Obtaining information about a plasma wave behind a wide opaque barrier

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The fluctuation level in a collisionless plasma behind a wide barrier may rise when an intense plasma wave is incident on the barrier. A nonlinear limitation on the fluctuations which arise in the case of plasma waves is found.

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Previous theoretical and experimental work on wave regeneration in inhomogeneous plasmas behind barriers[1–3] has dealt with cases in which the width of the opaque barrier, d, is not significantly greater than l_0, the scale length for the phase defocusing of the resonant particles. In the opposite limit, $d \gg l_0$, even in a collisionless plasma, information about the phase of the incident wave is evidently lost, and there is no nontunneling transmission. As we will show in the present letter, however, even in the case of wide barriers it is possible to detect the presence of an incident wave by measuring the fluctuation level in the plasma behind the barrier. We will be guided by the basic concepts and models used in Refs. 1 and 2.

Let us consider the one-dimensional problem in which a plasma wave $E_0 \cos(\omega t - kz)$ in a homogeneous plasma of density n and temperature T is incident on an opaque barrier of width d, with boundaries in the planes $z = 0$ and $z = d$. We assume $k^{-1} \ll l_0 \ll \gamma^{-1}$ where $l_0 = V_{\text{ph}}(m/ekE_0)^{1/2}$, $\gamma = -(\pi/2)(\omega_p/V_g) \times V_{\text{ph}}^2(\partial f_0/\partial \text{v})_{\text{v}=V_{\text{ph}}}$ is the collisionless spatial damping factor, and V_{ph} and V_g are the phase and group velocities of the wave.

At $z = 0$ the plasma wave is reflected from the barrier, and the modulated beam of resonant particles passes through the region $0 \leqslant z \leqslant d$; as shown in Ref. 2, if the barrier is not too wide, $d \lesssim 5 l_0$, this particle beam excites in the plasma behind the barrier a wave identical to the incident wave. The effect may be interpreted as a nontunneling transmission. In the opposite limit, $d \gtrsim 5 l_0$, the phase defocusing of the resonant particles causes a smoothing of the distribution function $f(\text{v},t,z)$. We will not go into the details of this process here; we will content ourselves with a phenomenological high-frequency cutoff of the spectrum $f(\text{v},t,z)$. Using the results of Refs. 2 and 4, we then find the following distribution at the exit side of the barrier ($z = d$):

$$f(\text{v}, t, z) = f_0(V_{\text{ph}}) - \frac{1}{\pi}(\gamma/k)\,\text{v}_E V_g\,(u/V_{\text{ph}})^3, \tag{1}$$

where $u = (\text{v} - V_{\text{ph}})/\text{v}_E$ and $\text{v}_E = 2(eE_0/mk)^{1/2}$ (we are assuming $u \ll 1$, since it is the effect of the resonant particles which is of particular interest).

Direct application of the results of the linear theory of fluctuations shows that for this behavior of the electron velocity distribution the ratio of the spectral distribution of the fluctuations of the electric field to its thermal level, which we denote by η, tends toward infinity at $\omega/k = V_{\mathrm{ph}}$:

$$\eta = v_E^2 (V_{\mathrm{ph}} - \omega / k)^{-2} . \tag{2}$$

To eliminate the divergence, we must incorporate a nonlinear limitation on the fluctuation amplitude. Let us assume that the plasma behind the barrier is nonisothermal ($T_e \gg T_i$), and let us assume that a plasma-wave fluctuation which is excited can decay into a plasma-wave fluctuation and an ion acoustic fluctuation: $l \rightleftarrows l' + s$.

We use the kinetic equation for the waves in the form

$$\frac{\partial N_{\mathbf{k}}}{\partial t} = -2\delta_{\mathbf{k}} N_{\mathbf{k}} + 4\pi \Sigma | V_{\mathbf{k}\mathbf{k}'\mathbf{k}''} |^2 \Big\{ N_{\mathbf{k}'} N_{\mathbf{k}''} - N_{\mathbf{k}} N_{\mathbf{k}'} \operatorname{sign}(\omega_{\mathbf{k}} \omega_{\mathbf{k}''}) - N_{\mathbf{k}} N_{\mathbf{k}''} \operatorname{sign}(\omega_{\mathbf{k}} \omega_{\mathbf{k}'}) \Big\} \times \delta(\omega_{\mathbf{k}} - \omega_{\mathbf{k}'} - \omega_{\mathbf{k}''}) \delta_{\mathbf{k},\mathbf{k}'+\mathbf{k}''} + \Delta_{\mathbf{k}} , \tag{3}$$

where we have used the standard notation: $N_{\mathbf{k}}\delta(\omega - \omega_{\mathbf{k}}) = (\langle E^2 \rangle_{\omega,\mathbf{k}}/16\pi)(\partial \Lambda_{ij}/\partial \omega) e_i e_j^*$, $\Lambda_{ij} = \mathbf{k}^2 c^{2*}/\omega^2 (k_i k_j/\mathbf{k}^2 - \delta_{ij}) + \epsilon_{ij}(\omega,\mathbf{k})$, e_i is the polarization vector, $V_{\mathbf{k}\mathbf{k}'\mathbf{k}''}$ is the matrix element of the corresponding wave–wave interaction, and $\delta_{\mathbf{k}}$ is the quasilinear damping factor. The term $\Delta_{\mathbf{k}}$, which incorporates the spontaneous emission of waves by the plasma particles, was found in Ref. 5:

$$\Delta_{\mathbf{k}} = \frac{2\pi^2 \, \mathrm{Sp} \lambda \langle j_i j_j \rangle^0_{\omega,\mathbf{k}} e_i e_j^*}{\omega^2 \left| \dfrac{\partial \Lambda}{\partial \omega} \right|} . \tag{4}$$

Here $\lambda_{ij}\Lambda_{ji} = \Lambda \delta_{ij}$, and the spectral distribution of the current-density fluctuations is given by the following expression, if we ignore the interaction between particles:

$$\langle j_i j_j \rangle^0_{\omega,\mathbf{k}} = 2\pi e^2 \int d\mathbf{v} \, v_i v_j f(\mathbf{v}) \delta(\omega - \mathbf{k}\mathbf{v}) ,$$

where $f(\mathbf{v})$ is an arbitrary nonequilibrium distribution function.

System (3) yields the following estimate of the extent to which the spectral density of the electric-field fluctuations exceeds its thermal level:

$$\eta = \frac{18}{(2\pi)^{2.5}} \frac{na^3}{h(ka)^4} \exp\left(- \frac{1}{2(ka)^2} - \frac{3}{2} \right) , \tag{5}$$

where a is the electron Debye length, and $h \geqslant 1$ is a parameter which is a measure of the ion acoustic turbulence level ($h > 1$ if there are additional sources of ion acoustic fluctuations behind the barrier).

This result can be applied, for example, to the part of the ionosphere at heights of

4500–5000 km, where the so-called auroral kilometric emission is detected. Substituting the typical plasma parameters $n = 10^2$ cm^{-3} and $T_e = 10^4$ K into (5), and assuming a wavelength $\lambda = 3.2 \times 10^5$ cm, we find $\eta \approx 10^2$.

In summary, this nonlinear interaction leads to a decrease in the amplitude of the fluctuation fields, as expected, but now these fields may be much higher than the thermal noise. By measuring the fluctuation level with a detector at $z > d$, we can, at the very least, reach the conclusion that there is, or is not, a certain wave mode in the plasma in front of the barrier. It was previously believed that for barriers so wide that the resonant-particle flux is completely smeared by phase defocusing it would be impossible to obtain information about the state of the plasma in front of the barrier.

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