Magnetic interaction of self-focusing channels and fluxes of electromagnetic radiation: their coalescence, the accumulation of energy, and the effect of external magnetic fields on them

G. A. Askar'yan and S. V. Bulanov
Institute of General Physics, Russian Academy of Sciences, 117942, Moscow, Russia

F. Pegoraro
Dipartimento di Fisica Teorica dell' Universitá di Torino, 10125 Torino, Italy

A. M. Pukhov
Moscow Physicotechnical Institute, 41730 Dolgoprudnyĭ, Moscow Region, Russia

(Submitted 14 July 1994)
Pis'ma Zh. Eksp. Teor. Fiz. **60**, No. 4, 240–246 (25 August 1994)

A new effect has been observed in a numerical simulation of relativistic self-focusing and filamentation of light beams: a coalescence of channels. The reason for the interaction of the chemicals is identified: an attraction of currents which arise in the self-focusing channels. It is demonstrated that external magnetic fields affect the channels, causing them to bend. These effects cast new light on the phenomenon of self-focusing. They raise the possibility of combining the energy from several channels into one. A mechanism for the generation of ultrastrong quasistatic magnetic fields is identified. Corresponding force effects are possible for other mechanisms for generating drive currents: radiation pressure; thermoelectric and thermionic-emission currents in laser discharges, laser beams, and intense microwave fields.

1. Recent progress in laser technology, which has made it possible to generate ultrashort pulses of electromagnetic radiation with an intensity above 10^{18} W/cm^2 and pulse lengths of a few femtoseconds,[1] marks a transition to completely new regimes in the interaction of electromagnetic radiation with plasmas.

In the limit of relativistic amplitudes, the self-focusing of electromagnetic radiation,[2] which is one of the most vivid effects in the modern physics of nonlinear processes, takes on some new features. These new features are the subject of very active theoretical work,[3] and they are finding experimental confirmation.[4] Another remarkable nonlinear-optics effect is the generation of a quasistatic magnetic field in an interaction of intense radiation with a plasma.[5] As we move up to relativistic values of the intensity, we would expect magnetic effects to become more influential. In this letter we are reporting a study of the generation of magnetic fields and the effect of these fields on relativistic self-focusing.

We are concerned primarily with radiation which is intense in the relativistic sense, i.e., the limit

$$a=\frac{eE}{m\omega c}\gg 1, \quad (1)$$

so the oscillation energy of an electron in the field of the wave is greater than mc^2. Here E is the electric field amplitude in the wave, and $\omega$is the carrier frequency. The results of a numerical simulation of the effect of relativistically intense laser pulses on a plasma of supercritical density[6] demonstrate that a strong magnetic field arises in the plasma only near its boundary.

The primary mechanisms for the generation of quasistatic magnetic fields in a laser plasma are as follows: First, there is the thermal emf, which is excited because the gradients of the plasma temperature and density are not collinear.[7] As was pointed out in Ref. 8, however, this mechanism is not pertinent in the limit of very high field amplitudes. It is then possible to produce a magnetic field which is directed along the radiation propagation direction, by virtue of the inverse Faraday effect.[9] This effect occurs for circularly polarized waves. It arises because of the electric current set up by electrons orbiting in the wave. It occurs along the periphery of a wave beam which is not uniform in the transverse direction. A nonuniformity or time variation of the ponderomotive force can also serve as a source of magnetic fields. A theory for this mechanism is derived in Ref. 8 for the case in which the magnetic field is generated near the boundary of a plasma of supercritical density. The next mechanism stems from the Weibel instability.[10] This is an electromagnetic instability which develops in a plasma with an anisotropic electron velocity distribution. The nonlinear stage of this mode and its relationship with quasistatic magnetic fields in a laser plasma are discussed in Ref. 11.

