The photon structure function F_2 in QCD with nonlocal vacuum quark condensates

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The contribution from nonlocal vacuum condensates of quark fields to the hadronic part of the photon structure function $F_2(x)$ in the operator product expansion approach to QCD has been calculated. A substantial improvement of the agreement with experimental data for the standard value of the parameter $\lambda_q^2 \equiv \langle \bar{q} D^2 q \rangle / \langle \bar{q} q \rangle \approx 0.5$ GeV^2 has been obtained. © *1994 American Institute of Physics.*

1. In the present letter we calculate the hadronic part of the photon structure function by taking into account the nonlocal quark condensate and using the method suggested by Gorsky *et al.*[1] We also compare our data with experiments and briefly discuss the results.

A method for calculating the hadronic part of the photon structure function $F_2(x,P^2,Q^2)$ in photon–photon DIS ($Q^2 \gg P^2$) in the region of moderate values of x and Q^2 was proposed in Ref. 1. This approach is based on the operator product expansion (OPE) technique for a 4-current correlation function, with account of the nonzero vacuum condensate (VC) of gluon fields ($\langle GG \rangle$). The authors of this paper have managed to reproduce rather well the existing experimental data in the region $0.3<x<0.7$ for different values of Q^2 (Q^2=4.3, 5.3, 9.2., 23.0 GeV^2) *without introducing new phenomenological parameters.* As to VC of quark fields, it was asserted that their contribution (through the lowest-dimension operators, i.e., $:\bar{q}q:$) is proportional to the $\delta(1-x)$ and thus can be omitted in the consideration of the region $0.3<x<0.7$. (The contributions of quark VCs to PhSF in this region can also come from diagrams with radiative corrections to the usual box diagrams, which are of order α_s, and for this reason are not considered.)

However, the use of a nonlocal quark condensate,[2] which is equivalent to the summation of an infinite set of condensates of higher dimensions, leads to the result with new properties. We show that nonlocal quark VCs give rise to a smooth (over x) contribution to the photon structure functions. It is interesting to note that the essential diagrams are new. The corresponding contribution to the photon structure function ($\Delta^{\langle \bar{q}q \rangle} F_2$) is of the same order of magnitude as the one from the gluon VC ($\Delta^{\langle GG \rangle} F_2$) and is determined by the characteristic length of the quark vacuum correlations $1/\lambda_g$:

$$\Delta^{\langle GG \rangle} F_2(x) \sim \frac{1}{m_\rho^4} \left\langle \frac{\alpha_s}{\pi} GG \right\rangle, \quad \Delta^{\langle \bar{q}q \rangle} F_2(x) \sim \frac{\pi^4 \langle \bar{q} q \rangle^2}{\lambda_q^2}. \tag{1}$$

The latter estimate in (1) does not depend on the concrete form of the condensate distribution function[2] and reflects the essentially nonperturbative character of this contribution. The numerical values are $\Delta^{\langle \bar{q}q \rangle} F_2 \approx 0.35$, while $\Delta^{\langle GG \rangle} F_2 \approx -0.15$ in the central region of

x for the standard values[3] $\lambda_q^2 \approx 0.4$ GeV^2 ($\lambda_q^2 \equiv \langle \bar{q} D^2 q \rangle / \langle \bar{q} q \rangle \approx 0.4-0.6$ GeV^2). Careful analysis shows a substantial improvement of the agreement with experiment in the entire region $0.2 < x < 0.8$ only for these standard values of the correlation length $1/\lambda_q$. For larger values $\lambda_q^2 \approx 1.2$ GeV^2, which are typical of instanton liquid models,[4] the quark VC contribution is of no importance compared with the gluon contribution. It is clear, however, that the value of λ_q cannot be much less than the standard value, because of the "explosion" of $\Delta^{\langle \bar{q}q \rangle} F_2$ for $\lambda_q \to 0$. One can conclude that the photon structure function is quite sensitive to the parameters of the condensate structure; therefore, the extraction of F_2 with high precision from experiments on photon–photon interactions make it possible to independently evaluate the length of the correlations in the nonperturbative QCD vacuum.

