Mimicry of phase transitions and the large-river effect

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(Submitted 11 May 1994)

Pis'ma Zh. Eksp. Teor. Fiz. **60**, No. 2, 133–137 (25 July 1994)

A recently proposed mechanism of a "minicry" of second-order phase transitions by fluctuation-induced first-order phase transitions is discussed. This mechanism is associated with the passage of phase trajectories of the renormalization group through a flow minimum. The validity of applying the exact renormalization-group equation to the problem and also the existence of the necessary flow minimum are confirmed through the use of the "large-river effect," which is characteristic of nonlinear relaxation equations.

Zumbach[1] recently suggested a mimicry of second-order phase transitions by fluctuation-induced first-order phase transitions. In a nutshell, if the renormalization-group (RG) equation has no stable fixed point, or if the seed values of the parameters lie beyond the separatrices of the region of attraction to this point, the RG phase trajectories intersect the boundaries of the region in which a fourth-order form of the free-energy functional is positive. This circumstance is ordinarily interpreted as the induction of a first-order phase transition in the system by fluctuations. However, Zumbach[1] pointed out that the RG trajectories might pass through a region of a minimum of the rate of evolution of the parameters (a minimum of the flow). The behavior of physical quantities near the minimum is qualitatively analogous to that in the course of a second-order phase transition. The latter point is offered as an extremely plausible reason for the discrepancy between experiment and the theory which predicts a first-order phase transition for several systems and also the discrepancy between the results of different experiments.

The application of the φ^4 model to phase-transition theory gives rise to several fundamental problems. To avoid them, Zumbach[1] used the exact RG equation[2] in its local version, which is usually sufficient for a discussion of qualitative predictions of the theory:[3–5]

$$\dot{f}=\hat{R}f=df+\sum_i\left[f_{\varphi_i\varphi_i}-\frac{d-2}{2}f_{\varphi_i}\varphi_i-f^2_{\varphi_i}\right]. \qquad (1)$$

Here $f\{\varphi_i\}$ is the local density of the free-energy functional

$$\mathscr{H}[\varphi]=\int d^dr[\Sigma_i(\bar{\nabla}_{\varphi_i})^2+f\{\varphi_i\}], \qquad (2)$$

and the summation is over the n different components of the vector $\boldsymbol{\varphi}=\{\varphi_i\}$. Equation (1) was rewritten in Ref. 1 in the form

$$\dot{\mu}=\hat{R}\mu=d\mu\ln(\mu)+\sum_i\left[\mu_{\varphi_i\varphi_i}-\frac{d-2}{2}\mu_{\varphi_i}\varphi_i\right], \qquad (3)$$

where $\mu=\exp(-f)$. That form is more convenient in several cases. The flow intensity can be characterized by the norm $\|\dot{\mu}\|=\langle\dot{\mu}|\dot{\mu}\rangle^{1/2}$. The scalar product $\langle\psi|\xi\rangle=\int d^n\varphi\rho(\varphi)\psi(\varphi)\xi(\varphi)$, which contains the weight function $\rho(\varphi)=\exp[-(d-2)\varphi^2/4]$, is determined by the structure of the quasilinear operator

$$\hat{L}=\partial\dot{\mu}/\partial\mu=d\mu[1+\ln(\mu)]+\sum_i\left[\partial_{\varphi_i}\partial_{\varphi_i}-\frac{d-2}{2}\partial_{\varphi_i}\varphi_i\right].$$

The basic hypothesis of Ref. 1 is that there exists a minimum of the flow $\mu=\mu^*$ [for which we have $\partial\|\dot{\mu}\|^2/\partial\mu(\varphi')|_{\mu=\mu^*}=0$]. Expanding $\dot{\mu}$ in the basis ψ_k: $\dot{\mu}=\Sigma\beta_k\psi_k$, and then projecting in succession onto the vector ψ_j, we easily find that we have $\int d^n\varphi'\partial\|\dot{\mu}|^2/\partial\mu(\varphi')\psi_j(\varphi')=2\beta_j\lambda_j=0$ in this case, where λ_j are eigenvectors of the operator $\hat{L}$. Since we have $\beta_j\neq 0$ for at least certain values of j, then there exist $\lambda_j=0$. The evolution of the RG parameters in the corresponding directions is anomalously slow. Linearizing μ near μ^*, and carrying out the standard operations of fluctuation theory, we easily find the behavior of the physical quantities. It is similar to their behavior in the course of an ordinary second-order phase transition. However, since the use of RG equation (1) [or Eq. (3)], instead of the φ^4 model, generates invariants $O_k(\varphi^2)$ of arbitrarily high order in the local density $f(\varphi)=\Sigma_k^\infty\hat{g}_kO_k(\varphi^2)$, it is not clear at the outset to what extent the presence of higher vertices $\hat{g}_k$ will affect the pattern of phase trajectories. The assumption that a minimum exists must also be proved.

