Sum rules for dipole transitions in finite systems and the giant dipole resonance in nuclei

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An expression for the average energy of dipole photoabsorption is derived in the self-consistent theory of finite Fermi systems. This expression gives a correct description of both the position of the giant dipole resonance in nuclei (even the lightest) and the splitting of this resonance by static deformation. The approach developed here can be generalized in a natural way to other systems. In particular, it gives the average frequency of dipole transitions in an atom and in small metal particles.

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We have worked from the equation for the transition density matrix in the theory of finite Fermi systems[1] to develop a technique for calculating the moments $m_k = \Sigma_s \omega_s^k |\langle s|V_0|0\rangle|^2$ for arbitrary external fields V_0. Simple closed expressions can be derived for the moments m_1 and m_3, and the mean square transition energy in a field V_0 can be expressed in terms of these moments: $\overline{\omega^2} = m_3/m_1$. Let us consider a multicomponent system to which a field $V_0^i = c_i x \exp(i\omega t)$ is applied, along the x axis. If the interaction $\mathcal{F}$ is not retarded and does not contain velocity harmonics, we then have the following expression for the first moment ($\hbar = 1$):

$$m_1 = \frac{1}{2} \sum_i \frac{N_i c_i^2}{m_i}, \tag{1}$$

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where N_i is the number of particles of species i with mass m_i, and c_i is the "charge" of these particles with respect to the field V_0. Expression (1) corresponds to the standard classical sum rule. For the third moment we find, using the consistency conditions,[2]

$$m_3 = \frac{1}{2} \sum_{ik} \left[\frac{c_i}{m_i} \frac{c_k}{m_k} - \frac{c_i^2}{m_i^2} \right] \left(\frac{d\rho^i}{dx} \mathcal{F}^{ik} \frac{d\rho^k}{dx'} \right) \tag{2}$$

(here and below, the parentheses denote an integration over all the coordinates). We see that only the "nondiagonal" interactions ($i \neq k$) make a direct nonzero contribution to m_3; the "diagonal" components of $\mathcal{F}$ affect the densities ρ^i through the consistency condition. For a nucleus, the charges are $c^p = eN/A$ and $c^n = -eZ/A$, and the average dipole-photoabsorption frequency, which is associated with the energy of the giant dipole resonance (GDR), is

$$\omega_{\text{GDR}} = (\overline{\omega_x^2})^{1/2} = \left[-\frac{1}{m} \frac{A}{NZ} \left(\frac{d\rho^n}{dx} \mathcal{F}^{np} \frac{d\rho^p}{dx'} \right) \right]^{1/2}. \tag{3}$$

This result, derived by a definitely quantum-mechanical approach, corresponds to the classical picture[3]: The giant dipole resonance is an oscillation of two subsystems of nucleons with respect to each other with a transition density $\rho_{\text{tr}}^i \sim d\rho^i/dx$ and a frequency determined by the classical mass coefficient and with a stiffness which depends on the neutron-proton interaction $\mathcal{F}^{np}$ (at the nuclear surface). In contrast with the interpretation adopted in Ref. 4, the isovector amplitude $\mathcal{F}^-$ associated with the symmetry energy does not appear explicitly in the result, and the oscillation is a surface oscillation, rather than volume oscillation. We also note that expression (3) has the collective dipole frequency vanishing in the limit $\mathcal{F}^{np} \to 0$. In the non-self-consistent approaches, this limiting case does not occur, although it is obvious from the physical standpoint: If there is no interaction between the different components in the system, the corresponding subsystems can separate without contributing energy to the internal excitations, as in the case of a shift of the common center of mass ($c^n = c^p$ and $m_3 = 0$). In the limit $A \to \infty$ we find from (3) the asymptotic behavior

$$\omega_{\text{GDR}} = \frac{2}{3} \left(\frac{9}{8\pi} \right)^{1/6} \epsilon_F \left(-\frac{f_{\text{surf}}^{np}}{p_F d} \right)^{1/2} A^{-1/6} (1 + \alpha A^{1/3})^{1/2}, \tag{4}$$

