Generalized twistor dynamics of superparticles

A. Yu. Nurmagambetov and V. I. Tkach
National Science Center, Kharkov Physicotechnical Institute, 310108 Kharkov, Ukraine

J. J. Rosales
University of Guanajota C. P. 37150 Leon, Mexico

(Submitted 22 June 1994)
Pis'ma Zh. Eksp. Teor. Fiz. **60**, No. 3, 145–148 (10 August 1994)

A generalization of the twistor-shift procedure to the case of a superparticle interacting with a background Maxwellian supermultiplet is analyzed. © *1994 American Institute of Physics.*

1. The twistor dynamics of relativistic particles was generalized in Ref. 1 by adding to the standard Lagrangians some additional terms which depend on only the twistor components and their derivatives with respect to the proper time. Since there exists a nontrivial transformation of variables (a twistor shift), the generalized dynamics of free particles is equivalent to the standard dynamics. When particles are interacting with external fields, however, this equivalence is lost, as is manifested through the introduction in the theory of an interaction nonlocality parameter l and the appearance of an infinite series in powers l^n of nonminimal-interaction terms, including the Maxwell stress tensor and its higher derivatives in the case of an electromagnetic external field.

The present letter is devoted to a further development of generalized twistor dynamics. We examine the manifestations of a twistor shift in a system consisting of a superparticle and a background Maxwellian supermultiplet in $D=2+1$.

2. The generalized dynamics of a superparticle interacting with a background Maxwellian field is described by the action

$$S=\int d\tau d\eta(iE^{-1}DE^{\alpha}DE^{\beta}(\gamma_m)_{\alpha\beta}DE^{m}+lE^{-1}E^{\alpha}DE_{\alpha}+DE^{A}\mathcal{A}_{A}), \tag{1}$$

where $D=\partial_\eta+i\eta\partial_\tau$ is a covariant derivative of "small" supersymmetry on a world trajectory, which is parametrized by the proper time τ and its Grassmann superpartner η; $E^A_M(z)$ is a standard superdreibein of a plane superspace with coordinates $z^M=(X^m,\Theta^\alpha)$, which are superfields that are scalars with respect to the small supersymmetry, where

$$X^m=x^m+i\eta\chi^m, \quad \Theta^\alpha=\theta^\alpha+\eta\lambda^\alpha, \tag{2}$$

χ^m is the superpartner of the boson coordinate x^m, and λ^α is a commutating spinor which represents the first half of the twistor; $E^{-1}=e^{-1}-i\eta(\hat{\psi}/e^2)$ is an analog of 1D supergravity on the world line; $\mathcal{A}_{A}(z^M)$ is a superfield of the background Maxwellian supermultiplet; and l is a parameter with the dimensionality of a length.

The twistor shift is generated by a transformation to the new space–time coordinates

$$\hat{X}^{\alpha\beta}=X^{\alpha\beta}+\frac{l}{2\tilde{\mu}D\Theta}(\tilde{\mu}^{\alpha}D\Theta^{\beta}+\tilde{\mu}^{\beta}D\Theta^{\alpha}), \tag{3}$$

where $\tilde{\mu}^{\alpha}=\mu^{\alpha}+\eta d^{\alpha}$ is an even superfield, which includes the second component of the twistor, μ^{α}, and its superpartner d^{α}. Here

$$\tilde{\mu}_{\alpha}=iX_{\alpha\beta}D\Theta^{\beta},$$

$$X_{\alpha\beta}=X_m(\gamma^m)_{\alpha\beta},\quad \tilde{\mu}D\Theta\equiv\tilde{\mu}_{\alpha}D\Theta^{\alpha}.$$

Taking the equations of motion into account, after a twistor shift, in first order in l, we find the following action (modulo the term $D\Theta^{\alpha}\mathscr{A}_{\alpha}$, which will be discussed below):

$$\tilde{S}=\int d\tau d\eta\left(iE^{-1}D\Theta\gamma_m D\Theta D\hat{X}^m+\hat{\Omega}^m\mathscr{A}_m+\frac{1}{2}l\hat{\Omega}^{\alpha\beta}(\sigma^{mn})_{\alpha\beta}\mathscr{F}_{mn}\right), \tag{4}$$

where $\hat{\Omega}^m$ is a form which is invariant under local transformations of the small supersymmetry and global transformations of the supersymmetry in the large superspace, given by

$$\hat{\Omega}^m=D\hat{X}^m+i\Theta\gamma^m D\Theta+iD\Theta\gamma^m\Theta.$$

In $D=3$, a vector field enters the spinor supermultiplet [2]

$$\mathscr{A}_{\alpha}=l_{\alpha}+B\Theta_{\alpha}+V_{\alpha\beta}\Theta^{\beta}+h_{\alpha}\Theta\Theta, \tag{5}$$

where $V_{\alpha\beta}=V^m(\gamma_m)_{\alpha\beta}$ and h_{α} are a vector gauge field and its superpartner, respectively.