In Ref. 5 we demonstrated the following: Because of the quadratic (isotropic) asymptotic behavior $f^*\propto\varphi^2/2$ of the physical branch of the solution of the equation $\hat{R}f^*=0$, fixed solutions ("points") of Eq. (1) (and the separatrices passing through them) can be derived through transformations of the symmetry group of the system applied to an "Ising" fixed point, which is the sum of the noninteracting solutions, $f_I^*=\Sigma_i f_I^*(\varphi_i)$. In the same paper we used the example of a tetragonal (two-component) system to demonstrate that the phase diagrams of Eq. (1) and of the φ^4 model are qualitatively related. It is thus convenient to restrict our analysis to the neighborhood of the integral $f_I(\varphi;t)=\Sigma_i f_I(\varphi_i;t)$ and to demonstrate, on the basis of this integral, that (a) the higher vertices are unimportant for the qualitative picture and (b) there exists a flow minimum in the case of a small deviation of the seeds from $f_I(\varphi;t)$.

The critical surface for

$$f_I(\varphi_i;t)=\sum_k^\infty g_{2k}(t)(\varphi_1/2)^{2k}$$

can be found by the shooting method.[6,7] In this case the coefficients of the Taylor series, $g_{2k}(t)$, are calculated numerically. Figure 1 shows projections of several RG trajectories onto the (g_4,g_6) plane. For all seeds, the coefficients g_{2k} initially approach a certain universal curve rapidly; they then evolve slowly along this curve toward the fixed point $f_I^*(\varphi_i)$. A similar phenomenon was recently observed on the basis of a slightly different version of the local RG equation and was called[7] the "large-river effect." This phenomenon is rather typical of relaxation nonlinear equations; Eq. (1) [or (3)] falls in that category. In particular, the attainment of a dissipation minimum on steady-state (attractor) trajectories in physical-kinetics problems is associated with that phenomenon.[8,9] In gen-

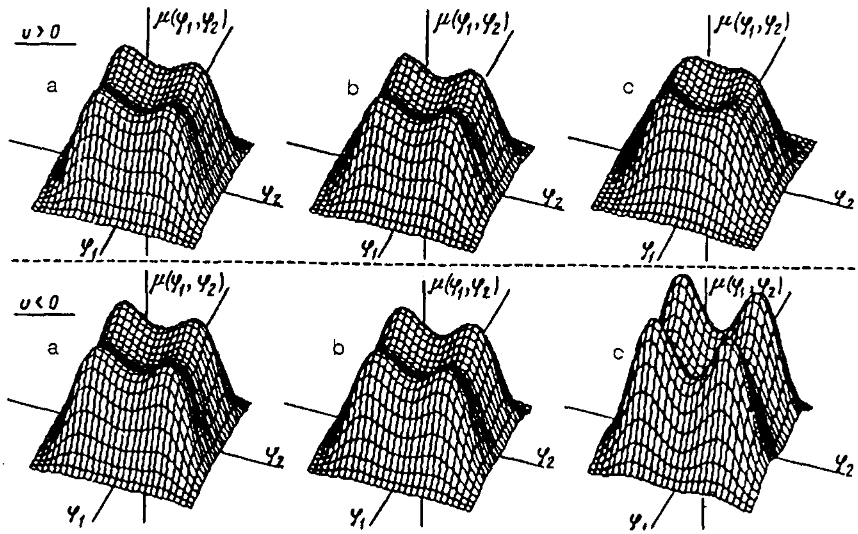

FIG. 1. Projection of phase trajectories onto the (g_4, g_6) plane. The points are analytic estimates for a "large-river" trajectory. Shown in the insets is the time evolution of the combinations $A = g_2 - g_2^*$ and $B = g_4 - g_2(2 - g_2)/3$, along with plots of $\ln(A)$ and $\ln(B)$ versus $\ln t$ and t, respectively.

eral, this phenomenon reflects a rapid "drainage" of phase trajectories into a (quasi-) potential valley, along which the variables then slowly "creep" toward the fixed point. For saddle points, this creep is anomalously slow (a power-law instead of exponential behavior). However, the fixed RG points are always saddle points, since the RG trajectories, passing through the flow minimum, always move away from the fixed point in the case of small deviations of the temperature from the critical surface.

The large-river trajectory can be estimated analytically. The equation for $g_{2k}(t)$ is

$$\partial g_{2k}/\partial t = [d-(d-2)k]g_{2k} - \sum_{m=2}^{k+1} m(k+1-m) g_{2m} g_{2(k+1-m)} + (2k+1)(k+1) g_{2(k+1)}/2. \tag{4}$$

The flow minimum is reached under the condition $\partial g_{2k}/\partial t \simeq 0$. In this region we have

$$g_{2(k+1)} \approx 2\left[[(d-2)k-d] g_{2k} + \sum_{m=1}^{k+1} m(k+1-m) g_{2m} g_{2(k+1-m)} \right] / [(2k+1) \times (k+1) g_{2(k+1)}]. \tag{5}$$

Recurrence relations (5) express $g_{2k} = g_{2k}(g_2)$ as a function of the sole parameter g_2 on a curve passing through the fixed point:[5] $g_0 \approx 0.076$, $g_2 \approx -0.456$, $g_4 \approx 0.373$, $g_6 \approx -0.141$, $g_8 \approx 0.067$, ... Figure 1 shows the projection of this curve onto the (g_4, g_6) plane. The points in Fig. 1 show the projection of this curve onto the (g_4, g_6) plane; this projection obviously lies extremely close to the large-river trajectory found