2. In order to identify the mechanism by which the magnetic field is generated and to clarify the effect on the course of the radiation–plasma interaction, one needs detailed information on the space–time behavior of the field, the electron distribution function, and the very nonlinear stage of this process, including the limit of relativistic radiation amplitudes. For this purpose we present here the results of a numerical simulation of 2D regimes of the propagation of electromagnetic radiation through a plasma. We use a highly efficient, completely relativistic, 2D electromagnetic code which is a realization of the particle-in-cell method. By "2D" here we mean that all properties depend on the coordinates x and y and the time. The complete system of Maxwell's equations is solved. In general, the momenta of the particles have three components, as do the electric and magnetic fields. Calculations were carried out on a grid of 128×256 cells. The number of particles in one cell was on the order of 10, so the total number of particles in the calculations reached 200 000. The boundary conditions along the coordinate y were periodic. Along the coordinate z, the parameters of the wave incident on the plasma were specified at the $x=0$ boundary. There was no reflection of the radiation escaping from the computation region at either boundary. The spatial dimensions were expressed in wavelengths of the electromagnetic radiation in vacuum, λ. The unit of time was the wave period $2\pi/\omega$. The plasma initially filled the region $x>5$. The ion background was homogeneous with immobile ions. The initial electron temperature was zero.

3. Figures 1–4 show the results of a numerical simulation of the interaction of a semi-infinite electromagnetic beam of finite transverse width with a plasma. In the version shown in Fig. 1 the plasma density corresponds to the ratio of the Weibel frequency

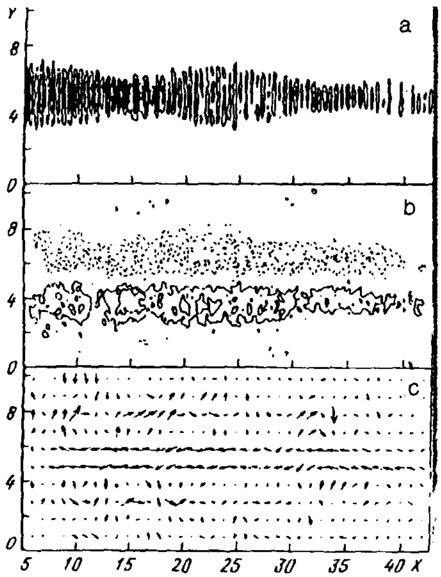

FIG. 1. Relativistic self-focusing of electromagnetic radiation in a plasma and generation of a magnetic field. $\omega t/2\pi = 50$. a: Contour map of the electromagnetic energy density. b: Contour map of the z component of the quasistatic magnetic field. Solid curves—Positive field; dotted curves—negative field. c: Distribution of the vector electric current density.

without relativistic effects to the carrier frequency of the laser light, $(\omega_{pe}/\omega)=0.75$. The dimensionless amplitude of the electromagnetic wave is $a=5$. The initial transverse dimension of the electromagnetic beam is 5λ. The wave is linearly polarized with electric field along the z axis. For these parameters, the threshold for relativistic self-focusing is exceeded. At a distance on the order of 10λ from the plasma boundary, the radiation is focused into a narrow channel with a transverse dimension comparable to the wavelength (Fig. 1a). The quasistatic magnetic field (the z component) is localized near the channel. This field vanishes at the channel axis and reaches a maximum at the edges (Figs. 1b and

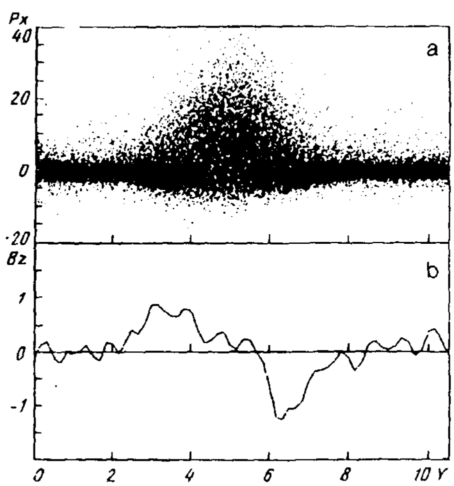

FIG. 2. a—The (p_x,y) electron phase plane averaged along the coordinate x; b—profile of the magnetic field along y at $x=15\lambda$.