The imposition of the Wess–Zumino gauge $l=B=0$ and the additional constraints

$$\mathscr{F}_{\alpha\beta}(\hat{X},\Theta)=0, \tag{6}$$

$$T^a_{\alpha\beta}(\hat{X},\Theta)=2i\gamma^a_{\alpha\beta} \tag{7}$$

singles out an irreducible submultiplet of the physical fields of the theory. Action (4) can then be put in the form

$$\tilde{S}=\int d\tau d\eta iE^{-1}D\Theta\gamma_m D\Theta D\hat{X}^m-\frac{i}{2}\hat{\Omega}^m\left(V_m+\frac{1}{2}\Theta_{\beta}(\gamma_m)^{\alpha\beta}h_{\alpha}-\frac{i}{2}\Theta\Theta\varepsilon^{kl}_{m}F_{kl}\right)$$
$$+i\frac{1}{2}l\varepsilon^{nmk}\hat{\Omega}_k\left(F_{nm}+\Theta_{\alpha}(\gamma_n)^{\alpha\beta}\partial_m h_{\beta}-\frac{i}{2}\Theta\Theta\varepsilon^{kl}_{m}\partial_n F_{kl}\right), \tag{8}$$

where ε^{nmk} is the Levi–Civita density, and F_{nm} the Maxwell stress tensor.

Integrating over η, and fixing the solution of the equation of motion with respect to $\hat{\psi}$ in the form

$$\chi^m=-(\theta\gamma^m\lambda+\lambda\gamma^m\theta), \tag{9}$$

we find a completely component form of Lagrangian (8). This choice of solution, associated with the first term in action (8), was discussed in Ref. 3.

We turn now to the last term $D\Theta^{\alpha}\mathscr{A}_{\alpha}$ in action (1). By writing this expression in component form and using the equation of motion in terms of e, we easily see that this term doubles certain terms of action (8), thereby redefining the coefficients in them. The total Lagrangian after the twister shift thus takes the following form:

$$\tilde{L}=-\left(e^{-1}\lambda\gamma_m\lambda-\left(V_m+\frac{1}{2}\theta_\beta\gamma_m^{\beta\alpha}h_\alpha-\frac{i}{4}\theta\theta\varepsilon_m^{kl}F_{kl}\right)\right.$$

$$\left.+l\varepsilon_m^{nk}\left(F_{nk}+\frac{1}{2}\theta_\alpha\gamma_n^{\alpha\beta}\partial_k h_\beta-\frac{i}{4}\theta\theta\varepsilon_m^{kl}\partial_n F_{kl}\right)\right)\omega^m, \tag{10}$$

$$d\omega^m=d\hat{x}^m-i\theta\gamma^m d\theta+id\theta\gamma^m\theta+2\lambda\gamma^m\lambda d\tau. \tag{11}$$

Lagrangian (10) is invariant under the following transformations of the global supersymmetry on the mass shell:

$$\delta V_m=-\varepsilon_\beta(\gamma_m)^{\beta\alpha}h_\alpha,\quad \delta\Theta_\beta-\varepsilon_\beta,\quad \delta h_\alpha=\frac{i}{2}\varepsilon^\beta(\sigma^{nm})_{\beta\alpha}F_{nm},$$

with an odd parameter ε_β.

By making it possible to eliminate the unwanted term $\dot{\theta}\dot{\theta}$ in the action (when we go over to field theory, this term disrupts the relationship between the spin and the statistics), the supertwistor-shift procedure thus gives rise to an infinite series in powers of the nonlocality parameter l in the boson sector. This series includes the Maxwell stress tensor and its higher derivatives. The fermion sector, which is present in the case of a superparticle, is also modified from the minimal field–current scheme to a nonminimal scheme, with the onset of derivatives of order equal to the order of the expansion in l of the superpartner of the vector field.

This scheme can be generalized in a natural way to higher dimensionalities: $D=4.6$. Unfortunately, the case $D=10$, the most interesting one, does not fit into this construction, because of the onset of a tachyon sector in the course of the twistor shift. However, work in this direction is actively continuing.

Let us take a brief look at the application of this scheme to the case of field theory. A field theory describing the interaction of a scalar matter field in $D=3$ with a modified vector field $\tilde{A}_m=A_m+l\varepsilon_m^{nk}F_{nk}$ was studied in Ref. 4. That theory incorporates a Chen–Simons term in the action. The behavior of this system was studied for the case of a spontaneous breaking of $U(1)$ gauge symmetry. A supersymmetric generalization of this model, including a Wess–Zumino multiplet in $D=3$ interacting with a modified vector potential and effects of spontaneous symmetry breaking, will be discussed in a paper which will appear in the near future.

We are sincerely indebted to D. B. Volkov for comments offered as this paper was being written. We also thank D. P. Sorokin for his interest and for stimulating discussions.

[1] V. A. Soroka *et al.*, Int. J. Mod. Phys. A **7**, 5877 (1992).
[2] S. J. Gates *et al.*, *Superspace or One Thousand and One Lessons in Supersymmetry* (Benjamin-Cummings, 1983).
[3] D. V. Volkov and A. A. Zheltukhin, Nucl. Phys. B **335**, 723 (1990).
[4] S. M. Latinskiĭ and D. P. Sorokin, JETP Lett. **53**, 177 (1991).

Translated by D. Parsons