tau$. The sign of the flux flow Hall conductivity [Eq. (16)] can be compared with the normal-state Hall conductivity in low fields, $\omega_c\tau\ll 1$. For the model with an isotropic scattering by impurities we have[15]

$$\sigma_n^{(H)}=\frac{He^3\tau^2}{(2\pi)^2c}\left[\int_{\rm q.e.}v_\perp^2dp_z-\int_{\rm q.h.}v_\perp^2dp_z\right]. \tag{17}$$

We see that the signs of the Hall conductivities in the normal and the mixed superconducting states may be different, since the functions under the integrals in Eqs. (16) and (17) have different momentum dependences. Therefore, the sign of the Hall effect can change after the transition from the normal to the superconducting state. The sign reversal depends on the shape of the Fermi surface; it is absent for a simple parabolic spectrum, $\epsilon_p=p^2/2m^*$, when the flux flow and the normal-state Hall conductivities have the same sign.[7] This conclusion differs from the models of Refs. 16 and 17 which predict a mixed-state Hall effect of the same sign as that in the normal state. The latter theories assume the quasiparticles in the vortex core to be the same as in the normal state. However, the localized quasiparticles have a different energy spectrum and their behavior therefore differs from that in the normal state.

The sign of the Hall conductivity of clean superconductors at low temperatures can be compared with the results of Refs. 18 and 19 for gapless superconductors in the dirty limit $l\ll\xi_0$. For the latter case the sign of the Hall effect depends on the average energy derivative of the density of states at the Fermi surface which, in principle, may be different from that in Eqs. (16) or (17). Therefore, the sign reversal depends on the purity of the sample and on the shape of the Fermi surface. The transition between presence or absence of the sign reversal for the same doping level occurs in the range $l/\xi_0\sim 1$.

In summary, we found that (1) in the superclean limit, $l\gg\xi E_F/\Delta$, the Hall conductivity at low temperatures is the same as that for high fields in the normal state. (2) In the moderately clean case, $\xi\ll l\ll\xi E_F/\Delta$, the sign of the Hall effect in the mixed state at low temperatures may differ from that in the normal state, depending on the particular structure of the Fermi surface. (3) The sign of a clean superconductor may differ from that of a dirty compound in the gapless regime. This result indicates that the sign reversal may depend on the impurity content in the region $l\sim\xi$. The sign reversal is therefore very sensitive to the Fermi surface structure, to the doping level, and to the impurity content.[1,20]

I am grateful to M. Feigelman and V. Vinokur for many useful discussions. This work was supported by the Russian Fund for Fundamental Research through Grant No. 94-02-03121-a.

[1] S. J. Hagen, A. W. Smith, M. Rajeswari *et al.*, Phys. Rev. B **47**, 1064 (1993).
[2] A. K. Niessen, F. A. Staas, and C. H. Weijsenfeld, Phys. Lett. **25** A, 33 (1967).
[3] K. Noto, S. Shinzawa, and Y. Muto, Solid State Commun. **18**, 1081 (1976).
[4] N. B. Kopnin, J. Low Temp. Phys. (to be published).

[5] Y. Matsuda, N. P. Ong, Y. F. Yan *et al.*, Phys. Rev. B **49**, 4380 (1994).
[6] J. M. Harris, Y. F. Yan, O. K. C. Tsui *et al.*, Phys. Rev. Lett. (to be published).
[7] N. B. Kopnin and V. E. Kravtsov, JETP Lett. **23**, 578 (1976).
[8] N. B. Kopnin and M. M. Salomaa, Phys. Rev. B **44**, 9667 (1991).
[9] N. B. Kopnin, Phys. Rev. B **47**, 14354 (1993).
[10] A. I. Larkin and Yu. N. Ovchinnikov, in *Nonequilibrium Superconductivity*, ed. by D. N. Langenberg and A. I. Larkin (Elsevier Science Publishers, 1986), p. 493.
[11] A. G. Aronov, Yu. M. Galperin, V. L. Gurevich, and V. I. Kozub, Adv. in Phys. **30**, 539 (1981).
[12] L. Kramer and W. Pesch, Z. Phys. **269**, 59 (1974).
[13] C. Caroli, P. G. de Gennes, and J. Matricon, Phys. Lett. **9**, 307 (1964).
[14] A. I. Larkin and Yu. N. Ovchinnikov, JETP Lett. **23**, 187 (1976).
[15] A. A. Abrikosov, *Principles of the Theory of Metals*, Nauka, Moscow, 1987.
[16] J. Bardeen and M. J. Stephen, Phys. Rev. **140** A, 1197 (1965).
[17] P. Nozières and W. F. Vinen, Philos. Mag. **14**, 667 (1966).
[18] A. T. Dorsey, Phys. Rev. B **46**, 8376 (1992).
[19] N. B. Kopnin, B. I. Ivlev, and V. A. Kalatsky, JETP Lett. **55**, 750 (1992); J. Low Temp. Phys. **90**, 1 (1993).
[20] M. V. Feigel'man, V. B. Geshkenbein, A. I. Larkin, and V. M. Vinokur, in *Proc. of the IVth Int. Conf. on Materials and Mechanisms of Superconductivity. High-Temperature Superconductors,* M^2S-HTSC-IV. Grenoble, France. July 1994 (to be published).

Published in English in the original Russian journal. Reproduced here with stylistic changes by the Translation Editor.