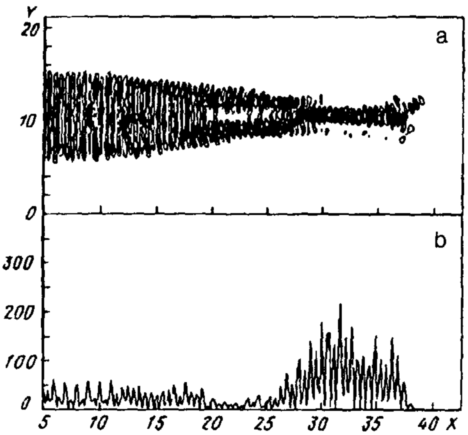

FIG. 3. Development of the filamentary instability and coalescence of filaments in the nonlinear stage of this instability for $\omega t/2\pi = 50$. a—Contour map of the electromagnetic energy density; b—longitudinal profile of the electromagnetic energy density at the channel axis.

2). The field structure corresponds to an electric current flowing along the channel (Fig. 1c).

The characteristic value of the magnetic field can be estimated in the following way in the limit of ultrarelativistic radiation intensities and therefore relativistic energies of the particles in the plasma. This estimate method is relatively insensitive to the details of the particular mechanism responsible for the generation of the current in the plasma. We denote by R the transverse dimension of the wave beam. In the ultrarelativistic limit, the electron velocity is limited by the velocity of light in vacuum, c, so the electric current density is approximately equal to $-enc$, where n is the particle density in the channel. Omitting the numerical factors, we then find the magnetic field to be $B \approx enR$. If the transverse dimension R is determined by the condition for channeling of the radiation, then it must satisfy the inequality $R > (ca/\omega_p)$. In other words, the dimension is on the order of the plasma wavelength calculated for the case with a relativistic increase in the electron mass. Here we allow for the circumstance that, in a relativistically strong electromagnetic wave, an electron acquires an energy of $mc^2a^2/2$ in the frame of reference in

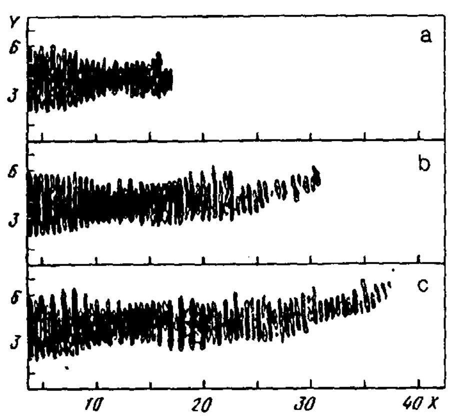

FIG. 4. Contour map of the electromagnetic energy density upon a deflection of the channel in a uniform external magnetic field for $\omega t/2\pi = 15$, 30, and 50 (parts a—c, respectively).

which it was at rest before the arrival of the wave.[12] Consequently, the magnetic field is given in order of magnitude by $B \approx enca/\omega_p$ or, in dimensionless form,

$$\frac{eB}{m\omega c} \approx a \frac{\omega_p}{\omega} . \tag{2}$$

This relation shows that the quasistatic magnetic field in a plasma of subcritical density is weaker by a factor of ω_p/ω than the field amplitude in the electromagnetic wave, and the field increases with increasing plasma density, in proportion to $n^{1/2}$.

It can be seen from Fig. 2a that the electric current is set up along the channel by a beam of fast particles which are accelerated by the electromagnetic radiation or by plasma waves excited in this region. It follows from the distribution with respect to the longitudinal momentum along the coordinate y (Fig. 2a) that the channel is filled with fast particles. By virtue of plasma quasineutrality, the fast-particle current must be cancelled by a current of the opposite sign, flowing along the periphery of the channel. This current, the replacement current, is carried by electrons of the cold component of the plasma. The oppositely directed currents repel each other. This effect is associated with a maintenance of the modulation of the current density in the transverse direction and the magnetic-field structure that arises. The case which we are discussing in the present letter corresponds to a field which has only a z component. Figure 2b shows its profile along the coordinate y. The field reaches the values given by expression (2).

