# Nature of current states in metals

N. M. Makarov and V. A. Yampol'skiĭ
*Institute of Radio Physics and Electronics, Academy of Sciences of the Ukrainian SSR*

(Submitted 22 January 1982; resubmitted 2 April 1982)

Pis'ma Zh. Eksp. Teor. Fiz. **35**, No. 10, 421–424 (20 May 1982)

A theory is derived for the current rectification in a metal irradiated by large-amplitude radio waves $\mathscr{H}$ in a parallel magnetic field $h_0$. In the anomalous skin effect, the hysteresis loop of the induced magnetic moment, plotted as a function of $h_0$, is predicted to lie in the interval $(-2\mathscr{H}, 2\mathscr{H})$ and to have a universal behavior. The induced magnetic field is comparable to the amplitude of the incident wave.

PACS numbers: 72.15.Gd, 75.60.Ej, 78.70.Gq

1. "Current states" have recently been discovered[1-5] in several metals: In pure samples irradiated with large-amplitude radio waves, a rectified current arises and excites a constant magnetic field **h**. The induced magnetic moment exists even in the absence of an external constant field $\mathbf{h}_0$, and there is hysteresis in its behavior as a function of $h_0$. The kinetic coefficients which depend on the magnetic field exhibit a corresponding hysteresis.

Babkin and Dolgopolov[6] have explained the excitation of current states in the case $\mathscr{H} \gg h_0, h$. They noted that the nonuniform field which shapes the paths of the electrons is the sum of the magnetic field of the wave, $\mathbf{H}(x,t)$, and the uniform, constant magnetic field $\mathbf{h}_0$ [we are assuming here that the vectors $\mathbf{H}(x,t)$ and $\mathbf{h}_0$ are collinear, that the induced field $\mathbf{h}(x)$ is the average value of $\mathbf{H}(x,t)$ over a period of the incident wave, that the $x$ axis is directed into the metal, and that the $z$ axis is parallel to the magnetic field vectors]. The variable component of the field $H(x,t)$ oscillates and is damped rapidly over a distance on the order of the skin thickness $\delta$, while the induced field $h(x)$ changes from zero at the $x=0$ boundary to $h(\infty)$. Accordingly, the nature of the paths of the effective electrons depends on the mutual orientation of the resultant field at the surface of the metal, $H(0,t)+h_0=2\mathscr{H}\cos\omega t+h_0$, and that in the interior of the metal, $H(\infty,t)+h_0=h(\infty)+h_0$. During those half-periods in which $\mathbf{H}(0,t)+\mathbf{h}_0$ and $\mathbf{H}(\infty,t)+\mathbf{h}_0$ are parallel, the paths are similar to that shown in Fig. 1a; when these fields are antiparallel, they are similar to that shown in Fig. 1b. We thus see that the conductivities are different in the different half-periods, and a rectified current arises at the surface of the sample.

For current states to exist, the path of an effective electron in the skin, $L\sim(4cp_F\delta/e\mathscr{H})^{1/2}$, would have to be much shorter than the mean free path $l=v_F/\nu$; furthermore, the phase of the wave must remain constant over the mean free time,

$$4\,cp_F\,\delta\,/\,el^2 \ll \mathscr{H}, \quad \omega \ll \nu . \tag{1}$$

2. We have derived a theory for current states over a broad range of the magnetic field $h_0$ under the conditions

  

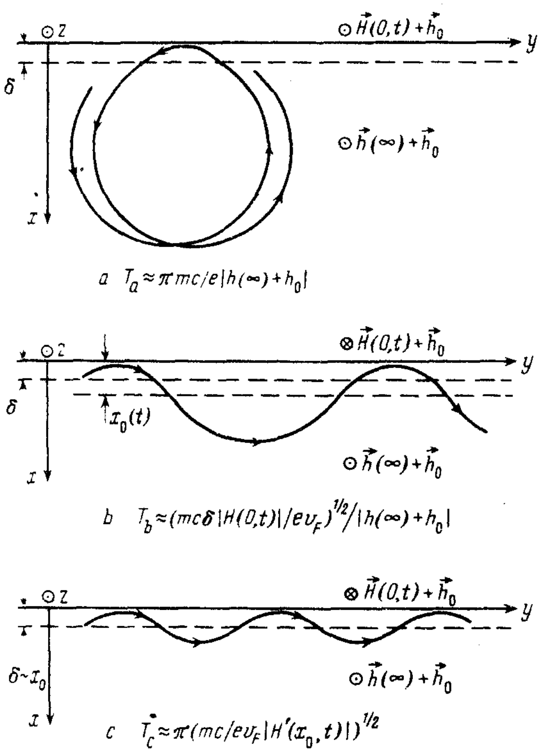


FIG. 1.

$$b \ll \frac{|h(\infty)+h_0|}{2\mathcal{H}}, \quad b = \left(\frac{4cp_F\delta}{el^2\mathcal{H}}\right)^{1/2} \ll 1. \tag{2}$$

These conditions are consistent with $|h(\infty)+h_0|\sim 2\mathcal{H}$.