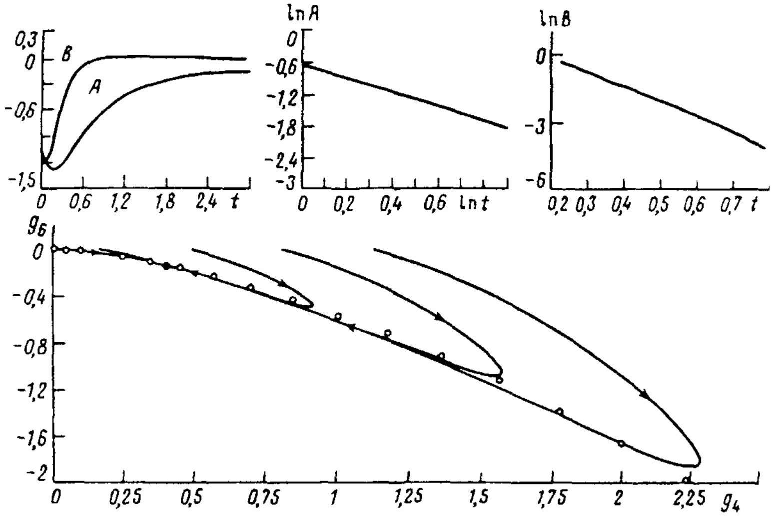

FIG. 2. Evolution of the surface $\mu(\varphi_1,\varphi_2)$ with the RG time under the conditions $v>0$ and $v<0$.

numerically. The insets in Fig. 1 also show the time evolution of the combinations $A=g_2-g_2^*$ and $B=g_4-g_2(2-g_2)/3$, along with plots of $\ln(A)$ and $\ln(B)$ versus $\ln t$ and t, respectively. These plots illustrate the fast exponential decay of the difference $[B \propto \exp(-0.15t)]$ and the slow, power-law ($A \propto t^{-4/3}$) attraction of this system toward the fixed saddle point. The attractor nature of the large river leads to a "universalization" of the critical behavior for various physical seeds in a nonperturbative approach based on the use of the exact RG equation.[6,7] In addition, this circumstance means that the effect of the higher vertices on the qualitative structure of the phase diagram is unimportant, since these vertices rapidly "adjust to" changes in the important vertex g_2 (and changes in some of the vertices $\hat{g}_4$, which are also important in anisotropic models). In particular, the qualitative picture found in the φ^4 model for the behavior of the RG trajectories near the saddle point, $f_l^*=\Sigma_i f_l^*(\varphi_i)$, should be preserved. This picture leads to an obvious flow minimum in this vicinity. The latter point was tested through a direct numerical integration of Eq. (3) for a tetragonal system with a seed function $\mu(\varphi_1,\varphi_2)=\exp[-f(\varphi_1,\varphi_2)]$ which depends on the invariant $u(\varphi_1^4+\varphi_2^4)$, for the case of a small increment in the interaction $v(\varphi_1^2+\varphi_2^2)^2$ (where $u \propto 10^{-2}v$).

Figure 2 illustrates the time evolution of the surface $\mu(\varphi_1,\varphi_2)$. In the initial stage ($t\leqslant 2$), both surfaces, which start on different sides of the separatrix, rapidly reach the large river, along which they are slowly attracted toward the fixed point $f_l^*=\Sigma_i f_l^*(\varphi_i)$ ($2\leqslant t\leqslant 25$). During this period, the two surfaces are essentially indistinguishable from each other, and they change only slightly in shape from that reached in the stage $t\leqslant 2$ (this point is actually illustrated by the first two pairs of figures, a–b). However, after the flow minimum is passed, the evolution of the parameters quickly speeds up, and the surfaces start to become greatly different. At $v>0$ the surface $\mu(\varphi_1,\varphi_2)$ becomes isotropic. This circumstance corresponds to an attraction of the system to a stable $O(n=2)$-symmetry point ("asymptotic symmetry"[10]). At $v<0$, the extrema of $\mu(\varphi_1,\varphi_2)$ instead move away from each other and become larger. For the local density $f(\varphi_1,\varphi_2)$ this situation corresponds to a deepening of the minima and to a withdrawal of these minima from the point $\varphi=0$. It can be shown that this effect leads to a fluctuation-induced collapse of the second-order phase transition to a first-order phase transition. Since the system passed through a clearly expressed minimum of the rate of evolution of the parameters on the way to this state, it can be concluded that the "mimicry" of a second-order phase transition by a fluctuation-induced first-order phase transition proposed in Ref. 1 can indeed occur.

I wish to thank J. P. Badiali for hospitality and collaboration at Pierre and Marie Curie University (Paris VI). It was in the course of this visit that the idea of using the "large-river effect" in various problems of the stability of phase transitions was formulated. I am also indebted to C. Bagnuls and C. Bervillier for useful discussions and for graciously furnishing recent results on the exact RG equation.

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Translated by D. Parsons