Soliton bound states in long Josephson junctions

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An analysis shows that stable localized states of fluxons can exist in a long Josephson junction with local inhomogeneities. A qualitative study is made of their spectrum. A change in the external magnetic field at the edge of the junction may cause, in addition to gradual changes in these states, abrupt changes to other states near bifurcation points in the spectrum.

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We do not have an adequate theory for the interaction of solitons (fluxons) in long Josephson junctions with inhomogeneities, which are of considerable interest for applications.[1-3] An exact solution has been derived for the problem of the static distributions of the magnetic flux $\Phi(x)$ in a uniform long Josephson junction of finite size,[4] and perturbation theory has been used to study the interaction of a fluxon with an inhomogeneity which is a repulsive barrier.[1,3] In the present letter we will show that static states of long Josephson junctions [i.e., $\Phi(x)$ distributions] of a new type arise if the junctions contain attractive inhomogeneities. These new states have some physical properties of applied interest. They cannot be derived by perturbation theory.

It has been established elsewhere that the propagation of fluxons in a long Josephson junction is described by the equation ($\phi_{xx} = \partial^2\phi/\partial x^2$, etc.)

$$L(L^{-1}\phi_x)_x - LC\phi_{tt} - \lambda_J^{-2}(x)\sin\phi = \overline{\alpha}\,\phi_t - \beta\phi_{xxt} + \overline{\gamma}, \tag{1}$$

where $\phi(x, t) = 2\pi\Phi(x, t)/\Phi_0$; Φ_0 is the quantum of magnetic flux; $\lambda_J^2 = \Phi_0/2\pi L I_0$; L, C, and I_0 are respectively the inductance, capacitance, and critical Josephson current per unit length; the parameters $\bar{\alpha}$ and $\bar{\beta}$ describe the dissipation; and $\bar{\gamma}$ describes the external current (see Ref. 1 for a detailed description of long Josephson junctions, a derivation of this equation, and the standard notation). Equation (1) must be supplemented with boundary conditions. For an infinite junction, for example, we would use $\phi(\pm\infty, t) = 2\pi N_\pm$, where $N_\pm$ are integers; for a semi-infinite junction $(0 \leqslant x < +\infty)$, we would use $\phi_x(0, t) = h_0$ and $\phi(+\infty, t) = 2\pi N_+$, where h_0 is the external field at the end of the junction. We consider inhomogeneities which are determined by the local change in $\lambda_J(x)$ over intervals short in comparison with the value of λ_J in the homogeneous regions. Such inhomogeneities could be produced by arranging local changes in I_0 at a constant value of L. In this case, a static $(\phi_t \equiv 0)$ state of the junction with $\bar{\gamma} = 0$ can be described by the equation

$$\phi'' = \left\{1 - \sum_{i=1}^{n} \mu_i \delta(x - x_i)\right\} \sin\phi, \qquad x_i < x_{i+1}, \tag{2}$$

where $\phi' = \phi_x$, $\phi'' = \phi_{xx}$, and λ_J is adopted as the unit of length. With $\mu_i < 0$ (an increase in I_0), an inhomogeneity is a "microscopic short circuit," while at $\mu_i > 0$ (a decrease in I_0) the inhomogeneity is a "microscopic resistance," and we will assume below that the condition $\mu_i < 1$ holds. In perturbation-theory terms we might say that the microscopic short circuit repels a fluxon, while the microscopic resistance attracts it.

Using the known exact solutions of Eq. (2) for the homogeneous regions[4] along with the conditions on the jump in the magnetic field ϕ' at the points x_i, we can reduce the problem of solving (2) with arbitrary boundary conditions to the problem of determining the roots of a transcendental equation—a very complicated one, in general. Corresponding to each of these roots is a state of the long Josephson junction, and the number of roots depends in a nontrivial way on the parameters x_i, μ_i, and h_0. As these parameters are changed, new states may be generated (a bifurcation). New branches appear in the energy spectrum, determined from