According to Ref. 6, the nature of the electron motion and thus the fact that current states appear depend on whether there exists a plane $x=x_0(t)$ in the sample on which $H(x_0,t)+h_0=0$. In those time intervals in which there is no such plane, the electrons move along paths like that in Fig. 1a. These paths differ little from a closed Larmor orbit in the field $h(\infty)+h_0$, and the period of the motion along these paths, $2T_a$, depends on only $|h(\infty)+h_0|$. If $|h_0|\leqslant 2\mathcal{H}$ there is a time interval during the wave period $2\pi/\omega$ in which the spatial distribution of $H(x,t)+h_0$ is a sign-changing distribution. Under condition (2) the $x=x_0(t)$ plane lies near the surface of the metal $(x_0\sim\delta)$, and the paths in Fig. 1b become symmetric with respect to the paths $x=x_0(t)$ (Fig. 1c) in the magnetic field $H'(x_0,t)\delta$. The period of the motion along these wavy orbits, $2T_c\sim L/v_F$, is much smaller than both $2T_a$ and $l/v_F$, by virtue of (1). The conductivity of the metal in the sign-changing field is thus significantly higher than that in a constant-sign field; the ratio of these conductivities is

$$a = \frac{l}{L}\,\mathrm{th}\,(\nu T_a) \gg 1. \tag{3}$$

This result is one of the qualitative distinctions between the case in (2) and the situation in Ref. 6. Another distinction is that now the length of the time interval during which there is an $x = x_0(t)$ plane in the sample is not equal to the half-period of the wave and is a function of the induced field $h(\infty)$. All these differences lead to new dependences of $h(\infty)$ on $h_0$ and $\mathscr{H}$ and to a new shape for the hysteresis loop.

3. We will omit the details and proceed to the basic results of the calculations. The magnetic-field distribution in the metal, $H(x,t)$, is described by

$$H(x,t) = \sum_{n=-\infty}^{\infty} H_n \exp\left[-\frac{x}{\delta_n} - in\xi(t)\right]. \qquad (4)$$

The function $\xi(t)$ satisfies the condition $\xi(t+2\pi/\omega) = \xi(t) + 2\pi$ and, along with $\delta_n$, is determined from Maxwell's equations. The conductivity of the $n$th component of the current density is

$$\sigma_n(t) = \frac{3\pi}{4}\sigma_0 \frac{\delta_n}{l} \operatorname{cth}(\nu T_a)\tilde{\sigma}(t), \qquad (5)$$

$$\tilde{\sigma}(t) = \theta\left(\frac{2\mathscr{H}\cos\omega t + h_0}{h(\infty) + h_0}\right) + a\theta\left(-\frac{2\mathscr{H}\cos\omega t + h_0}{h(\infty) + h_0}\right).$$

Here $\sigma_0$ is the static conductivity of the metal, and $\theta(x)$ is the Heaviside unit step function. For simplicity, the spatial dispersion is taken into account in (4) and (5) in the noneffectiveness model (see Ref. 7, for example), which yields correct results within constant, real factors. In accordance with (5), we find the following expressions from Maxwell's equations:

$$\delta_n = \left(\frac{c^2 l \operatorname{th}(\nu T_a)}{3\pi^2\omega\sigma_0}\frac{\rho}{|n|}\right)^{1/3} \exp\left(\frac{i\pi}{6}\frac{n}{|n|}\right), \qquad (6)$$

$$\xi(t) = \frac{\omega}{\rho}\int_0^t \frac{dt'}{\tilde{\sigma}(t')},$$

$$\rho = \frac{\omega}{2\pi}\int_0^{2\pi/\omega}\frac{dt}{\tilde{\sigma}(t)} \approx \frac{1}{\pi}\arccos\left(-\frac{h_0}{2\mathscr{H}}\operatorname{sign}\frac{h(\infty)+h_0}{2\mathscr{H}}\right).$$

The coefficients $H_n$ in (4) are found from the boundary condition $H(0,t) = 2\mathscr{H}\cos\omega t$:

$$H_n = \frac{\mathscr{H}\omega}{\pi\rho}\int_0^{2\pi/\omega}\frac{dt}{\tilde{\sigma}(t)}\exp[in\xi(t)]\cos\omega t. \qquad (7)$$

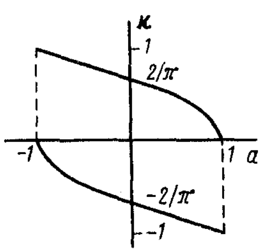


FIG. 2.

