The pinch effect explains turbulent transport in tokamaks

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The pinch effect of the particles, which is observed in all tokamaks, furnishes the key to the selection of dominant turbulence modes. The concept of a turbulent uniform distribution of particles over some phase-space surfaces specified by the geometry of the magnetic field and by invariants is introduced. Large-scale electrostatic modes lead to a turbulent uniform distribution $nq = \text{const}$ with a maximum particle density at the center of the column. They also lead to a natural explanation of the self-consistency of profiles. These modes can be transformed to the greatest extent by radial electrostatic fields, in accordance with the phenomenology of L–H–VH transitions.

Many years of effort on the part of experimentalists and theoreticians have been rewarded with tokamak working regimes which are free of the most dangerous large-scale MHD instabilities. Unfortunately, transport processes are still anomalous. This anomalous transport is the primary obstacle to the ignition of a fusion reaction. The number of turbulent-transport coefficients, including off-diagonal ones, is several tens. Since a set of instabilities contribute to each of these coefficients, it is natural to attempt to guess at the primary loss channel by working from experimental data. Any instability leads to an intensification of transport processes, so the diagonal coefficients are of no assistance in choosing a model for the turbulence. From this standpoint, a pinch effect is an excellent test, since only a few models explain this effect. This puzzling phenomenon can be summarized as follows: A profile of the particle density in a tokamak is bell-shaped, with a prominent maximum at the center, although the source of particles is at the periphery of the plasma column, and the normal diffusion is also directed outward. During pulsed heating of the hydrogen, the plasma itself collects at the center. The Weir collisional pinch effect is slight in comparison with turbulent diffusion and cannot explain the experiments. Phenomenologically, the pinch effect is described by introducing a convective particle flux nv_r directed toward the center:[1]

$$Q=-D\,\frac{\partial n}{\partial r}+nv_r. \tag{1}$$

Here D is the coefficient of turbulent diffusion. The convective particle flux is on the order of the diffusion flux in all tokamaks, in all regimes. Consequently, the two fluxes are governed by a common mechanism, the same for all tokamaks, and it is useful to rewrite the resultant flux in the form

$$Q=-D_1\frac{\partial}{\partial r}\left(\frac{n}{n_0}\right), \quad (2)$$

where $n_0(r)$ is the turbulent uniform distribution of the density. We will show below that this distribution is governed by invariants and does not depend on the intensity of the turbulence, which determines the diffusion coefficient D_1.

Our explanation is based on the idea that large-scale electrostatic modes lead to a uniform distribution on some surfaces in phase space which are specified by adiabatic invariants. This situation corresponds to a profile of the particle density along the minor radius, which is in approximate agreement with experiment. Nontrivial turbulent uniform distributions are well known for the atmosphere, in which turbulence forms adiabatic temperature profiles in place of isothermal ones. [2] The particular equilibrium is governed by the invariants and by the specification of the coordinates, in which the divergence of the phase flux vanishes. In a tokamak, these coordinates are governed by a magnetic field of complex geometry. We will accordingly begin our analysis with the simpler case of a z-pinch at absolute zero with a magnetic field and an average density which depend on only the radius. Following Ref. 3, we consider a z-pinch with electrostatic cells $m=0$, which mix the frozen-in plasma. From the frozen-in condition it follows that the quantity nr/B_p is conserved along a trajectory, where B_p is the magnetic field, which has only a poloidal component in a z-pinch. The conservation of nr/B_p means that the turbulent uniform distribution of the particle density is

$$n_0\sim\frac{B_p}{r}, \quad (3)$$

and the relaxation to this equilibrium is described by the diffusion flux $Q=-D_T\partial(nr/B_p)/\partial r$. Conceptually, a turbulent uniform distribution is analogous to the plateau regime in the case of quasilinear diffusion of particles involving waves, and relation (3) arises when the turbulent uniform distribution is projected onto ordinary space.

