Mass of the t quark and the number of quark–lepton generations

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The SU(5) grand unified model with a "horizontal" SU(3) symmetry between quark–lepton generations is used to calculate the masses of the d, s, and b quarks; to calculate the Kobayashi–Maskawa mixing angles; and to predict the mass of the t quark (on the basis of the mass difference between K_L and K_S). The prediction for a model with three quark generations is $m_t = 200$ (600) GeV, and that for a model with six generations is $m_t = 20$ (40) GeV. The masses of the d, s, and b down quarks change only very slightly.

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The existence of three identical quark–lepton generations

$$(u, d, \nu_e, e), (c, s, \nu_\mu, \mu), (t?, b, \nu_\tau, \tau) \tag{1}$$

may possibly reflect an additional "horizontal" SU(3) symmetry between generations which becomes an exact symmetry at very short ranges. Let us examine two versions of the $SU(5)\otimes SU(3)_H$ model: (A) a model with three generations of quarks and leptons [in the spirit of the standard SU(5) symmetry[1]]; (B) a model with six generations of the type in (1) [as is predicted by the SU(8) symmetry with constituent quarks and leptons[2]].

The fermion sector of model A is formed by the multiplets (left-hand-helicity members; the index "L" is omitted)

$$\psi^{i\alpha}\ (\bar{5},\bar{3}),\ \psi^{\alpha}_{[ij]}\ (10,\bar{3}),\ \psi^{(N)}_{\alpha}\ (1,3), \tag{2}$$

where $i, j, k = 1, \ldots, 5$ [the SU(5) indices]; $\alpha, \beta, \gamma = 1, 2, 3$ [the SU(3) indices]; and $N = 1, \ldots, 15$. The first two multiplets in (2) correspond to the generations in (1); the other 15 fields $\psi^{(N)}_\alpha$ are introduced in order to make the theory free of anomalies in the $SU(3)_H$ horizontal group. These fields receive large Marjorana masses (more on this below) and do not affect the observed spectrum of quarks and leptons in (1).

The scalars $SU(5)\otimes SU(3)$ include the scalars of the standard SU(5) symmetry,

$$\Phi^i_j\ (24, 1),\ H_i\ (5, 1), \tag{3}$$

which cause SU(5) to "collapse" to $SU(3)_c\otimes U(1)_{EM}$; the scalars

$$\xi_\alpha\ (1,3),\ \eta_\alpha\ (1,3),\ \chi_{\{\alpha\beta\}}(1,6), \tag{4}$$

whose vacuum expectation values destroy $SU(3)_H$ and make the horizontal fermions $\psi^{(N)}_\alpha$ heavier; and, finally, the scalars

$$\omega_{i\{\alpha\beta\}}(5,6),\ \rho^i_{\{\alpha\beta\}}(\bar{5},6),\ \zeta^{i\alpha}\ (\bar{5},\bar{3}),\ \sigma^{[ij]\alpha}_k\ (\overline{45},\bar{3}), \tag{5}$$

which act through Yukawa couplings of the general form (G and G_1-G_3 are constants)

$$G\psi^{a}_{[ij]}C\psi^{\beta}_{[kl]}\,\omega_{m\,\{\alpha\beta\}}\,\epsilon^{ijklm},\quad C=i\gamma_2\gamma_0\ , \tag{6}$$

$$\psi^{,i\alpha}C\psi^{\beta}_{[jk]}\left[G_1\,\delta^j_i\,\rho^{k}_{\{\alpha\beta\}}+\epsilon_{\alpha\beta\gamma}\left(G_2\,\delta^j_i\,\zeta^{k\gamma}+G_3\sigma_i^{[jk]\gamma}\right)\right], \tag{7}$$

to generate mass matrices for the up quarks $U=(u,c,t)$ [Eq. (6)] and for the down quarks $D=(d,s,b)$ and leptons $L=(3,\mu,\tau)$ [Eq. (7)].