The repulsion of the oppositely directed currents and the increase in the nonuniformity in the distribution of the current density in the plasma are the manifestations of the Weibel instability,[10] which was studied in detail in Ref. 11 that was cited above (see also the papers cited in Ref. 11). The characteristic transverse scale of the field distribution is found by equating the magnetic-energy density to the density of the transverse kinetic energy of the particles, $B^2/8\pi \approx nmc^2a$. In the linear approximation, one can estimate the threshold for this instability in a similar way. In the ultrarelativistic limit, this estimate puts the size of the field-localization region on the order of ca/ω_p. This figure agrees in order of magnitude with the radius of an electromagnetic filament found from the condition for channeling of the radiation. The growth rate of the Weibel instability in the nonrelativistic case is $\omega_p(v/c)$, where v is a characteristic electron current velocity. In the limit of relativistic particle energies, this growth rate can be estimated to be ω_p/a. These estimates are in accordance with the characteristic dimensions of the uniformity of the magnetic field in the directions across and along the electromagnetic filament. We should point out that there may be only an order-of-magnitude agreement, since the force of the ponderomotive pressure is predominant in the versions of the calculations presented above. This force arises because of a nonuniformity of the electromagnetic radiation. Nevertheless, the magnetic interaction has an important effect on the propagation of radiation in the plasma, as follows from the results of the numerical simulation presented below.

Figure 3a shows a distribution of the electromagnetic energy density which forms as the result of a filamentation instability of an electromagnetic beam with a fairly large transverse dimension. In this version of the calculations, the plasma density corresponds to a ratio $\omega_{pe}/\omega = 0.75$. The dimensionless amplitude of the electromagnetic wave is $a = 5$. The transverse dimension of the electromagnetic beam is 10λ. The wave is linearly polarized with electric field along the z axis. At a distance on the order of 15λ from the plasma boundary, the radiation breaks up into two well-separated filaments, with transverse dimensions comparable to the wavelength. The filaments are separated by a distance greater than their radii. (This figure shows the stage of the process corresponding to $\omega t/2\pi = 50$. The time evolution of the instability will be discussed in a separate paper.) The filaments later coalesce into a single channel. Nearly all the radiant energy is in the channel through which the wave is propagating. This process leads to an extreme accumulation of energy. It can be seen from the x profile of the electromagnetic energy density at the channel axis, $y=0$ (Fig. 3b), that the energy density in this case is eight times the value at the plasma boundary.

4. The coalescence of the filaments can be explained on the basis of a magnetic interaction between them. To demonstrate the importance of the magnetic field, we carried out a simulation of the effect of a uniform external magnetic field on radiation self-focusing. Figure 4 shows the deflection in the transverse dimension of a channel in which a beam with the parameters corresponding to the version in Fig. 1 is focused. The magnetic field is directed along the z axis and has a magnitude $B = \omega mc/e$. In other words, it is on the order of the characteristic field excited in the plasma by the electromagnetic beam, (2). We see a deflection of the channel in accordance with the sign of the field and of the fast-electron current. A change in the sign of the field leads to a deflection in the opposite direction. The curvature of the channel is due to the transverse deflection of electrons as they move in the magnetic field. The relativistic electrons overtake the leading edge of the wave, which is propagating at the group velocity; this velocity is lower than the velocity of light in vacuum because of the plasma and because the channeled radiation is propagating in a narrow effective waveguide. Ahead of the pulse front, the electrons increase the refractive index of the medium because of their relativistic increase in mass. This increase in refractive index leads in turn to a change in the propagation direction of the radiation. Since the Larmor radius of the fast electrons is greater than that of the electrons of the cold component, the deflection of the latter in the opposite direction is not as large. The longitudinal modulation which can be seen in Fig. 4 can be attributed to an interference of individual modes of the narrow waveguide.

