Pion form factor and quantum-chromodynamics sum rules

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A new approach is outlined for studying the electromagnetic form factor of the pion in quantum chromodynamics. This new approach is based on the method of quantum-chromodynamics sum rules. A theoretical curve derived for $F_\pi(Q^2)$ agrees well with the experimental data available.

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The application of perturbative quantum chromodynamics to large-momentum-transfer elastic processes in the late 1970s (see Ref. 1, for example, and the review articles in Refs. 2 and 3) was an important step in the development of the quantum chromodynamics of hard processes. In particular, the asymptotic freedom of quantum chromodynamics makes it a simple matter to reproduce the familiar quark counting rules for the hadron form factors in the asymptotic region.[4] In the range of momentum transfers Q^2 presently attainable, however, the experimental data available on the pion and proton form factors are not described satisfactorily by the existing theory.[1-3] This disagreement should not be interpreted as evidence against quantum chromodynamics, since the question here is one of asymptotic formulas, and their extrapolation to moderately large Q^2 would generally not be legitimate. For the pion, for example, hard rescattering is predominant in the limit $Q^2 \to \infty$ (Fig. 1a). A detailed analysis[5] for the region $Q^2 \lesssim 20\ \mathrm{GeV}^2$, however, shows that the average virtuality of the gluon in the diagram in Fig. 1a does not exceed $(300\ \mathrm{MeV})^2$, and in such a situation we cannot rely on perturbation theory because nonperturbative effects are predominant at such small virtualities. One of the previous authors has attempted previously to incorporate such effects in a model.[6] In the present letter we will use the analysis of the pion form factor as an example to outline a new approach to the study of exclusive processes in quantum chromo-

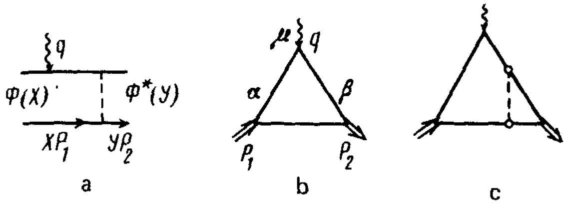

FIG. 1.

dynamics. This new approach is based on the systematic use of the quantum-chromodynamics sum rules,[7] in which nonperturbative effects are taken into account by incorporating in the theory the nonvanishing average gluon and quark vacuum fields.

Proceeding in the spirit of Ref. 7, we consider the three-point amplitude

$$T^{\mu\alpha\beta}(p_1,p_2)=i^2\int e^{ip_1x-ip_2y}<0|T(j^{+\beta}(y)J^{\mu}(0)j^{\alpha}(x))|0>d^4x\,d^4y \qquad (1)$$

(Fig. 1b), where J^{μ} is the electromagnetic current, $j^{\alpha}=\bar{d}\gamma_5\gamma_{\mu}u$ is the axial current (which has a nonvanishing projection on the one-pion state $|P\rangle$: $\langle 0|j^{\alpha}(0)|P\rangle=if_{\pi}P^{\alpha}$, where $f_{\pi}\cong 133$ MeV). The invariant amplitudes T_i in $T^{\mu\alpha\beta}$ depend on the three variables $p_1^2, p_2^2, q^2=(p_1-p_2)^2$. Because of the asymptotic freedom of quantum chromodynamics, we can evaluate $T_i(p_1^2,p_2^2,q^2)$ in the Euclidean region $p_1^2, p_2^2, q^2<-\mu_0^2\sim-(1\text{ GeV})^2$. To extract information on the form factors of physical states, we use the double dispersion relation

$$T_i\ (p_1^2,p_2^2;q^2)=\frac{1}{\pi^2}\int_0^{\infty}ds_1\int_0^{\infty}ds_2\frac{\rho(s_1,s_2,q^2)}{(s_1-p_1^2)(s_2-p_2^2)}+\dots, \qquad (2)$$

where the terms which have not been written out explicitly are polynomials in $p_1{}^2$ and/or $p_2{}^2$. These terms vanish after the Borel procedure, described in Ref. 7, is applied to (2) with respect to $p_1{}^2$ and $p_2{}^2$:

$$\Phi_i\ (M_1^2,M_2^2;Q^2)=\frac{1}{\pi^2}\int_0^{\infty}\frac{ds_1}{M_1^2}\int_0^{\infty}\frac{ds_2}{M_2^2}\rho(s_1s_2;q^2)e^{-\frac{s_1}{M_1^2}-\frac{s_2}{M_2^2}}, \qquad (3)$$

where $Q^2=-q^2$, and Φ_i is the (double) Borel transform of the amplitude T_i. To avoid introducing an asymmetry between the initial and final states, we set $M_1=M_2=M$; it is then possible to rewrite (3) as an integral over $s\equiv s_1+s_2$ and $\xi=s_1/s$.