The fields Φ^i_j generate large vacuum expectation values ($V\sim10^{15}$ GeV) and determine the basic "vertical" SU(5) scale. The condition for a "grand" hierarchy for the scalar H_i - $\nu=\mathcal{O}(10^{-13})V$ is adopted here in the same form as in the standard SU(5) symmetry.[1] All the other scalars are free of this condition, so that their masses naturally "shrink" to the scale of V through the terms corresponding to the interaction with the field Φ. In general, therefore, the scale for the violation of $SU(3)_H$—the horizontal scale V_H—is specified by the scalars in (4), $V_H\sim V$. The structure of their vacuum expectation values[3] can always be chosen to satisfy

$$<\xi_\alpha>=p\delta_{\alpha 1},\ <\eta_\alpha>=q\,\delta_{\alpha 3},\ <\chi_{\{\alpha\beta\}}>=\mathrm{diag}\,(0,0,r_3)_{\alpha\beta} \tag{8}$$

with a "soft"hierarchy of vacuum expectation values, $r_3/p\ \sim\ p/q\ \sim 5-10, r_3\equiv V_H$. It is then easy to see that the cross couplings of the fields ξ,η, and χ in the Higgs polynomial, which are linear in the components χ_{11} and χ_{22} of the field χ(h_1 and h_2 are constants),

$$\tilde{P}(\xi,\eta,\chi)=[\,h_1\,\xi_\alpha\xi_{\alpha'}+h_2\eta_\alpha\eta_{\alpha'}]\,\chi_{\{\beta\beta'\}}\,\chi_{\{\gamma\gamma'\}}\epsilon^{\alpha\beta\gamma}\epsilon^{\alpha'\beta'\gamma'}+h.c.\ , \tag{9}$$

cause the field χ to also have a vacuum expectation value corresponding to the component $\langle\chi_{22}\rangle\equiv r_2$; in the next step (after r_2 has appeared), they cause this field to also have a vacuum expectation value corresponding to the component $\langle\chi_{11}\rangle\equiv r_1$. Here we have

$$r_2\sim h_1\frac{p^2r_3}{m_\chi^2}\sim O\left[h_1\left(\frac{p}{r_3}\right)\right]p,\ r_1\sim h_2\,\frac{q^2r_2}{m_\chi^2}\sim O\left[h_2\left(\frac{q}{r_3}\right)^2\right]r_2\ , \tag{10}$$

which finally determines the complete horizontal hierarchy of vacuum expectation values:

$$|r_3|\gg|p|\gg|q|\sim|r_2|\ggg|r_1|\,. \tag{11}$$

For the scalars in (5) which generate the masses of the quarks and leptons, the vacuum expectation values are $\mathcal{O}(\nu)$, although the squares of their masses are positive and have values $\mathcal{O}(V^2)$. In the complete Higgs polynomial of the fields (3)-(5), there are necessarily couplings which contain the fields in (5) in a linear way (k_ω is a constant),

$$\tilde{P}(\Phi,H,\chi;\omega)=k_\omega\Phi^i_j\,\bar{H}^j\bar{\chi}^{\{\alpha\beta\}}\omega_{i\,\{\alpha\beta\}}+\ h.c.\,, \tag{12}$$

with corresponding couplings for the fields ρ,ζ, and σ. It is now simple to see that the vacuum expectation values for the fields in (5) are of the natural order of magnitude

$$< \omega_{i\{\alpha\beta\}} > \cong \delta_i^5 \frac{k_\omega < \Phi_5^5 > < \chi^{\{\alpha\beta\}} > \mathrm{v}}{m_\omega^2} \sim \mathcal{O}(\mathrm{v}). \tag{13}$$

Substituting these vacuum expectation values into the Yukawa couplings in (6) and (7), we find the mass matrices

$$M_U = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix} \quad M_D = \begin{pmatrix} O(C_1), & A, & \mathcal{O} \\ A, & C_2 e^{i\lambda}, & B \\ 0, & -B, & C_3 \end{pmatrix}, \quad M_L = \frac{1}{r} \begin{pmatrix} \mathcal{O}(C_1), & A, & 0 \\ A, & C_2 e^{i\lambda}, & -3B \\ 0, & -3B, & C_3 \end{pmatrix} \tag{14}$$

with a "stepped" hierarchy in the matrices for the down quarks, M_D, and leptons, M_L: $C_3 \gg B \gg A \sim C_2 \ggg C_1$ (see (11)]. The parameter $r(\mu)$ in the matrix M_L is a factor which results from the relative renormalization of the masses of the quarks and the leptons from their values in the exact SU(5) symmetry to our energies μ (Ref. 1). For $\mu = 10$ GeV, the two-loop approximation in r^4 leads to

$$r^{(3)} \cong 2.7, \quad r^{(6)} \cong 7.5 \quad (a_s (10\ \mathrm{GeV}) \cong 0.14) \tag{15}$$

for three and six generations of quarks, respectively.