We also establish the breakdown of the factorization theorem[5] for the discontinuities (i.e., imaginary parts) of the diagrams with nonlocal VC on the cut line. This effect depends on the decay rate of the vacuum correlations at large distances: e.g., the Gauss decay of quark VC [$\langle \bar{q}(z) q(0) \rangle \sim \exp(-\gamma z^2)$ for $|z| \to \infty$] violates the factorization, but the exponential decay [$\sim \exp(-\gamma |z|)$] does not violate it. At the same time, our conclusion about the necessity to take into account δ-function distributions is not related to the nonlocality of the VCs: The distribution concentrated near the border of some area cannot be used properly for treating local problems in the center of the area. In this case the appearance of a border-concentrated distribution, like the δ-function from the OPE, indicates the deficiency of that expansion. Our method, which uses another type of distribution, avoids this problem.

2. The nonlocal VC seems to be introduced for the first time in Ref. 6, and the exponential decay in the coordinate representation was obtained in the lattice calculations.[7] The nonlocal VC was effectively used to explain the different dynamical hadron properties in the exclusive processes.[8] We use a δ-shaped ansatz for the distribution functions $f(\alpha)$ of nonlocal scalar [$M(z)$] and vector [$M_\mu(z)$] quark VCs:[9]

$$M(z) \equiv \langle \bar{q}(0) \hat{E}(0,z) q(z) \rangle = \langle \bar{q}(0) q(0) \rangle \int_0^\infty e^{\alpha z^2/4} fs(\alpha) d\alpha, \tag{2}$$

$$M_\mu(z) \equiv \langle \bar{q}(0) \gamma_\mu \hat{E}(0,z) q(z) \rangle = -i z_\mu A \int_0^\infty e^{\alpha z^2/4} fv(\alpha) d\alpha, \tag{3}$$

$$f_S^{\text{mod}}(\alpha) = \delta(\alpha - \Delta^2), \quad f_V^{\text{mod}}(\alpha) = \delta'_\alpha(\alpha - \Delta_V^2), \tag{4}$$

where $A = 2/81 \pi \alpha_S \langle \bar{q} q \rangle^2$, and the scales $\Delta^2 \equiv \lambda_q^2/2$ and $\Delta_V^2 \equiv \alpha_V \lambda_q^2/2$ are not always equal, $\langle \sqrt{\alpha_S} \bar{q} q \rangle^{1/3} \approx 0.23$ GeV. We select the value $\alpha_V = 0.7$, consistent with the Taylor expansion of the VC in the lowest orders.[2]

Let us examine the diagrams in Fig. 1 (crossed graphs are included; the contribution of diagram 1A is marked by index S, and that of 1B by V) and take into account the contributions of the lowest twist; i.e., we ignore terms like P^2/Q^2, $\exp[-Q^2/(\Delta^2 x)]$. Denoting

$$C_{\text{norm}} \equiv \frac{3 \alpha_{em} \Sigma_q e_q^4}{\pi}, \quad \gamma_S \equiv \frac{2xP^2}{\Delta^2}, \quad \gamma_V \equiv \frac{\bar{x} P^2}{\Delta_V^2},$$

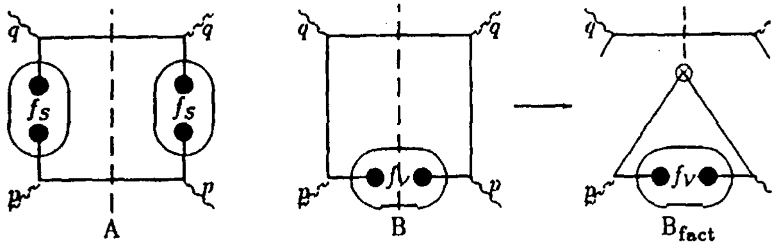

FIG. 1. Diagrams with nonlocal VCs, essential for the photon structure function $F_2(x)$. The diagram with the scalar VCs is on the left-hand side; in the middle is the original diagram with the vector VC; to the right is its factorized form.

we obtain

$$\frac{1}{C_{\rm norm}}\,\Delta_S^{\langle\bar qq\rangle}F_2^T(x;P^2)=\frac{8\pi^4}{9\Delta^6}\,\langle\bar qq\rangle^2(x\bar x)\exp(-\gamma_S), \tag{5}$$

$$\frac{1}{C_{\rm norm}}\,\Delta_S^{\langle\bar qq\rangle}\,F_2^L(x;P^2)=\frac{8\pi^4}{9\Delta^6}\,\langle\bar qq\rangle^2\left(\frac{x^2}{\gamma_S}\right)\,\exp(-\gamma_S), \tag{6}$$