We will not attempt to determine the diffusion coefficient, which is subject to the effects of many factors. Our goals are turbulent uniform distributions which are universal for a given type of turbulence. Taking account of the thermal drift of the particles at a velocity v_T (the center of the Larmor circle is moving), we can describe the fluxes in the z-pinch by the equations

$$\frac{v_zB_p}{c}=-\frac{\partial\phi}{\partial r}+\frac{v_TB_p}{c}, \quad \frac{v_rB_p}{c}=\frac{\partial\phi}{\partial z}. \quad (4)$$

Since the thermal drift is directed along an invariant direction, it does not alter equilibrium (3), although it does affect the diffusion coefficient. Distribution (3) can also be derived from the Liouville theorem, according to which the density in phase space is conserved along trajectories:

$$\frac{n}{v_\perp^2v_\parallel}=\text{const.} \quad (5)$$

Substituting in $v_\perp^2$ from conservation of the magnetic moment, $\mu = v_\perp^2/2B_p$, and $v_\parallel$ from conservation of angular momentum, $L = v_\parallel r$, we again find $n_0(r) \sim B_p/r$. An increase in the density in regions of elevated magnetic field is caused by a slowing of the particle drift; the $1/r$ arises from the cylindrical geometry. If we assume that collisions lead to a locally Maxwellian distribution, then the temperature profile should adiabatically adjust to the density: $T \sim n^{2/3}$.

Let us consider equilibrium distributions of the particle density in tokamaks under the influence of large-scale electrostatic modes. The passing particles effectively average the drift, and we will ignore their contribution to the transport. We will describe the trapped particles by means of drift equations averaged over the bounce period. These equations are exactly the same as in a z-pinch [Eq. (4)], except that now the velocity of the thermal toroidal drift, v_T, is governed by the magnetic moment of a particle, μ, and by its longitudinal invariant $J = \int v_\parallel dl$, and the coordinate z corresponds to the direction along the torus. The toroidal magnetic field appears in only the magnetic moment, and the electric potential enters as an average over the bounce period. This statement means that the trapped particles in a tokamak relax toward an equilibrium which is analogous to the equilibrium in a z-pinch. The passing particles do not contribute to the transport, but they dominate the density. If the rate of Coulomb collisions is much lower than the bounce frequency, but much higher than the reciprocal of the revolution time in a convection cell, then the density and temperature of the passing particles simply adjust to the trapped particles by means of Coulomb collisions. In this case we have

$$n_0 \sim \frac{B_p}{r} \sim \frac{B_p}{B_T}\frac{R}{r} = \frac{1}{q}, \quad T \sim n^{2/3}. \tag{6}$$

Here q is the safety factor introduced by Kruskal and Shafranov, and B_T and R are assumed constant. The relaxation to density equilibrium (6) is described by a diffusion equation in the form in (2), in total agreement with the phenomenology of the experiments.[1]

As in the case of a z-pinch, a turbulent uniform distribution can be found by analyzing the invariants of motion and the Liouville theorem. In the absence of perturbations, a particle conserves its energy, its toroidal moment, and the two adiabatic invariants which we have already used. We assume that the turbulence changes the energy and the toroidal moment, but that it leaves the two adiabatic invariants unchanged. According to the Liouville theorem, the density in phase space [Eq. (5)], is constant along trajectories. From conservation of the magnetic moment, with the change in the magnetic field ignored, we find $v_\perp^2 = \text{const}$. From conservation of the longitudinal invariant $J \sim v_\parallel qR$ we find $v_\parallel \sim 1/q$ and $n_0(r) \sim v_\perp^2 v_\parallel \sim 1/q$. When Coulomb collisions are taken into account, we obtain an adiabat for the temperature.

The third and most graphic way to construct the pinch effect is to examine the radial drift of the trapped particles under the influence of a constant toroidal electric field. The velocity of this drift is inversely proportional to the poloidal magnetic field. The particles spend a long time in regions of an elevated poloidal field, so (also taking the cylindrical geometry into account) we immediately find $n_0(r) \sim 1/q$. A constant vortical electric field is suppressed by the good conductivity of the passing particles, but the potential field of the large-scale electrostatic cells does not contradict an ideal conductivity, and it does not aggravate the mixing.