In the limit $x \to \infty$, only a single term, that with $n = 0$, remains in expression (4) for $H(x,t)$. Therefore, $h(\infty) = H_0$. As a result, we find two nontrivial solutions for the induced constant magnetic field in the interior of the sample according to (7):

$$\frac{h(\infty)}{2\mathcal{H}} = \pm \left(1 - \frac{h_0^2}{4\mathcal{H}^2}\right)^{1/2} \Big/ \arccos\left(\mp \frac{h_0}{2\mathcal{H}}\right), \tag{8}$$

and we find a single trivial solution, $h(\infty) = 0$, at $h_0 = 0$. Figure 2 shows a plot of $h(\infty)$ against $h_0$ in units of $2\mathcal{H}$; here $\kappa = h(\infty)/2\mathcal{H}$ and $a = h_0/2\mathcal{H}$. In contrast with the results of Ref. 6, the $\kappa(a)$ dependence is a universal function, which contains no parameters. In particular, $\kappa(a)$ does not depend on the electrodynamics of the metal. The reason is the following circumstance: The quantity $\tilde{\sigma}(t)$ appears in the denominators of the integrands in (6) and (7). Therefore, those time intervals during which the field is sign-changing do not play a role. What is important here is the fact that there is a significant increase in the conductivity—not the exact value of the conductivity. For this reason, in deriving Eq. (8) it is not necessary to incorporate spatial-dispersion effects correctly. In the second and fourth quadrants in Fig. 2, in neighborhoods of the points $a = \mp 1$ which are small to the extent that $b$ is small, condition (2) is violated. In these neighborhoods there is a transition from the solution found here, (8), to that which was analyzed in Ref. 6.

The excitation of current states occurs after a threshold is reached: Specifically, the hysteresis loops appear only after the amplitude $\mathcal{H}$ becomes larger than the critical value $\mathcal{H}_{cr}$. Since the results of the theory derived here apply to the case of a well-developed hysteresis ($b \ll 1$), $\mathcal{H}_{cr}$ cannot be determined in this theory. The loop shown in Fig. 2 occurs at amplitudes $\mathcal{H}$ far from the threshold. One possibility for experimentally testing these calculations would be to measure the quantity $h(\infty) = \pm 4\mathcal{H}/\pi$ at $h_0 = 0$.

We wish to thank É. A. Kaner, V. F. Gantmakher, and V. T. Dolgopolov, who drew our attention to the problem of current states in the first place, for many useful discussions and stimulating criticism. We are also indebted to L. M. Fisher, V. Ya. Demikhovskiĭ, V. A. Gasparov, and I. F. Voloshin for interest in this work and for a discussion of the results.

1. V. T. Dolgopolov, Usp. Fiz. Nauk **130**, 241 (1980) [Sov. Phys. Usp. **23**, 134 (1980)].
2. V. T. Dolgopolov and L. Ya. Margolin, Pis'ma Zh. Eksp. Teor. Fiz. **17**, 233 (1973) [JETP Lett. **17**, 167 (1973)].
3. V. T. Dolgopolov, Zh. Eksp. Teor. Fiz. **68**, 355 (1975) [Sov. Phys. JETP **41**, 173 (1975)].
4. V. T. Dolgopolov and S. S. Murzin, Pis'ma Zh. Eksp. Teor. Fiz. **23**, 213 (1976) [JETP Lett. **23**, 190 (1976)].
5. V. V. Boĭko, L. V. Ovchinnikova, and G. N. Landysheva, Pis'ma Zh. Eksp. Teor. Fiz. **32**, 432 (1980) [JETP Lett. **32**, 408 (1980)].
6. G. I. Babkin and V. T. Dolgopolov, Solid State Commun. **18**, 713 (1976).
7. É. A. Kaner, N. M. Makarov, V. L. Fal'ko, and V. A. Yampol'skiĭ, Zh. Eksp. Teor. Fiz. **73**, 1400 (1977) [Sov. Phys. JETP **46**, 737 (1977)].

Translated by Dave Parsons
Edited by S. J. Amoretty