$$\frac{1}{C_{\rm norm}}\,\Delta_V^{\langle\bar qq\rangle}F_2^T(x;P^2)=-\frac{32\pi^3}{81\Delta_V^6}\,\alpha_S\langle\bar qq\rangle^2x\,\exp(-\gamma_V)$$
$$\times\left(x^2+\bar x^2-\gamma_V\left(x^2+\frac{x+1}{2}\right)+\gamma_V^2\,\frac{x}{2}\right), \tag{7}$$

$$\frac{1}{C_{\rm norm}}\,\Delta_V^{\langle\bar qq\rangle}F_2^L(x;P^2)=\frac{32\pi^2}{81\Delta_V^6}\,\alpha_S\langle\bar qq\rangle^2x\bar x\,\exp(-\gamma_V)$$
$$\times\left(\frac{2x-1}{2\gamma_V}-3x-\frac{1}{2}+\gamma_V\,\frac{5x+1}{2}-\gamma_V^2\,\frac{x}{2}\right). \tag{8}$$

It should be noted that the methods used for calculating these contributions are different: for the scalar VCs the Cutkosky's method was used (it is equivalent to the double Borel transform method; for details see Ref. 10), and for the vector VC we used the factorization of large and small distance contributions directly—the discontinuation diagram is determined by the coefficient function of the process (i.e., at short distances). This difference can be explained as follows. Exact calculation of the imaginary part of diagram $1B$ with the nonlocal vector VC, with the δ-shaped ansatz form (3), gives the exact zero. This is a simple consequence of the more general

Proposition:

If the weight $f(\alpha)$ of a line of the diagram in α-space is concentrated on the bounded support, then the contribution to discontinuity of this diagram from the cut through this line is zero.[3)]

This can be proved by direct calculations in the case of "box" diagrams. The distribution $f_V^{\rm mod}(\alpha)$ [Eq. (4)] used by us satisfies the conditions of this proposition. On the other hand, there is a whole class of ansatzes $h_n(\alpha)$, for which the nonlocal VCs

- decay in the large $|z|$ limit exponentially [$\sim\exp(-\gamma|z|)$];
- have the unbounded support in α-space, e.g., $$h_n(\alpha)=\exp\left(\frac{-m_n^2}{\alpha}\right)\frac{\alpha^{-n}(m_n^2)^{n-1}}{\Gamma(n-1)};$$
- imitate the δ-shaped ansatz for large values of the parameter n.

The imaginary part of diagram $1B$ for these ansatzes is not zero. This quantity, however, is essentially ansatz-dependent. For this reason we use the factorization method, which is weakly sensitive to the specific choice of the ansatz [we could use the ansatz $h_n(\alpha)$ in all the calculations, but it would produce very complicated expressions and numerically similar results].

We see that the nonlocal quark VCs give smooth (over x) contributions in the entire region $0<x<1$ and the nonlocality parameters is located in the denominators of common factors, which suggests nonperturbative nature of these corrections. We wish to emphasize that in the limit $P^2\to 0$ only the contributions to F_2^L are singular, while those related to F_2^T are regular.

3. Treatments of the SF of a real photon requires an extrapolation of the result obtained for $P^2\gg\Lambda_{\rm QCD}^2$ to the region $P_2\to 0$. First of all, we recall that in this limit the physical SF F_2^T and F_2 coincide, and physical $F_2^L\to 0$ (for details see Ref. 1). We will therefore consider below only the transverse part of the photon structure function, i.e., F_2^T. We can then extrapolate to $P^2=0$ our results for the nonlocal quark VC contributions without any problem. Note that we do not pretend to describe the region of large $P_2\gg\lambda_q^2$ in (5)–(8), because this asymptotic regime is determined by the unknown details of the distribution function $f(\alpha)$ in the large α region. For treating singularities like $1/P^4$ (from the gluon VC) and $\log(Q^2/P^2)$ (from the perturbative part), which appear in the OPE calculations in QCD, we used the method and model of Ref. 1. Using this method, we can represent the photon structure function by means of the dispersion relations in p^2 in terms of the contributions of the physical states [vector meson (ρ)+continuum]. The parameters of the model are chosen in such a way that they correctly reproduce all the terms of the OPE calculations:

$$\frac{1}{C_{\rm norm}}F_2(x)=x\left\{-1+6x\bar{x}+[x^2+\bar{x}^2]\log\left(\frac{Q^2}{x^2p_0^2}\right)\right.$$
$$+\left[\frac{8\pi^4}{9\Delta^6}\langle\bar{q}q\rangle^2\,\bar{x}-[x^2+\bar{x}^2]\,\frac{32\pi^3}{81\Delta^6}\,\alpha_S\langle\bar{q}q\rangle^2\right]$$
$$\left.-\frac{p_0^4}{2m_\rho^4}\left[x^2+\bar{x}^2+\frac{8\pi^2}{27p_0^4x^2}\left\langle\frac{\alpha_S}{\pi}\,GG\right\rangle\right]\right\}, \qquad (9)$$

where $p_0^2\approx 1.5$ GeV is the standard value of the continuum threshold in the QCD sum rule calculations of the ρ-meson properties, and m_ρ is the ρ-meson mass. The quark contri-