Here is a list of the simplifying assumptions which are supported (more or less) by the small parameters. 1. The heat flux away from the center of the plasma column is appreciable. It strongly perturbs the equilibrium. We would thus naturally expect that the temperature would deviate to a greater extent than the density from turbulent uniform distribution [Eq. (6)] in an experiment. 2. In analyzing the longitudinal adiabatic invariant we ignored the dependence on the bounce angle, retaining only the most important dependence, on q. 3. In analyzing the magnetic adiabatic invariant we ignored the dependence on the magnitude of B. 4. Collisional exchange of passing and trapped particles leads to an adiabat under some extremely vague limitations. Excursions from the adiabat lead to the appearance of fluxes in the spirit of Eq. (2.39) of the review by Yushmanov. [4]

Let us compare the predicted turbulent uniform distribution with experiment. In tokamaks, $1/q$ usually increases toward the center, as does the density. Indeed, most of the decrease in density occurs in the region with most of the decrease in $1/q$.

Since the source of energy, in contrast with the source of particles, is inside the plasma, there is an unavoidable energy loss outward, and we would be less likely to expect an equilibrium energy distribution. We can only assert that the specific entropy is greater at the center; i.e., we have $\partial(\ln T)/\partial(\ln n)>2/3$. This inequality is typical of experiments. The circumstance that the temperature gradient is greater than the adiabatic one constitutes a source of energy for turbulence. For the energy flux inward we would more naturally expect a 2/3 adiabat. This value may be consistent with some experiments [5,6] which have been reported with nonaxial heating and with a thermal pinch effect.

The transport coefficients measured in dynamic experiments are slightly larger than the corresponding static ones. This contradiction can be removed or at least weakened if the turbulent fluxes are reckoned from inhomogeneous, uniform distributions in which there are no fluxes at all. In simple terms, the static transport coefficients are lowered by the use in the calculations of a relaxation to Coulomb equilibria, which are spatially uniform and which do not correspond to the situation.

It is useful to compare our methods with existing theories of relaxed states.[7,8] A common factor is the idea of using different rates for the decay of the invariants; a distinction is that we do not minimize the hydrodynamic energy under additional constraints. We instead maximize the entropy of the particles, while conserving the adiabatic invariants. Interestingly, Kadomtsev[8] takes a $p=q^{-2}$ dependence of the plasma pressure on the safety factor to be an experimentally obtained result. Here we find essentially the same dependence, $p=q^{-5/3}$, theoretically. The sign of the deviation from equilibrium agrees with the direction of the energy flux. The self-consistency of the profiles, first noted by Coppi,[9] finds a simple and natural explanation.

The primary value of the proposed uniform distributions is that one can work from them to reconstruct a turbulent-loss channel and to partially answer the question of whether electrostatic or magnetic perturbations are responsible for the transport.[10] The answer is the electrostatic perturbations. Although small-scale electrostatic modes could in principle lead to a pinch effect, [11–13] large-scale modes are better at conserving the longitudinal invariant, and they make a major contribution to the transport. In the atmosphere, for example, the primary fluxes are governed by global convection cells. A small-scale turbulence, a secondary effect, arises in large gradients caused by the global cells. It is natural to assume that the same thing is going on in a tokamak plasma. Large-scale electrostatic modes come in various types,[14] but the measured flattening of the profiles of the electron temperature on rational surfaces[15] naturally emphasizes the role of rational cells. Such rational cells are subject to the effects of radial electric fields,[16] so L–H–VH transitions may correspond to a suppression of convection on rational surfaces. The source of energy for the convection is the longitudinal energy of the trapped particles. It can be seen from conservation of the longitudinal invariant that the particles spread out along the magnetic field as they move toward the periphery. The role played by the passing particles reduces to one of maintaining a Spitzer longitudinal conductivity.[3,17,18] This assumption was used implicitly when we ignored the time variation of the magnetic field.

Judging from the complexity of the theory of global cells in the atmosphere, we conclude that the theory in a tokamak is very complicated, since a nonlinear competition in the case of kinetic convection cannot be ignored. An analysis of turbulent uniform distributions based on invariants makes it possible to skip over the details, and it describes the correct structure of the transport equations. The formation of profiles similar to the equilibrium profiles in (6) may promote an experimental realization of regimes with improved confinement. This model can be extended to other cases, in which the turbulent equilibrium is dictated by other invariants. A numerical study of turbulent uniform distributions by the particle method would provide stronger support for the hypothesis presented here, or would at least refute it.

The qualitative correspondence between the concept of turbulent uniform distributions and experiments, particularly regarding the pinch effect, is, in my view, more than a random coincidence.

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