A unified description of confinement and superconductivity in terms of the vacuum correlation functions

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It is argued that both confinement and superconductivity can be described on the basis of the vacuum-correlation function method. The close similarity of these phenomena is stressed. The fundamental vacuum correlation functions D and D_1 are expressed in terms of the current correlation functions (ordinary and monopole-like). © *1994 American Institute of Physics.*

It is a widely accepted point of view that confinement in SU(N) gluodynamics and QCD is a dual Meissner effect.[1–3] There is a strong support for this opinion from recent Monte Carlo studies on the basis of the so-called Abelian-projection method.[4–8] Even the profile of the QCD string is similar to that of the Abrikosov vortex line.[9–13]

At the same time, there are strong objections against the total similarity of the underlying dynamics. Specifically, in superconductivity one can work out all dynamical equations for fields as classical equations, following from, e.g., the Landau–Ginzburg Lagrangian; i.e., there are classical configurations underlying the Abrikosov line and the superconducting vacuum. In contrast to QCD [or SU(N) gluodynamics], it is unlikely that field configurations are classical, since stable classical solutions are topological and the net topological charge of vacuum is zero. In lattice calculations the nonperturbative physics is ensured by a rather large set of configurations, which can be checked by the so-called cooling method.[14,15]

It is therefore necessary to formulate both superconductivity and confinement using a single and most general language which does not depend on the classical equations of motion. We suggest in this letter the use of vacuum field correlation functions (current correlation functions) to describe both phenomena and to demonstrate explicitly which correlation functions are necessary for this purpose and what duality means in this language. Remarkably, the same correlation functions are responsible for confinement in the Abelian and non-Abelian theory. As a result, it is possible to obtain a purely quantum superconductivity, which is described by quantum correlation functions, rather than by classical equations.

1. The Wilson loop and correlation functions

We consider the Abelian theory, like QED with possible admission of magnetic monopoles, since all final equations are applicable to the non-Abelian case, with obvious insertion of parallel transporters, traces, etc. Confinement is usually characterized by the area law of the Wilson loop along the trajectory of the charges e, $-e$:

$$\langle W(C)\rangle=\left\langle \exp ie\int_C A_\mu dx_\mu\right\rangle=\left\langle \exp ie\int F_{\mu\nu}\,d\sigma_{\mu\nu}\right\rangle=\exp(-\sigma S), \tag{1}$$

where S is the area of the contour C in the 14 plane, and the string tension σ is expressed in terms of the field strength correlation functions (FSC) $\langle F_{\mu\nu}(x)F_{\rho\lambda}(y)\rangle$,[16,17]

$$\sigma=\tfrac{1}{2}e^2\int d^2x\langle F_{14}(x)F_{14}(y)\rangle+\cdots, \tag{2}$$

where the dots imply the contribution of higher-order correlation functions $\langle FFFF\rangle$, etc., which are unimportant for our purpose here.

On general grounds of Lorentz invariance, the FSC can be expressed in terms of two basic scalar functions[17] $D(x)$ and $D_1(x)$, and we shall write separately the FSC for electric and magnetic fields

$$\langle E_i(x)E_j(y)\rangle=\delta_{ij}\left(D^E+D_1^E+h_4^2\frac{\partial D_1^E}{\partial h^2}\right)+h_i\,h_j\frac{\partial D_1^E}{\partial h^2}, \tag{3}$$

$$\langle H_i(x)H_j(y)\rangle=\delta_{ij}\left(D^H+D_1^H+h^2\frac{\partial D_1^H}{\partial h^2}\right)-h_i\,h_j\frac{\partial D_1^H}{\partial h^2}, \tag{4}$$

where $h=(h_\mu\,h^\mu)^{1/2}$, and $h_\mu=x_\mu-y_\mu$.

For Lorentz-invariant vacuum, like that of QED or QCD, we have $D^E=D^H=D$ and $D_1^E=D_1^H=D_1$. However, for the same theories but at nonzero temperature T, the electric and magnetic correlation functions do not coincide. For a superconducting material the Lorentz invariance is also violated, and again $D^E\neq D^H$ and $D_1^E\neq D_1^H$.

