Polarizability of π mesons

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The amplitude for the reaction $\gamma\gamma\to\pi\pi$ is calculated in a nonlocal quark model. Numerical values are found for the electric (α_π) and magnetic (β_π) polarizabilities of the pions: $\alpha_{\pi^\pm}=0.40\alpha/m_\pi^3$, $\beta_{\pi^\pm}=-0.39\ \alpha/m_\pi^3$, $\alpha_{\pi^0}=0.1\alpha/m_\pi^3$, $\beta_{\pi^0}=-0.4\alpha/m_\pi^3$.

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Collisions of mesons with photons offer a unique opportunity for studying the electromagnetic properties of mesons. At low energies these properties are characterized by a finite number of parameters, which are determined by expanding the amplitudes for meson-photon precesses in the frequencies of the γ rays. In particular, the electric and magnetic polarizabilities, α_π and β_π, respectively, of the mesons are introduced for a description of the effective interaction of hadrons with an external electromagnetic field[1,2] :

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$$V_{int} = -\frac{\alpha_\pi}{2}E^2 - \frac{\beta_\pi}{2}H^2. \qquad (1)$$

The low-energy characteristics of mesons have recently attracted significantly more interest. One reason is the development of more sophisticated methods for measuring meson properties such as the electromagnetic radius and the polarizability,[10] and another is the development of new theoretical models and approaches in the theory of elementary particles (see the review article by Petrun'kin[3]).

Theoretical predictions of the polarizabilities α_π and β_π for π mesons can be found from the amplitude for the reaction $\gamma\gamma \to \pi\pi$. The calculation of this amplitude presupposes an understanding of the strong interactions of pions. Since we do not have such a theory, a variety of low-energy assumptions and approximations have been used in the literature to calculate α_π and β_π. For example, Terent'ev[1] calculated the amplitude for the reaction $\gamma\gamma \to \pi\pi$ on the basis of current algebra and the partial conservation of axial-vector current. Terent'ev also determined the relationship between this amplitude and the polarizabilities of the π meson. Volkov, Pervushin, and Ebert[4,5] have used quantum field theory with chiral-invariant Lagrangians to study the reaction $\gamma\gamma \to \pi\pi$. Several calculations of the π-meson polarizability[6,7,9] have been carried out in various versions of the σ model. We might also mention Refs. 2, 3, 8, and 11, where α_π and β_π were studied by methods of dispersion sum rules and nonrelativistic quark models.

In the present letter we will use a nonlocal quark model[12] to calculate the electric (α_π) and magnetic (β_π) polarizabilities of the π mesons. Figure 1 shows the diagrams corresponding to the reaction $\gamma\gamma \to \pi\pi$ in this model. The low-energy approximation allows us to restrict our discussion of the $\gamma\gamma \to \pi\pi$ amplitude to terms of zeroth and second orders in the external momenta.

Taking the Born term into account, we can write the contribution of the diagrams in Fig. 1 to the $\gamma\gamma \to \pi\pi$ amplitude as follows (the calculation technique was described in Ref. 13):

$$\langle \pi^a(K_3)\,\pi^b(K_4)|S|\gamma^{\lambda_2}(K_2)\gamma^{\lambda_1}(K_1)\rangle = C^{\lambda_1\lambda_2}_{\mu\nu}\, T^{\mu\nu}_{ab}(K_3, K_4|K_1, K_2). \qquad (2)$$

Here

$$T^{\mu\nu}_{ab} = 2e^2\left\{(\delta_{ab} - \delta_{a3}\delta_{b3})\left[g_{\mu\nu} - \frac{K_{4\nu}K_{3\mu}}{K_1K_4} - \frac{K_{4\mu}K_{3\nu}}{K_1K_3} + \gamma_{\pi^\pm} d_{\mu\nu} + \frac{\gamma^V_{\pi^\pm}}{m_\pi^2} f_{\mu\nu}\right] - \delta_{a3}\,\delta_{b3}\left(\gamma_{\pi^\circ} d_{\mu\nu} - \frac{\gamma_{\pi^\circ}}{m_\pi^2}\rho_{\mu\nu}\right)\right\},$$

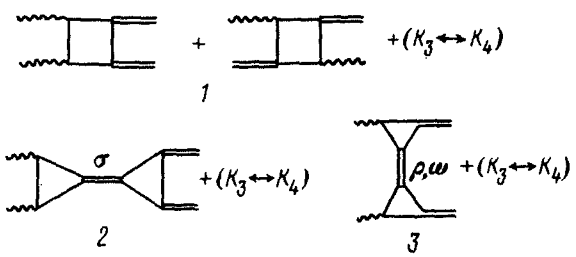

FIG. 1.