where $\epsilon_F(p_F)$ is the Fermi energy (or momentum), d is the diffuseness parameter in the distribution of the nuclear density, $f_{\text{surf}}^{np} \approx (f_{\text{in}}^{np} + f_{\text{ex}}^{np})/2$, and f_{in}^{np} and f_{ex}^{np} are the constants of the theory of finite Fermi systems. If the amplitudes $\mathcal{F}^{np}$ does not depend on the density, then the coefficient α in (4) is $\alpha = 0$, and for this coefficient to reach a value of order unity we would have to introduce a strong interpolation; i.e., we would have to change f^{np} substantially from the internal value f_{in}^{np} to the vacuum value f_{ex}^{np}. Figure 1 shows the energies ω_{GDR} found from (3) with the interaction $\mathcal{F}^{np} = C_0 f_{\text{surf}}^{np} \delta(\mathbf{r}-\mathbf{r})$, where $C_0 = 300$ MeV · F^3 and $f_{\text{surf}}^{np} = -2.0$ (in agreement with the values found for f_{in}^{np} and f_{ex}^{np} through a description of the ground and low-lying collective states of nuclei in the self-consistent theory of finite Fermi systems with a density-dependent interaction[5]). As the ρ^i we use the shell-model densities in the Woods-Saxon potential and the universal parameters from Ref. 6. It is interesting to note the good overall agreement between this rough calculation (the crosses) and the experimental data of Ref. 7 (the circles); the slow

oscillations of $\omega_{GDR}(A)$ are reproduced in a qualitative way. Their amplitude can be smoothed and the agreement with experiment improved if we incorporate pairing in the model for the density ρ^i (these results are indicated by the triangles, which were calculated with a pairing-correlation parameter $\Delta = 1$ MeV). With rare exceptions, the calculated values lie within the experimental errors (usually ±100–250 keV; these errors are not indicated in Fig. 1).

Account of the velocity harmonics in $\mathcal{F}$ leads to an additional factor in (3), which reduces to a renormalization of f_{surf}^{np}. In the simple model with three- and four-particle forces, which lead to linear and quadratic dependences of the effective interaction on the density, we find the following relationship between f_{surf}^{np} (= -2.0) and the basic parameters of the nuclear matter:

$$f_{surf}^{np} = -\left(1 - \frac{2}{3} f_1^{np} \frac{m}{m^*}\right)\left[\frac{m^*}{m}\left(\frac{1}{20}K - \frac{3}{5}\mu + \frac{3}{2}\beta\right)\frac{1}{\epsilon_F^0} + \frac{7}{25}\right]. \quad (5)$$

Here m^* is the effective mass, f_1^{np} is the constant of the first (velocity) harmonic $\mathcal{F}^{np}$, K is the compressibility parameter, μ is the chemical potential, and $\epsilon_F^0 = 38.7$ MeV. The position of the giant dipole resonance is thus determined by not only the symmetry energy β but also other parameters of the mass formula.

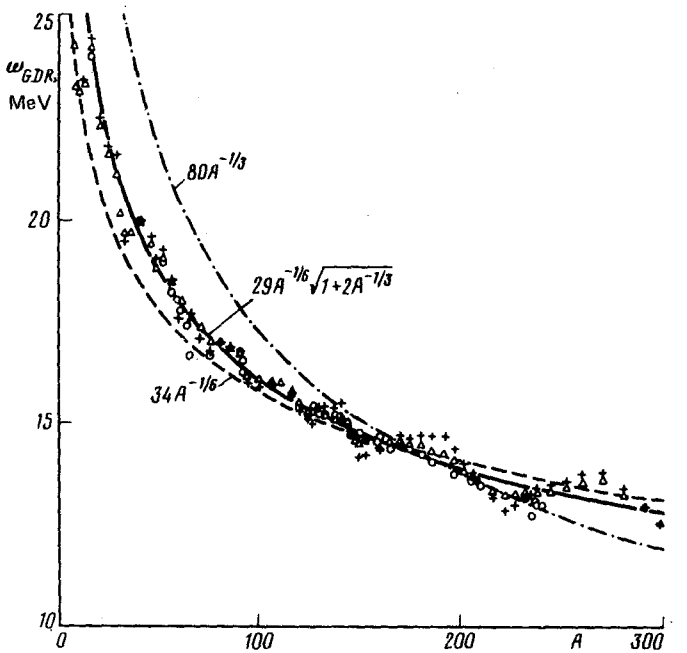

FIG. 1. Dependence of the calculated position of the giant dipole resonance on the mass number A and comparison with the experimental data from Ref. 7 (shown by the circle). Crosses–Results calculated with shell-model densities and the Woods-Saxon potential[6]; triangles–results calculated with allowance for pairing. Asymptotic curves are also shown: Dashed curve–For the Goldhaber-Teller model; dot-dashed curve–for the Steinwedel-Jensen model[8]; solid curve–results of the present calculations.