For the contour C in the 14 plane we obtain the following relation from (3):

$$\sigma=\frac{e^2}{2}\int d^2\,xD(x)+\cdots\,. \tag{5}$$

Let us now consider a magnetic charge g of a type-II superconductor. By analogy with (2), we introduce the Wilson loop operator

$$\langle W^*(c)\rangle=\left\langle \exp ig\int F^*_{\mu\nu}\,d\sigma_{\mu\nu}\right\rangle=\exp(-\sigma^*S). \tag{6}$$

Here σ^* is expressed in terms of the dual fields F^*_{14} as in (2), but since $F^*_{14}=\epsilon_{1234}F_{23}=H_1$, we have, due to (4), the expression

$$\sigma^*=\frac{g^2}{2}\int d^2xD_1^H(x). \tag{7}$$

Comparison of (5) and (7) gives us the exact meaning of the notion of dual Meissner effect, without reference to the underlying equations of motion. We need first to define $D(x)$ and $D_1(x)$ more explicitly, together with integrals (5) and (7), which may diverge. As can be seen in (4), $D^E(x)$ can exist only due to the magnetic monopoles.[17] Applying ∂_i to both sides of (4), we have

$$\langle \text{div }H(x),\text{div }H(y)\rangle=-\partial^2D^H(x-y). \tag{8}$$

Hence $D^H(x)$ does not contain purely perturbative contributions (at least in lowest orders). In contrast, $D_1(x)$ may contain perturbative contributions, which should be subtracted from it in (7). To lowest order we have

$$\bar{D}_1(x)=D_1(x)-\frac{4e^2}{x^4}, \tag{9}$$

and D_1 should be replaced with $\bar{D}_1$ (7). The phenomenon of the Abrikosov string therefore depends on the nonperturbative contents of $\bar{D}_1$, i.e., on the possibility of creating a mass parameter which characterizes the size of the string.

To see the mechanism of this mass creation, we can use the Ginzburg–Landau equations to derive[1)]

$$D_1^{LG}(x-y)=(e^2|\phi|^2-\partial^2)_{xy}^{-1}, \tag{10}$$

for the region outside the string core, $r\gg\xi$ (ξ is the coherence length[18]), when $|\phi|=\phi_0=\text{const}$, yields exponential decay at large distances, with the mass parameter

$$D_1(x)\sim e^{-mx}, \quad m=e|\phi_0|=1/\delta, \tag{11}$$

where δ is the London (Landau) penetration depth.

It is interesting to compare this behavior with that of $D(x)$, which was measured recently in SU(3) gluodynamics,[19]

$$D(x)\sim e^{-\mu|x|}, \quad \mu\sim 1 \text{ GeV}. \tag{12}$$

We see in (11) and (12) that the notion of duality of the QCD string and the Abrikosov string has a more detailed correspondence. To see more of this correspondence, we can compare profiles (off-string density distributions) for the QCD string and the Abrikosov string.

2. String profiles

In the first case we can probe the field inside the QCD string using the so-called connected ρ^c and disconnected ρ^{disc} plaquette averages around the Wilson loop[11–13]

$$\rho^c=\frac{\langle W(C_\sigma)\rangle-\langle W(C)\rangle\langle W(\sigma)\rangle}{\langle W(c)\rangle}, \tag{13}$$

where C_σ is the contour formed by connecting $\Delta\sigma$ and the Wilson loop C, and $W(\sigma)$ is the Wilson loop for $\Delta\sigma$ contour. On the basis of the vacuum correlation-function method (VCM) we can obtain the following expression for the Wilson loop:

$$W(C_\sigma)\simeq\langle W(c)\rangle\left(1+e^2\int d\sigma_{14}(y)\Delta\sigma_{\mu\nu}\langle E_1(y)\Phi F_{\mu\nu}(x)\Phi^+\rangle\right)+\cdots, \tag{14}$$

where the dots represent higher-order cumulants and the $O((\Delta\sigma)^2)$ terms. Here Φ denotes the parallel transporter $\Phi(x,y)=P\exp ie\int_y^x A^\mu\, dz_\mu$.

As one can see, the ρ^c quantity measures the spatial distribution of the components of the field strength tensor $F_{\mu\nu}$ in presence of charges. The MC simulation show that the electric field in the confinement phase is squeezed into flux tubes with an exponential[13] behavior of the flux tube profile inside the string. In the case of a very long string, we obtain a simple analytic result for ρ_{14}^c. The transverse shape measured at the middle is given by

$$\rho_{14}^c=\frac{2\pi a^2}{\mu^2}\left[D(0)(1+\mu x)-D_1(0)\frac{1}{2}(\mu x)^2\right]e^{-\mu x}, \tag{15}$$

where the parameters obtained by us are

$\mu\approx 0.190382\text{fm}, \quad a^2 D(0)\approx 3.91468\times 10^7, \quad D_1(0)=D(0)/3,$

m from (11) is given by $m\approx\mu$, with a $\chi/d.o.f.=0.17$.

We see that such behavior of ρ_{14}^c is in good agreement with the dual Meissner effect, when the asymptotic limit of the field distribution of the vortex line is exponential:

$$H(r)=\frac{\phi_0}{(8\pi\ r\ \delta^3)^{1/2}}\exp(-r/\delta), \tag{16}$$

where δ=const as $\xi\to 0$ limit, and $\delta=\delta_{\text{eff}}=\int_0^\infty H(r)dr/H(\infty)$ as δ goes to zero.