5. Corresponding force effects will be observed for intense microwave beams or laser beams, because of the drive currents which arise from the "radiation pressure" on the electrons that are scattered by ions,

$$I_{\text{light}} = \frac{e}{m\nu_s c}\,\frac{\partial P}{\partial z} \simeq \frac{r_0}{e}\,\frac{\omega_p^2}{\omega^2+\nu_s^2}P \lesssim \frac{r_0\omega_p^2}{e\omega}{}_2 P \lesssim \frac{r_0}{e}P$$

for $\omega > \omega_p \gg \nu_s$ (r_0 is the classical radius of an electron, P is the total power, and ν_s is the collision rate), and also because of thermoelectric or thermionic-emission currents. For example, for the currents from the radiation pressure alone the force per unit length due to the external field H is $F_1 \simeq (1/c)IH \simeq (r_0/ec)PH$. Equating this force to the force per unit length of the hydrodynamic resistance, $F_{\text{HD}} \simeq C\rho_0 u^2 a$, where C is a form factor of the flow around an object ($\simeq 0.1-1$), ρ_0 is the density of the surrounding gas, a is the channel radius, and u is the velocity of the current-carrying channel), we find $u \gtrsim 10^4$ cm/s for $H \gtrsim 10^4$ Oe and $P \simeq 100$ MW. In other words, these interactions of fluxes and their curvature or deflection by external fields can be extremely effective.

This study had financial support from the Ministry of Science and Technological Policy of the Russian Federation and also from the Russian Fund for Fundamental Research (Grant 94-02-16922).

[1] G. Mourou and D. Umshtadter, Phys. Fluids B **4**, 2315 (1992).
[2] G. A. Askar'yan, Zh. Eksp. Teor. Fiz. **42**, 6 (1962) [Sov. Phys. JETP **15**, 8 (1962)]; Usp. Fiz. Nauk **111**, 249 (1973) [Sov. Phys. Usp. **16**, 680 (1973)].
[3] A. G. Litvak, Zh. Eksp. Teor. Fiz. **57**, 629 (1969) [Sov. Phys. JETP **30**, 344 (1969)]; C. Max *et al.*, Phys. Rev. Lett. **33**, 209 (1974); L. A. Abramyan, *et al.* Zh. Eksp. Teor. Fiz. **102**, 1816 (1992) [Sov. Phys. JETP **75**, 978 (1992)]; A. B. Boris ov *et al.*, Phys. Rev. A **45**, 5830 (1992); X. L. Chen and R. N. Sudan, Phys. Rev. Lett. **70**, 2082 (1993); S. V. Bulan ov *et al.*, Phys. Rev. Lett (in press).
[4] A. B. Borisov *et al.*, Phys. Rev. Lett. **68**, 2309 (1992).
[5] V. V. Korobkin and R. V. Serov, JETP Lett. **3**, 194 (1966); G. A. Askar'yan *et al.*, JETP Lett. **5**, 93 (19 67).
[6] S. C. Wilks *et al.*, Phys. Rev. Lett. **69**, 1383 (1992).
[7] J. Stamper *et al.*, Phys. Rev. Lett. **26**, 1012 (1971).
[8] R. N. Sudan, Phys. Rev. Lett. **70**, 3075 (1993).
[9] A. D. Steiger and C.H. Woods, Phys. Rev. A **5**, 1467 (1971); V. Yu. Bychenkov *et al.*, Zh. Eksp. Teor. Fiz. **105**, 118 (1994) [JETP **78**, 62 (1994)].
[10] E. W. Weibel, Phys. Rev. Lett. **2**, 83 (1959).
[11] V. Yu. Bychenkov *et al.*, Zh. Eksp. Teor. Fiz. **98**, 1269 (1990) [Sov. Phys. JETP **71**, 709 (1990)].
[12] H. M. Lai, Phys. Fluids **23**, 2373 (1980).

Translated by D. Parsons