Observation of the Aaronov-Bohm effect in hollow metal cylinders

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The oscillatory dependence of the resistance on the magnitude of the magnetic flux in the cross section of a specimen with period $hc/2e$ and negative longitudinal magnetoresistance are observed in cylindrical lithium films at helium temperatures. The phase of the oscillations and the sign of the magnetoresistance are opposite to those observed for magnesium,[4] which is attributed to the smallness of the spin-orbital interaction in lithium. The results agree well with the theoretical predictions.

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It was shown in Ref. 1 that the conductivity of a thin-walled cylinder made of normal metal must be an oscillatory function of the magnetic flux Φ, which penetrates the cylinder due to the dependence of the electron wave functions on the vector potential of the magnetic field (Aaronov-Bohm effect[2]) even when the electrons collide often with lattice defects, i.e., when $l \ll 2\pi r$, where r is the average radius of the cylinder and l is the mean free path of electrons with elastic scattering. The period of these oscillations is equal to $\Phi_0 = hc/2e$, the superconducting flux quantum. The low sensitivity to the degree of disorder in the specimen and a period that is half as long distinguishes this phenomenon from the oscillations predicted in Ref. 3, which could occur only if $l \gg 2\pi r$. The condition for observing the effect[1] is a sufficiently long diffusion length $L_\varphi = \sqrt{D\tau_\varphi}$ compared to $2\pi r$ (τ_φ is the phase interruption time of the electron wave function due to inelastic processes or due to scattering by paramagnetic impurities, $D = 1/3\, v_F^2 \tau$ is the diffusion coefficient of electrons, v_F is the Fermi velocity, and τ is the momentum relaxation time). If $L_\varphi \sim 2\pi r$, then the amplitude of the oscillations in the total conductivity G of a hollow cylinder with height b is in order of magnitude

$$\frac{e^2}{\pi^2 \hbar}\,\frac{2\pi r}{b} = 2.47 \times 10^{-5}\,\frac{2\pi r}{b}\ \Omega^{-1}.$$

In order to observe this effect, cylindrical magnesium films with a diameter of about 1.5 μm were investigated[4] at helium temperatures. The resistance of these specimens in a longitudinal magnetic field oscillated with a period corresponding to the quantum flux Φ_0. The phase of the oscillations corresponded to the minimum in the resistance for $H = 0$. In addition to oscillations, a positive longitudinal magnetoresistance was observed. (The difference in the magnitude of the monotonic part of the magnetoresistance for the two specimens studied in Ref. 4 may possibly be the result of the inaccurate alignment of the specimens along the field.) These characteristics indicate that the spin-orbital interaction plays an important role in the observed effect.[5,6] Taking into account this interaction with the spin rotation time τ_{so}, the Maki-Thompson corrections,[7] and the finite thickness of the film a (Ref. 8), we can write[1)] the following expression for the quantum corrections to the conductivity of the cylinder:

$$\Delta G = \frac{e^2}{2\pi^2 \hbar} \frac{2\pi r}{b} \left[\left(\frac{1}{2} + \beta \right) Z_\Phi \left(L_\varphi(H) \right) - \frac{3}{2} Z_\Phi \left(\widetilde{L}_\varphi(H) \right) \right], \qquad (1)$$

where

$$Z_\Phi(L_\varphi) = \ln \frac{L_\varphi}{l} + 2 \sum_{n=1}^{\infty} K_0 \left(n \frac{2\pi r}{L_\varphi} \right) \cos 2\pi n \frac{\Phi}{\Phi_0} .$$

Here

$$\frac{1}{L_\varphi^2(H)} = \frac{1}{D\tau_\varphi} + \frac{1}{3} \left(\frac{aeH}{\hbar c} \right)^2, \quad \frac{1}{\widetilde{L}_\varphi^2(H)} = \frac{1}{L_\varphi^2(H)} + \frac{2}{D\tau_{so}},$$

$\Phi = \pi r^2 H$, β a correction which takes into account the interaction of electrons, was calculated in Ref. 7, and $K_0(x)$ is the Macdonald function. As τ_{so} is reduced, $Z_\Phi(\widetilde{L}_\varphi)$ decreases and the phase of the oscillations changes sign when the amplitude goes through zero. The amplitude decreases with increasing magnetic field because the magnetic fluxes which penetrate different electronic trajectories are not the same when the value of a is finite.