The invariant amplitude [which we will denote below as $\Phi(M^2,Q^2)$] which is most important for the analysis of the pion form factor is related to the structure $P^{\mu}P^{\alpha}P^{\beta}$, where $P=p_1+p_2$. This amplitude can be distinguished easily by rotating $T^{\mu\alpha\beta}$ with $n_{\alpha}n_{\beta}n_{\mu}$, where n is a lightlike vector having the properties $n^2=0$, $(np_1)=(np_2)\neq 0$, $(nq)=0$. If the quark masses are ignored ($m_{u,d}\lesssim 10$ MeV $\ll\mu_0$), the diagram in Fig. 1b makes the following contribution to Φ:

$$\Phi^{(1b)}(M^2,Q^2)=\frac{3}{4\pi^2M^2}\int_0^1 x\ (1-x)\exp\left\{-\frac{Q^2}{2M^2}\frac{x}{1-x}\right\}dx$$

$$=\frac{1}{2\pi^2M^4}\int_0^{\infty}\frac{s^2(2s+3Q^2)}{(2s+Q^2)^3}e^{-\frac{s}{M^2}}ds. \qquad (4)$$

In the first form given here for the contribution, the variable x is the fraction of the pion momentum which corresponds to the passive quark (in a system of infinite momentum). This representation is most convenient in analyzing the behavior of $\Phi^{(1b)}$ in the limits $Q^2 \to \infty$ and $Q^2 \to 0$. The second form given here for the contribution $\Phi^{(1b)}$ has the spectral representation (3), where an integration is carried out over ξ. With regard to the spectral density $\rho(s_1, s_2; q^2)$, we will assume, as is customary in our approach,[7] that ρ is the sum of a resonant (pion) contribution and a "background." Beginning at a certain value of s, which we will call s_0, this background becomes equal to the free value,[1)] which we adopt in the form required by (4):

$$\rho(s_1, s_2; q^2) = \pi^2 f_\pi^2 F_\pi(Q^2)\,\delta(s_1 - m_\pi^2)\,\delta(s_2 - m_\pi^2) + \theta(s - s_0) \times \frac{s\,(2s + 3Q^2)}{2\,(2s + Q^2)^3}. \tag{5}$$

An important achievement of the method of quantum-chromodynamics sum rules is that the duality interval s_0 is not a free parameter but is instead determined by corrections to (4) which are proportional to powers of $1/M^2$. Taking into account the contributions proportional to $(\alpha_s/\pi)\langle G^\alpha_{\mu\nu} G^\alpha_{\mu\nu}\rangle \cong 0.012$ GeV^4 and $\alpha_s\langle \bar{q}q\rangle^2 = 1.83 \cdot 10^{-4}$ GeV^6 (the numerical values are taken from Ref. 7), and using (3)-(5), we find the following representation for the form factor:

$$f_\pi^2 F_\pi(Q^2) = -\frac{M^2 s_0^2}{2\pi^2}\,\frac{(2s_0 + 3Q^2)}{(2s_0 + Q^2)^3}\exp\left(-\frac{s_0}{M^2}\right) + \frac{3M^2}{4\pi^2}\int_0^{2s_0/(2s_0+Q^2)} x\,(1-x) \times \exp\left\{-\frac{Q^2}{2M^2}\,\frac{x}{1-x}\right\}dx + \alpha_s \langle G^a_{\mu\nu} G^a_{\mu\nu}\rangle / (12\,\pi M^2) + \frac{176\,\pi\,\alpha_s \langle \bar{q} q\rangle^2}{81 M^4} \times \left(1 - \frac{2}{11}\,\frac{Q^2}{M^2}\right). \tag{6}$$