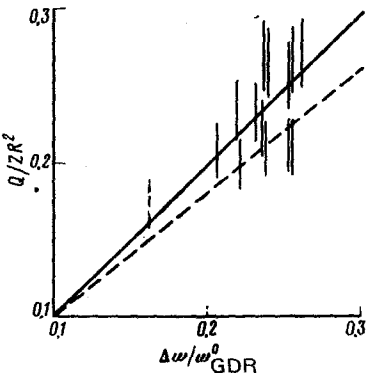

FIG. 2. The quadrupole moment Q_0, in units of ZR^2, as a function of the relative splitting of the giant dipole resonance in deformed nuclei. Solid line (bisector)--Predictions of the present paper; dashed line--predictions of the hydrodynamic model.[9] The vertical line segments show the scatter in the values of Q_0/ZR^2 due to the uncertainty in the values of $R = r_0 A^{1/3}$ ($1.15 \leqslant r_0 \leqslant 1.25$ F). The experimental data on Q_0 and $\Delta\omega/\omega^0_{GDR}$ are taken from Ref. 7 for the following nuclei: ^{150}Nd, ^{152,154}Sm, ^{153}Eu, ^{159}Tb, ^{160}Gd, ^{165}Ho, ^{175}Lu, ^{181}Ta, ^{186}W, ^{232}Th, ^{235,238}U, and ^{237}Np.

The approach can also be used to describe the splitting of the giant dipole resonance in deformed nuclei. It can be seen from (3) that the oscillation frequency (or the stiffness) depends on the direction of the external field. For axisymmetric nuclei, in first order in the deformation, we find the relationship

$$\Delta\omega/\omega^0_{GDR} = Q_0/ZR^2, \tag{6}$$

where Q_0 is the quadrupole moment of the ground state, $\Delta\omega$ is the distance between the maxima on the photoabsorption curve, and ω^0_{GDR} is the position of the resonance for the spherical nucleus of equivalent volume. It can be seen from Fig. 2 that this relation agrees satisfactorily with the data available. We can draw the general conclusion, in contradiction of a widely held opinion, that the giant dipole resonance in nuclei corresponds to a surface (shear) rather than volume (zero-sound) oscillation mode. In macroscopic terms, these results contradict the Steinwedel-Jensen hydrodynamic model,[8] but at the same time they may be regarded as a microscopic justification for the Goldhaber-Teller model.[3]

For systems having a Coulomb attraction between particles of different species (plasmas or metals) we find from (2)

$$m_3 = \frac{e^4}{2\mu^2}\left(\frac{d\rho^i}{dx}\,\frac{1}{|\mathbf{r}-\mathbf{r}'|}\,\frac{d\rho^e}{dx'}\right) = -\frac{e^3}{2\mu^2}\left(\rho^i\frac{d^2\phi^e}{dx^2}\right), \tag{7}$$

where $\mu = m_e m_i/(m_e + m_i)$ is the reduced mass of the electron and ion, and ϕ^e is the electrostatic electron potential. For simple geometries we can use the Poisson equation, $\Delta\phi^e = -4\pi e\rho^e$, directly, and for the average energy of the dipole transitions we find

$$\omega_D = \omega_p/\gamma, \tag{8}$$

where $\omega_p = (4\pi e^2\rho^e/\mu)^{1/2}$ is the plasma frequency, and γ is a geometric factor, equal to 1, $\sqrt{2}$, and $\sqrt{3}$ for planar, cylindrical, and spherical systems, respectively. These results are quite well known in the physics of aerosols.

In a corresponding way we can find the average frequency of the dipole transitions in an atom:

$$\omega_D = \left[\frac{4\pi}{3}\frac{e^2}{Z_e m_e}(\rho^p \rho^e)\right]^{1/2} \approx \left[\frac{4\pi}{3}\frac{e^2}{m_e}\rho^e(0)\frac{Z}{Z_e}\right]^{1/2}, \quad (9)$$

where Z_e is the number of electrons, Z is the number of protons in the nucleus, and $\rho^e(0)$ is the electron density at the center of the nucleus. For estimates we can use hydrogen-like wave functions and consider the contribution of only two s electrons to $\rho^e(0)$. In this case, for a neutral atom, we find

$$\omega_D \approx \sqrt{\frac{8}{3}} m_e e^4 Z^{3/2} \approx 44.4\, Z^{3/2}\ (\text{eV}). \quad (10)$$

The results of exact calculations will be reported in detail separately.

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