$$f_{\mu\nu} = K_{1\mu} q_\nu (K_2 q) + K_{2\nu} q_\mu (K_1 q) - g_{\mu\nu} (qK_1)(qK_2) - q_\nu q_\mu (K_1 K_2),$$

$$q = K_3 - K_2, \quad \gamma_\pi = \gamma_\pi^q + \gamma_\pi^\sigma + \gamma_\pi^V, \quad \gamma_{\pi^\pm}^q = -\frac{ia}{6\pi m_\pi^2} (Lm_\pi)^2 h_P \frac{N(\xi)}{e^2 L^2},$$

$$\gamma_{\pi^\circ}^q = \frac{40}{9e^2} \frac{\pi a}{m_\pi^2} (Lm_\pi)^2 h_P, \qquad \gamma_{\pi^\pm}^V = \frac{8\pi a}{m_\pi^2 e^2} \frac{m_\pi^2}{m_\rho^2 - m_\pi^2} (Lm_\pi)^2 h_P^2 K_{PV}^2(\xi),$$

$$\gamma_{\pi^\pm}^\sigma = \gamma_{\pi^\circ}^\sigma = \frac{20\pi a}{27 e^2 m_\pi^2} \left(\frac{m_\pi}{m_\sigma}\right)^2 256 h_P^2 V_5(\xi),$$

$$C_{\mu\nu}^{\lambda_1 \lambda_2} = \frac{i\delta(K_1 + K_2 - K_3 - K_4)}{(4\pi)^2 \sqrt{\omega_1 \omega_2 K_{03} K_{04}}} e_\nu^{\lambda_1} e_\mu^{\lambda_2}, \quad d_{\mu\nu} = g_{\mu\nu}(K_1 K_2) - K_{1\mu} K_{2\nu},$$

$$\gamma_{\pi^\circ}^V = \gamma_{\pi^\pm}^V + \frac{9 m_\pi^2}{m_\omega^2 - m_\pi^2} \frac{8\pi a}{m_\pi^2 e^2} (L m_\pi)^2 h_P^2 K_{PV}^2(\xi), \qquad a = e^2/4\pi \cong 1/137,$$

a and b are isotopic indices, K_1 and K_2 are the momenta of the photons, $e_\nu^{\lambda_1}$ and $e_\mu^{\lambda_2}$ are their polarization vectors, and K_3 and K_4 are the momenta of the π mesons. We have also taken into account the relations $K_1^2 \equiv K_2^2 = 0$, $K_3^2 = K_4^2 = m_\pi^2$, $K_1 l_1 \equiv K_2 l_2 = 0$. The parameters of the model are $L = (320\ \text{MeV})^{-1}$, $\xi = 1.4$, and $h_P = 0.13$; the functions $N(\xi)$, $V_5(\xi)$, and $K_{PV}(\xi)$, which arise in calculations by the nonlocal quark model, are given in Refs. 13 and 14.

If we were to ignore diagram 3 in Fig. 1 [i.e., if we were to set $\gamma_\pi^V = 0$ in (2)], the $\gamma\gamma \to \pi\pi$ amplitude would have the single gradient-invariant structure $d_{\mu\nu}$, so that the polarizabilities α_π and β_π in this approximation would be

$$a_{\pi^\pm} = -\beta_{\pi^\pm}^{(0)} = \frac{e^2}{m_\pi} (\gamma_{\pi^\pm}^q + \gamma_{\pi^\pm}^\sigma), \quad a_{\pi^\circ} = -\beta_{\pi^\circ}^{(0)} = \frac{e^2}{m_\pi} \left(\gamma_{\pi^\circ}^q + \gamma_{\pi^\circ}^\sigma\right). \tag{3}$$

The contributions to the $\gamma\gamma \to \pi\pi$ amplitude of diagrams with the exchange of ρ and ω mesons contain, in addition to $d_{\mu\nu}$, another gradient-invariant structure, $f_{\mu\nu}$. For this dia-

TABLE I.

Type of diagram	$a_{\pi^\pm} (a/m_\pi^3)$	$\beta_{\pi^\pm} (a/m_\pi^3)$	$a_{\pi^\circ} (a/m_\pi^3)$	$\beta_{\pi^\circ} (a/m_\pi^3)$
(1)	– 0.05	0.05	– 0.35	0.35
(2)	0.45	– 0.45	0.45	– 0.45
(3)	0	0.01	0	0.06
Resultant value	0.40	– 0.39	0.10	– 0.04

gram, α_π and β_π may therefore differ in both magnitude and sign. Analysis of the terms with γ_π^V in (2) shows that it makes no contribution to the electric polarizability, while its contribution to the magnetic polarizability is nonzero (it makes a substantial contribution to β_{π^0} because of ω exchange). The electric polarizability of the pions is thus determined from (3) according to the nonlocal quark model, while for the magnetic polarizability we have

$$\beta_{\pi^\pm} = \beta^{(0)}_{\pi^\pm} + \frac{e^2}{m_\pi}\gamma^V_{\pi^\pm}, \qquad \beta_{\pi^0} = \beta^{(0)}_{\pi^0} + \frac{e^2}{m_\pi}\gamma^V_{\pi^0}. \tag{4}$$

The results calculated for α_π and β_π from Eqs. (3) and (4) are listed in Table I. There is a large discrepancy between the values of α_π and β_π calculated in the different models, so that measurements of these polarizabilities are of fundamental importance for constructing a theory for the interaction of mesons with an electromagnetic field. We believe it particularly important to determine the sign of the electric polarizability of the neutral pion, α_{π^0}, since this quantity is the most sensitive to assumptions regarding the mechanism for strong interactions of π mesons at low energies.

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