A diagonalization of the matrices M_D and M_L leads to the mass formulas

$$m_b \cong r\,(m_\tau - m_\mu)\, m_d\, m_s \cong r^2 m_e\, m_\mu \tag{16}$$

and to a Kobayashi-Maskawa matrix with the angles θ_i ($i = 1, 2, 3$; $\sin\theta_i \equiv s_i$) and with the CP phase δ:

$$s_1 \cong \sqrt{\frac{m_d}{m_s}}, \quad s_2^{(\pm)} \cong \left[\frac{m_\mu}{m_\tau} \pm \frac{m_s}{m_b}\right]^{1/2}, \quad s_3^{(\pm)} \cong \frac{m_s}{m_b} s_2^{(\pm)}, \quad \sin\delta \sim \sin\lambda, \tag{17}$$

where the $s_{2,3}^{(\pm)}$ correspond to the two possible solutions of $\cos\lambda \approx \pm 1$. The masses of the quarks (in MeV) in Eqs. (15) and (16) are (for $m_d/m_s = s_1^2 \approx \sin^2\theta_C = 0.05$)

$$m_b \approx 4400, \quad m_s \approx 140, \quad m_d \approx 7 \quad (r^{(3)} (1\ \mathrm{GeV}) \cong 4.1), \tag{18}$$

in good agreement with experiment.[1] If s_2 and s_3 are known, we can determine the mass of the t quark and also the CP phase δ from the contribution of the t quark to K^0-$\bar{K}^0$ transitions,[1] to the difference between the masses of the K_L and K_S mesons ($\Delta_{LS} = 5 \times 10^9\ \mathrm{s}^{-1}$), and to K_1-K_2 mixing ($\epsilon = 2.3 \times 10^{-3}$):

$$m_t^{(\pm)} \cong \begin{cases} 200 \\ 600 \end{cases} \mathrm{GeV} \quad (m_c = 1.3\ \mathrm{GeV}); \quad \sin\delta \cong 0.1. \tag{19}$$

Such a large mass for the t quark must apparently be ruled out.[5,6]

We turn now to model B, with six generations of quarks and leptons[2]:

$$\psi_L^{ia}, \quad \psi_{[ij]L}^{a}; \quad \Psi_R^{ia}, \quad \Psi_{[ij]R}^{a}, \tag{20}$$

where the first two multiplets contain the generations in (1), while the "right-hand" Ψ multiplets contain three new generations, which have masses $\mathcal{O}$(100 GeV) and the $(V+A)$ weak-interaction structure (there are no anomalies). The generations ψ_L and Ψ_R, constructed from the "left-hand" and "right-hand" preons, respectively, do not mix with each other,[1)] so that the mass matrices of the generations in (1) can be analyzed independently. However, the fact that six families of quarks exist leads to a large value of $r = r^{(6)}$ [see (15)], so that the mass formulas in (16) cannot be used. If, however, we alter only the SU(5) structure in the scalars ρ, ζ, and σ in (5), which generate the M_D and M_L mass matrices [leaving the horizontal $SU(3)_H$ structure of the fields unchanged: all the 5-plets are replaced by 45-plets and vice verse], we can then work from the form of the vacuum expectation values of the 45-plets, without changing the matrices M_D, to find the new matrix $M_L^{(6)}$ in place of the matrix $M_L^{(3)}$ in (14); here $M_L^{(6)} = M_L^{(3)}$ $(r \to r/3, B \to B/3)$. Similarly, we replace (16) by

$$m_b \cong \frac{r}{3}\,(m_\tau + m_\mu),\; m_d\, m_s \cong \frac{r^2}{9}\; m_e\, m_\mu \tag{21}$$

which lead to essentially the same quark masses as in (18) $(r = r^{(6)})$, and in place of the angles (17) we find

$$S_1 = s_1,\;\; S_2^{(\pm)} = 3\,s_2^{(\pm)},\;\; S_3^{(\pm)} = 3\,s_3^{(\pm)}\,. \tag{22}$$

With the same values of Δ_{LS} and ϵ, these values lead to a t-quark mass $m_t = 20$ (40) GeV for $\cos\delta \approx 1$ (-1) and $\sin\delta \cong 0.1$ (as before).

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[1)]Gravitation causes a very slight mixing of ψ_L and Ψ_R (Ref. 7), so that there are no stable states in Ψ_R multiplets.

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