$$\mathcal{E} = \sum_{i=0}^{n} \int_{x_i}^{x_{i+1}} dx[\phi'^2/2 + 2\sin^2(\phi/2)] - \sum_{i=1}^{n} 2\mu_i \sin^2(\phi(x_i)/2)\,; \tag{3}$$

these new branches correspond to the appearance of at least two energy-degenerate solutions (some simple examples of this effect were studied in Refs. 5 and 6 on the basis of a different model). The few lowest-energy states are usually stable with respect to small fluctuations and may exist for an unlimited time. Of particular interest for applications is the possibility of continuously controlling these states, by (for example) changing the magnetic field h_0 far from the bifurcations and by arranging a rapid switching to a state with a lower energy at a bifurcation point. The details will be published separately; here we wish to discuss some simple examples which give a clear picture of the basic effects.

For a study of the qualitative nature of the states of the junction it is convenient to use a (ϕ, ϕ') diagram, as in Fig. 1, making use of the fact that the quantity

$$\phi'^2/4 - \sin^2(\phi/2) = k^2 - 1 = \epsilon \tag{4}$$

is conserved in the homogeneous regions. The state is described by a curve consisting of arcs of constant values of k. At the points x_i, the field ϕ' changes abruptly by an amount

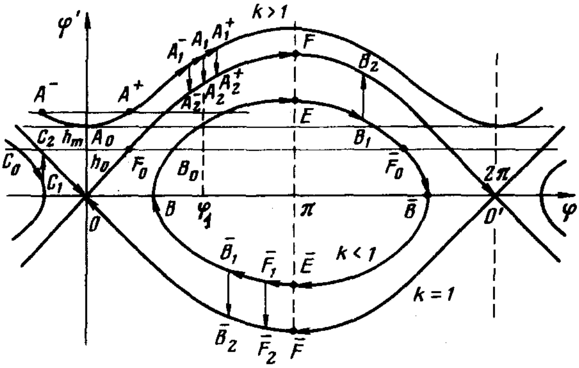

FIG. 1.

$-\mu_i \sin\phi(x_i)$, and there is a corresponding abrupt change in k. The solution in the homogeneous region can be written in the form $\cos(\phi/2) = -k\,\mathrm{sn}(x + x_0, k)$, where sn is the Jacobian elliptic sine.[7] For an infinite junction with a single microscopic resistance, μ_1 at the point $x = x_1 = 0$, the only solutions which exist (aside from $\phi \equiv 0$) are those corresponding to the curves OFO' and $O'\bar{F}O$, which describe a bound state of an undistorted fluxon or antifluxon, localized at the point $x = 0$ $[\phi(0) = \pi]$, with an energy $\mathscr{E}_0 - 2\mu_1$, where $\mathscr{E}_0$ is the energy of a free fluxon at rest. States of this sort also exist in a Josephson junction of finite size (curves $BE\bar{B}$ and $\bar{B}\bar{E}B$ in Fig. 1). For a microscopic short circuit, such solutions do exist formally, but the corresponding states are unstable: The fluxon is repelled and moves away from the microscopic short. Even this simple example demonstrates the important advantages of the microscopic resistance over the microscopic short circuit for applications involving the storage and switching of fluxons in long Josephson junctions (cf. Refs. 1 and 2). A fluxon can be localized precisely near a microscopic resistance; the position of its center is stable with respect to fluctuations, and the oscillations which arise have a small amplitude and are rapidly damped. In contrast, a fluxon trapped between two microscopic short circuits may oscillate with a significant amplitude around an equilibrium position. In order to suppress this oscillation, it will be necessary to increase the parameters $\bar{\alpha}$ and $\bar{\beta}$, but this measure would degrade the characteristics of the junction. Localized states in more complex systems with microscopic resistances have similar properties.