3. Two-point FSC in terms of currents

Let us consider U(1) electrodynamics with monopoles [there may be, for example, Dirac monopoles or topological defects in compact U(1) theory]

$$\partial_\mu F_{\mu\nu}=j_\nu, \quad \partial_\mu \overset{*}{F}_{\mu\nu}=\overset{*}{j}_\nu,$$

where the variables j_ν and j_ν describe the normal and the monopole-like current, respectively. In keeping with Ref. 20, we can express the observed field strength tensors $F_{\mu\nu}$ in terms of currents by redoubling field strength tensors $H_{\mu\nu}$ and $G_{\mu\nu}$ as follows:

$$F_{\mu\nu}=H_{\mu\nu}+\overset{*}{G}_{\mu\nu}, \quad \overset{*}{G}_{\mu\nu}=\tfrac{1}{2}\varepsilon_{\mu\nu\alpha\beta}G^{\alpha\beta}.$$

$H_{\mu\nu}$ and $G_{\mu\nu}$ satisfy Maxwell's equations

$$\partial_\mu H_{\mu\nu}=j_\nu, \quad \partial_\mu G_{\mu\nu}=\overset{*}{j}_\nu,$$

$$\partial_\mu \overset{*}{H}_{\mu\nu}=0, \quad \partial_\mu \overset{*}{G}_{\mu\nu}=0.$$

We can thus define the dual pair of potentials A_ν and $\overset{*}{A}_\nu$ in the usual way:

$$H_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu,$$

$$G_{\mu\nu}+\partial_\mu \overset{*}{A}_\nu-\partial_\nu \overset{*}{A}_\mu.$$

In the Lorentz gauge for the Fourier components we then have

$$A_\nu(k)=\frac{1}{k^2}j_\nu(k), \quad \overset{*}{A}_\nu(k)=\frac{1}{k^2}\overset{*}{j}_\nu(k).$$

For the field correlation function we easily obtain

$$\langle F_{\mu\sigma}(-k)F_{\nu\rho}(k)\rangle=\frac{1}{k^2}\Bigg[\langle\overset{*}{j}{}^2\rangle(k)\{\delta_{\mu\rho}\,\delta_{\sigma\nu}-\delta_{\mu\nu}\,\delta_{\rho\sigma}\}$$

$$\times\{\langle j^2+\overset{*}{j}{}^2\rangle(k)\}\left\{\frac{k_\mu k_\nu}{k^2}\,\delta_{\rho\sigma}-\frac{k_\nu k_\sigma}{k^2}\,\delta_{\mu\rho}+(\mu\nu\leftrightarrow\sigma\rho)\right\}\Bigg], \qquad (17)$$

while for the dual cumulant, which is responsible for the confinement of magnetic charge, we have the same expression with the replacement $j\leftrightarrow\overset{*}{j}$. From (17) we deduce expressions for the Fourier components of the $\tilde{D}$ and $\tilde{D}_1$ function

$$\frac{\partial\,\tilde{D}_1(k)}{\partial k^2}=\frac{1}{k^2}\,\frac{\langle j^2+\overset{*}{j}{}^2\rangle(k)}{k^2}, \quad \tilde{D}(k)+2\tilde{D}_1(k)=\frac{\langle\overset{*}{j}{}^2\rangle(k)}{k^2},$$

or, a formal way, for the space forms

$$D_1(h)=2\,\Box^{-1}\Box^{-1}\,\frac{\partial}{\partial h^2}\langle j^2+\overset{*}{j}{}^2\rangle(h),$$

$$D(h)=-2D_1(h)-\Box^{-1}\langle\overset{*}{j}{}^2\rangle(h).$$

Conclusion

We have described both confinement and superconductivity using the field correlation functions D and D_1. In the first case, due to the Lorentz invariance $D^E=D^H=D$ and this correlation function ensures confinement. In the case of superconductivity and in absence of the condensate of the magnetic monopoles only D_1^H is nonzero and is responsible for the confinement of magnetic charges and for the formation of the Abrikosov fluxes. The correlation functions D and D_1 are expressed in terms of the correlation functions of charge and monopole currents. We have shown that duality of confinement and superconductivity go beyond symmetric expressions for string tensions (5), (7) and manifest themselves also in the form of the field correlation functions (11), (12) and the string profiles (15), (16).

All the treatments above refer to the case of zero temperature. It would be very interesting to extend this approach of the field (current) correlation functions to nonzero temperatures and especially to the phase transition region.

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[1)]In general, it can be shown that a two-point correlation function (which describes the response of the vacuum to the source) coincides with the second derivative of the effective Ginzburg–Landau Hamiltonian: $\Delta^{-1}=[\delta^2 H_{\rm eff}/\delta H^2|_{H=H_0}]^{-1}$.

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