Confinement of toroidally trapped high-energy particles in a rippled magnetic field

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The joint effects of Coulomb collisions and the dynamic stochastization of trajectories cause a significant loss of banana-orbit particles with energies ~ 100 keV. This loss imposes an upper limit $\delta \sim 0.3\%$ on the ripple during injection.

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Plans for the next generation of tokamaks call for plasma heating through the injection of beams of neutral atoms at an angle of 70–80° from the main magnetic field.[1] After ionization, these particles are trapped on banana orbits and form a high-energy component with an energy of 100–200 keV. The efficiency of this beam heating of the plasma will depend on how efficiently these ions can be confined; in turn, this confinement will be determined largely by the ripple in the toroidal magnetic field.

Analysis of the ion loss on the basis of the banana-drift kinetic equation[2] leads to the prediction of a stabilization of the ripple diffusion coefficient at energies $w \sim 30$ keV; at lower energies, this coefficient increases $\sim w^{3/2}$ (Ref. 3). At $w > 30$ keV, the ripple loss of the high-energy banana-orbit particles in fact decreases, and this decrease has led to some extremely optimistic estimates of the maximum permissible field ripple: $\delta = (B_{\max} - B_{\min})/(B_{\max} + B_{\min}) \sim 1\%$. It is known, however, that if the ion energy reaches 10^6–10^7 eV, the transport rate of these ions increases again because of the dynamic stochastization of trajectories, reaching the level of the ripple plateau in the diffusion, $D^{\mathrm{RP}} \sim \rho_\theta^2 N \delta^2 v / R \epsilon^{1/2}$,[4] where $\rho_\theta = vq/\epsilon\omega_B$ is the gyroradius calculated from the field of the current, N is the number of longitudinal-field coils, R is the major radius of the torus, $\epsilon = r/R$ is the inverse aspect ratio, and q is the safety factor.

In order to resolve this disagreement and to correctly determine the loss of particles at energies $\gtrsim 10^5$ eV, it is necessary (first) to reexamine the banana-drift transport, since at such energies the orbit of a particle will precess a distance $\sim \tau_b U_{\mathrm{tor}}$ ($U_{\mathrm{tor}} \sim v^2 q / R\omega_B \epsilon$ is the toroidal precession velocity), which is greater than the distance between coils, in the time required for a single traversal of a banana orbit, $\tau_b \sim qR/v\epsilon^{1/2}$. The banana-drift kinetic equation itself will thus become inapplicable. Second, it is necessary to consider effects which lead to a dynamic stochastization of trajectories.

We will first determine the loss of the high-energy banana-orbit particles without dynamic stochastization. The ripple in the toroidal field perturbs a banana orbit, particularly effectively near the $v_\parallel = 0$ points. As a result, the tip of a banana orbit is displaced a distance

$$d \sim \frac{v\delta N^{1/2}}{\omega_B}\left(\frac{q}{\epsilon}\right)^{3/2} (\cos N\phi^{(1)} + \cos N\phi^{(2)}), \tag{1}$$

during a single traversal of the orbit, where $\phi^{(1)}$ and $\phi^{(2)}$ are the toroidal angles at the reflection or bounce points.[3] During the next traversal, because of the orbital precession, the displacement phases $N\phi$ acquire increments $\beta = N\tau_b U_{\rm TOR}/R$. The radial displacement of a banana orbit over several traversals is thus $\Delta r_\beta = \Sigma d = d/\sin\beta$. For most of the particles we have $\Delta r_\beta \sim d$, and only the resonant particles ($\beta = k\pi$) are displaced a large distance. There are, however, two factors which limit this displacement. The first is Coulomb collisions. Because of these collisions, the particles cannot be reflected in phase more than τ_c/τ_b times ($\tau_c \sim \epsilon/N^2q^2\nu$ is the scale time for collisional scattering of $N\phi$ through $\pi/2$), so that they cannot be displaced more than $\Delta r_c \sim d\tau_c/\tau_b$. The second factor restricting the displacement of the particles is the radial dependence of the orbital parameters. The most important point here is the radial dependence of the toroidal angle traced out by the particle as it moves along its banana orbit, $\phi_{\rm TOR} = \phi^{(1)} - \phi^{(2)}$. If, because of a radial displacement, $\Delta\phi_{\rm TOR}$ reaches π/N, then d vanishes or even changes sign, so that Δr is limited by the value $\Delta r_T \sim (Nd\phi_{\rm TOR}/dr)^{-1}$. Of the three displacements derived here (Δr_β, Δr_c, and Δr_T), it is the smallest, $\Delta r^2 = (\Delta r_\beta^{-2} + \Delta r_c^{-2} + \Delta r_T^{-2})^{-1}$, which appears in the diffusion coefficient $D \sim \Delta r^2/\tau_{\rm eff}$. Assuming that the stochastization of the phase $N\phi$ results from Coulomb collisions alone, we find

$$D \sim d^2[\sin^2\beta + (\tau_B/\tau_c)^2 + (dNd\phi_{\rm TOR}/dr)^2]^{-1}/\tau_c\,. \tag{2}$$

At low energies, under the condition $\beta \ll 1$, the diffusion coefficient in (2) converts into that calculated from the banana-drift kinetic equation.[2] At energies corresponding to $\beta \gtrsim 1$ the $D(w)$ dependence is a curve with sharp peaks ($\Delta w/w \sim \delta\sqrt{Nq}/\epsilon$) near the resonant regions:

$$\beta \equiv \frac{4N\sqrt{w}\,q^2}{\sqrt{m}\,R\,\omega_B\,\epsilon^{3/2}}\left[E - \frac{K}{2} + 2\frac{r}{q}\frac{dq}{dr}\left(E - \cos^2\frac{\theta_0}{2}K\right)\right] = n\pi\,, \tag{3}$$

where $E = E(\sin\theta_0/2)$ and $K = K(\sin\theta_0/2)$ are the elliptic integrals, and θ_0 is the poloidal angle at the bounce point.