In addition, some other effects arise in these more complicated systems. To illustrate these effects, we consider the case of a semi-infinite junction with a microscopic resistance μ_1 at the point x_1. If $h_0 = 0$, then (aside from the ground state, $\phi \equiv 0$) there can be only states of the type BB_1B_2O' and $\bar{B}\bar{B}_1\bar{B}_2O$, which arise at sufficiently large values $x_1 \geqslant x_m(\mu_1)$, and x_m is a bifurcation point. (As x_1 is increased further, solutions appear in which the magnetic field oscillates in the interval $0 < x < x_1$.) States of this type also exist in the case $h_0 \neq 0$ ($B_0B_1B_2O'$ and $\bar{F}_0\bar{F}_1\bar{F}_2 0$). In this case, the energy degeneracy of the states of the soliton and antisoliton types is lifted, and the energy of the antisoliton state $\bar{F}_0\bar{F}_1\bar{F}_2O$ may even be lower than the energy of the soliton state $B_0B_1B_2O'$. If $h_0 = 2/\cosh\chi_1$, there may exist a stable, bound fluxon state F_0FO for which $\phi(x_1) = \pi$ and which is localized at the point x_1. As h_0 is changed, this state changes smoothly into a state of the type $B_0B_1B_2O'$ or $A^+A_1^+A_2^+O'$. To show how a rapid switching may arise, we note that in the present case the transcendental equation written above takes the form

$$k_0 \mathrm{sn}(x_1 + x_0, k_0) = c(k_0) \equiv \cos(\phi(x_1)/2), \tag{5}$$

where $c(k_0)$ is the solution of the equation for the discontinuity ϕ' at the point x_1,

$$\mu_1 c(1 - c^2)(2 + \mu_1 c) = 1 - k_0^2 \equiv -\epsilon, \qquad |c| \leqslant 1, \tag{6}$$

and x_0 is eliminated with the help of the boundary condition $h_0 = 2k_0 \mathrm{cn}(x_0, k_0)$. There is a state corresponding to each root $k_0(x_1, \mu_1, h_0)$. The energy of each such state can be calculated easily from (3) and (4), by making use of the conditions $k = k_0$ at $x < x_1$ and $k = 1$ at $x > x_1$. Equation (6) has two roots, $c_\pm$, if $|\epsilon| < \epsilon_m$, where $\epsilon_m = \epsilon(c_m)$ and c_m is the root of the equation $d\epsilon/dc_m = 0$. With $\epsilon = \pm\epsilon_m$, the two roots merge, and at $|\epsilon| > \epsilon_m$ they vanish. The value $k_0^2 = k_m^2 = 1 + \epsilon_m$ corresponds to a nontrivial bifurcation point along h_0, at $h_0 = h_m$ if $\phi(0) = \pi$, so that the total flux across the junction is precisely equal to the quantum of flux, Φ_0. If μ_1 and x_1 are such that at $h_0 = h_m$ there is a state $A_0 A_1 A_2 O'$ corresponding to $k_0 = k_m$, then at $h_0 > h_m$ this state splits into $A^+ A_1^+ A_2^+ O'$ and $A^- A_1^- A_2^- O'$, and there is a rapid switching at $h < h_m$. Working in a similar way, we can find a bifuraction at which the solution $C_0 C_1 C_2 O$, with the minimum energy and magnetic field, vanishes. This state arises when, with increasing h_0, the total flux reaches the value $\Phi_0/2$ (the state $EB_1 B_2 O'$).

In summary, the most important distinctive property of long Josephson junctions with microscopic resistances is the existence of rigidly localized, bound states of the magnetic flux. An attempt might be made to observe these states directly, by measuring the distribution $\Phi(x)$ along the junction, or indirectly, by detecting the oscillations in a bound fluxon caused by small external perturbations. The fundamental frequency of this oscillation, ω_0, depends on only μ_i, x_i, and h_0 (for the state $F_0 F O'$, for example, with a small value of h_0 we would have $\omega_0 \sim \sqrt{\mu_1/2}$). The existence of bound states and the existence of bifurcations along h_0 distinguish the dependence of the maximum Josephson current $I_m(h_0)$ from the corresponding dependence for a homogeneous long Josephson junction.[4] The rapid switching at the bifurcation point upon a slow change in h_0 can lead to microwave emission. The characteristics of all these effects are determined by the parameters of the static solutions. It would also be interesting to study the voltage–current characteristics and microwave properties of this system. The corresponding calculations would require studying the solutions of the time-varying equation in (1) and using the static solutions which have now been found.

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