An estimate using (1) shows that the magnitude and sign of the effect[4] indeed can be determined by the spin-orbit interaction, in spite of its comparative smallness for such a light metal as Mg. According to Refs. 9 and 10, for an annealed film we have

$$\frac{a}{\tau_{so} v_F} = \epsilon \cong (aZ)^4, \qquad (2)$$

where a is the fine-structure constant, and Z is the atomic number of the element. For Mg we would have $\epsilon \cong 6 \times 10^{-5}$. The ratio τ_φ / τ_{so} can therefore be represented in the form

$$\frac{\tau_\varphi}{\tau_{so}} = \frac{3L_\varphi^2}{la} \epsilon . \qquad (3)$$

If it is assumed that $D\tau_{so} \ll L_\varphi^2(H)$, then the second term in (1) can be ignored. According to Ref. 4, $\beta < 0.1$ and can be dropped in making estimates. In this case, for example, for a Mg II specimen, we find from the period of the oscillations that $2\pi r = 5.2\ \mu m$, while from the magnitude of the first oscillation $L_\varphi(0) = 1.7\ \mu m$. Since the specific resistance of the specimen is

$$\rho = \frac{2\pi r a R}{b},$$

where R for a Mg II specimen is equal to 12.3 kΩ, and $b = 1$ cm, assuming, according to the free electron model, that $\rho l = 6.5 \times 10^{-12}\ \Omega\ cm^2$, we obtain $la = 1 \times 10^{-12}\ cm^2$.

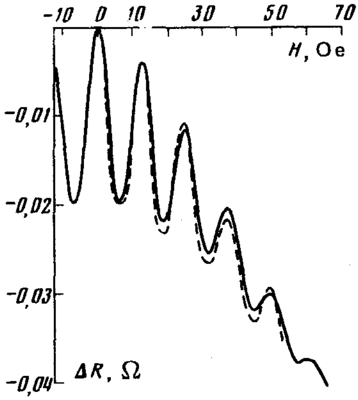

FIG. 1.

Thus, according to (3), $\tau_\varphi/\tau_{s0} = 5$; the second term in (1) can therefore be neglected and the phase of the oscillations coincides with the observed value.

In order for the spin-orbit effects to be insignificant, lithium, for which ϵ must be two orders of magnitude smaller than for Mg, was chosen as the material for the specimens.

The results of the experiment using a lithium film, similar to that described in Ref. 4, are shown in Fig. 1. A film with $R = 2$ kΩ and $b = 1$ cm was obtained by condensation of lithium with an initial purity of 99.95% on a quartz filament. The helium temperature in the experiment was 1.1 K. The measuring current, equal to 40 μA, heated the specimens by an amount of the order of 10^{-1} K.

The dashed line in the figure shows the results of a calculation using Eq. (1) with $\tau_{s0} = \infty$, $\beta = 0$, $r = 0.72$ μm, $L_\varphi = 2.32$ μm, and $a = 0.12$ μm. The quantity $2r - a = 1.32$ μm is close to the value of the diameter of the quartz filament 1.3 μm, determined with the help of an electron microscope. Another check of the theory is the agreement between the monotonic decrease and the damping of the oscillations, determined by the same quantity a. Thus, theory and experiment apparently are in good quantitative agreement.

It should be noted that it was possible to observe in these experiments the negative longitudinal magnetoresistance of thin films.

Confirming the validity of the basic ideas of the theory of weak localization in quasi-two-dimensional systems, experiments of the type described above also make it possible to study electron scattering mechanisms in thin films.

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1) An error in expression (6) in Ref. 1 should be corrected. The total coefficient should be $1/2\pi^2$, rather than $1/\pi^2$, and $2\pi L_\epsilon$ should be replaced by L_ϵ.

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