The diffusion coefficient in (2) does not, however, convert into $D^{\rm RP} \sim d^2/\tau_b$ at high energies. In order to describe the transition region, we must consider jointly the Coulomb effects and dynamic stochastization. This can be done by introducing a random change in the toroidal angle in the equations describing the relationship between the coordinates r and ϕ of the successive bounce points:

$$\begin{aligned}
r^{(1)}_{n+1} &= r^{(2)}_n + d\cos(N\phi^{(1)}_n),\\
\phi^{(2)}_{n+1} &= \phi^{(1)}_n + \phi_{\rm TOR}(r^{(1)}_{n+1}) + \Delta\phi_c + \beta/2,\\
r^{(2)}_{n+1} &= r^{(1)}_{n+1} - d\cos(N\phi^{(2)}_{n+1}),\\
\phi^{(1)}_{n+1} &= \phi^{(2)}_{n+1} - \phi_{\rm TOR}(r^{(2)}_{n+1}) + \Delta\phi_c + \beta/2,
\end{aligned} \tag{4}$$

where $\Delta\phi_c$ is a random quantity with a typical dispersion

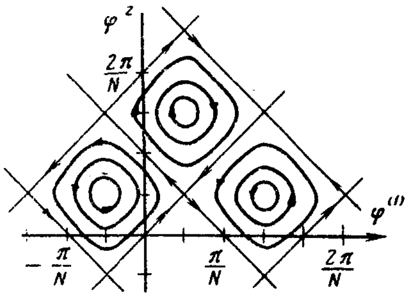

FIG. 1. Trajectories traced out by the solutions of (5) in the phase plane.

$\sim(\tau_b/N^2\tau_c)^{1/2}\sim(\nu Rq^3/v\epsilon^{3/2})^{1/2}$, which describes the change due to Coulomb collisions.

To analyze the behavior of the solutions of (4), we first consider the case $\Delta\phi_c=0$, $\beta=0$. Under the condition $w\ll w_{st}\sim mR^2\omega_B^2\epsilon^5/q^5\delta^2N^3$, such that $Nd d\phi_{\rm TOR}/dr\ll 1$, the solution of the difference equations is the same as that of the differential equations

$$\frac{d\phi^{(1)}}{dt}=d\frac{d\phi_{\rm TOR}}{dr}\cos(N\phi^{(2)}),$$
$$\frac{d\phi^{(2)}}{dt}=d\frac{d\phi_{\rm TOR}}{dr}\cos(N\phi^{(1)}),\qquad (5)$$

which constitute a family of closed trajectories separated by separatrices (Fig. 1). Dynamic stochastization occurs if some finite step (under the condition $dNd\phi_{\rm TOR}/dr\gtrsim 1$) causes a jump from one phase cell to another. The angle $N\phi$ is a random quantity in each reflection, so that the diffusion coefficient satisfies $D\sim d^2/\tau_b$. The diffusion described by Eq. (2) corresponds to a gradual transition from one trajectory to another as a result of Coulomb collisions. In addition to a gradual transition, collisions may also cause a crossing of the separatrix and provide another contribution to the transport.

All of these effects have been taken into account in a numerical analysis of the expansion of a beam of trajectories corresponding to Eqs. (4). Figure 2 shows the

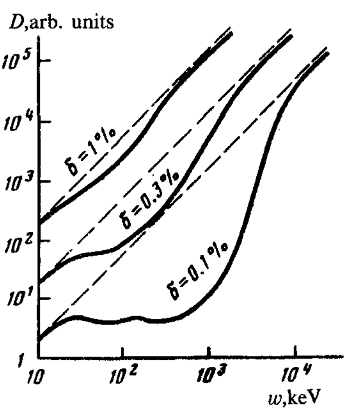

FIG. 2. Dependence of the diffusion coefficient D on the energy w. Dashed lines—The ripple-plateau diffusion, $D^{\rm RP}\sim\delta^2w^{3/2}$ (Ref. 3).

energy dependence found for the diffusion coefficient of the banana-orbit particles by this method. As parameters in the calculations we used the values for the INTOR device[1]: $n = 10^{14}$ cm^{-3}, $N \approx 12$, $R = 5.3$ m, $r = 0.7$ m, $q = 1.2$, $B = 5.5$ T, and $Nd\phi_{\rm TOR}/dr = 0.7$ cm^{-1}. Other values of $Nd\phi_{\rm TOR}/dr$ are considered by choosing different values of δ and holding the quantity $dNd\phi_{\rm TOR}/dr$ constant. Since the beam of trajectories is specified with a rather large spread in β, the peaks which should appear at $w \ll w_{st}$ turn out to be very rounded. As the energy is increased to $w \sim w_{st}$, D rapidly assumes the behavior $D^{\rm RP} \approx d^2/\tau_b \sim \delta^2 w^{3/2}$. This transition occurs sooner as δ increases, since $w_{st} \sim \delta^{-2}$. At $\delta = 1\%$ the region of ripple-plateau transport spans essentially the entire energy range. The loss of beam particles with $w \sim 100$ keV in this case is higher than the neoclassical value by a factor $N\delta^2 v/\sqrt{\epsilon} R\nu \sim 10^3$, and it becomes comparable to νr^2. This transport may cause a loss of hot ions before these ions manage to transfer energy to the plasma. Consequently, the ripple could hardly be allowed to exceed 0.3% during the beam injection.

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