The physical quantity $[F_\pi(Q^2)]$ obviously must not depend on the auxiliary parameter M^2, whose choice is entirely at our disposal. It is not difficult to show that at sufficiently large values of M^2 the M^2 dependence of the right side of (6) is very weak, but that value of the parameter M^2 at which the asymptotic regime is established depend on s_0. If we assume that the "actual" value of s_0 is that at which the region of insensitivity to changes in M^2 is broadest, then for $Q^2 = 2$ GeV^2 we find $s_0 = 1.0$ GeV^2 on this basis. This value is in excellent agreement with the value $s_0 = 1.05$ GeV^2 found from the requirement that the area of our "duality triangle" [equal to $s_0{}^2/2$, according to (5)] must be equal to $(s_0^{svz})^2$, where $s_0^{svz} \cong 0.75$ GeV^2 is the duality interval for the two-point amplitude calculated in Ref. 7.

In choosing M^2 we should take into account the fact that the contribution of the power-law corrections to (6) falls off with increasing M^2, but the contributions of the background [which is taken into account approximately in (5)] increases; conversely, the background contribution decreases with decreasing M^2, and the power-law corrections increase. We accordingly choose the minimum possible M^2 for which the power-law

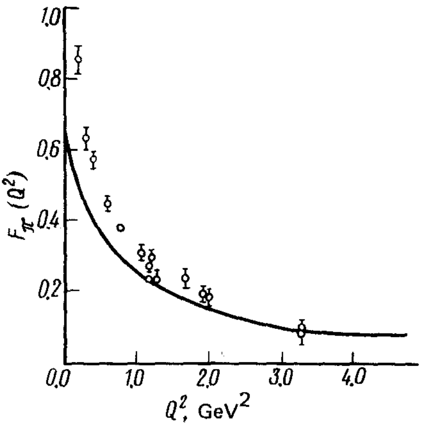

FIG. 2.

corrections (for $Q^2 = 2$ GeV^2) do not exceed 30% of the main contribution. On this basis we find $M^2 = 1.8$ GeV^2.

Representation (6) does not hold for arbitrary values of Q^2. In the region $Q^2 \lesssim m_\rho^2 = 0.6$ GeV^2 the power-law corrections in $1/Q^2$ are potentially dangerous, and on this basis we have the lower boundary $Q^2_{\min} = 0.6$ GeV^2. On the other hand, in the limit $Q^2 \to \infty$ the basic contribution to $\Phi^{(1b)}(Q^2, M^2)$ comes from the region $x \sim M^2/Q^2$, where the passive quark has a virtuality $k^2 \sim M^4/Q^2$. Accordingly, at $Q^2 > M^4/m^2_\rho$ (i.e., for $Q^2 \gtrsim M^4/m_\rho^2 = 5$ GeV^2) we should expect large corrections of the type Q^2/M^2. In fact, in the limit $Q^2 \to \infty$ the amplitude $\Phi^{(1b)}(Q^2, M^2)$ exhibits the behavior $1/Q^4$, while the contribution of order $\langle G^2 \rangle$, say, does not depend on Q^2 [see (6)]. Consequently, the power-law corrections to (6) (which amount to 30% at $Q^2 = 2$ GeV^2) reach nearly 100% at $Q^2 = 6$ GeV^2, and it becomes necessary to also consider some (if not at all) of the succeeding terms in the expansion in $1/M^2$. Figure 2 compares a theoretical curve plotted from (6) (for $s_0 = 1$ GeV^2, $M^2 = 1.8$ GeV^2) with the available experimental data.[8]

In principle, ordinary perturbative corrections must also be taken into account in addition to the power-law corrections (Fig. 1c). Their contribution, however, is suppressed by the factor α_s/π, and at $Q^2 \lesssim 10$ GeV^2 we estimate their contribution to be about 10%. In the limit $Q^2 \to \infty$, however, diagrams like that in Fig. 1c have a $1/Q^2$ behavior, which corresponds to the quark counting rules of Ref. 4 and the asymptotic quantum chromodynamics analysis of Refs. 1-3.

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[1)]This choice was suggested to us by B. L. Ioffe.

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