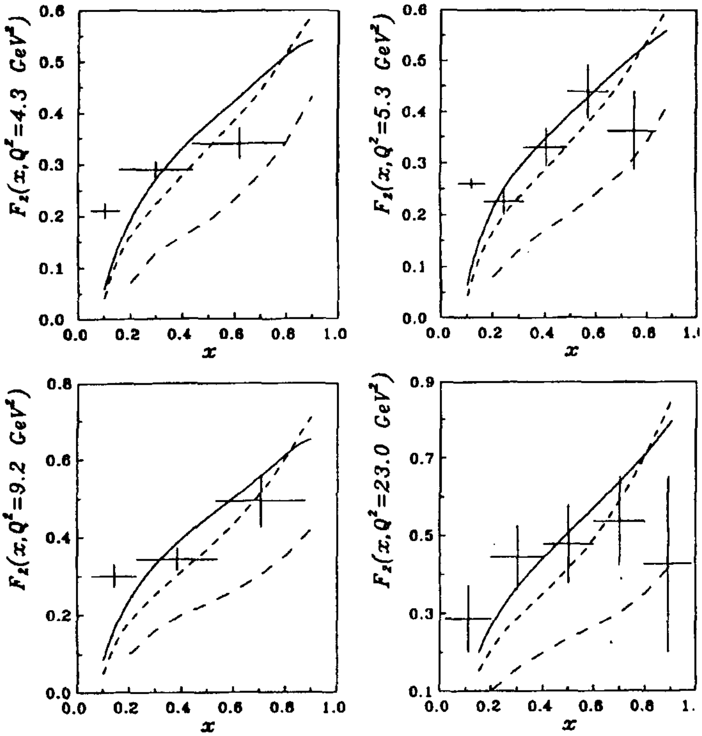

FIG. 2. Comparison of the prediction of the theoretical model with the experimental data for $Q^2=4.3\ \mathrm{GeV}^2$, $Q^2=5.3\ \mathrm{GeV}^2$, and $Q^2=9.2\ \mathrm{GeV}^2$ from Ref. 11 and for $Q^2=23.0\ \mathrm{GeV}^2$ from Ref. 12. Solid line—our results; dashed line—results of Ref. 1, and the lowest dashed line (long dashes)—hadronic part of the contribution.

bution to the hadronic part is represented by the first term of the second equation in (9). To compare the new result (9) with the experimental data.[11,12] we should include the evolution of the quark VC with Q^2: $\langle\bar{q}q\rangle^2$ $(\mu^2\sim\mathrm{GeV}^2)$ $\rightarrow\langle\bar{q}q\rangle^2(Q^2)$, $\langle\bar{q}g(\sigma G)q\rangle(\mu^2)\rightarrow\langle\bar{q}g(\sigma G)q\rangle(Q^2)$ on the basis of the standard one-loop evolution (e.g., Ref. 13). The quark terms lead to the growth of the hadronic part in the central region of x, giving a better agreement with the experiment, as shown in Fig. 2. In the region $x\geqslant 0.8$ there is a slight tendency, due to the quark VC corrections, to lower the growth of the curve. The best agreement with data is achieved for the value $\lambda_q^2=0.5-0.6$ GeV^2.

The experimental errors should nonetheless be reduced severalfold in order to compare the experiment with the theoretical curve carefully. Further theoretical progress can be obtained in the following three ways:

1. By improving the hadronic model of the photon structure function (see Ref. 1) through the introduction of new vector resonances. This step can improve the behavior of the theoretical curve at "large" $x\geqslant 0.7$.

2. By revising the expression for the gluon corrections, taking into account the

correlation length of the gluon condensate.[14,15] This will correct the theoretical curve in the region $x \sim 0.2$.

3. At $x \sim 0$, i.e., near the singularity in the t-channel, the OPE series diverges.[1] This series can be corrected, however, by reforming[4)] OPE through cancellation of the "long-range" contribution, using the approach of Ref. 16.

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