A possible self-resonant cyclotron acceleration of charges in an inhomogeneous plasma

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A cyclotron resonance automatically adjusts itself as a charged particle moves through inhomogeneous crossed electric and magnetic fields and in the electric field of waves in a plasma. This self-adjustment results from the mutual cancellation of the changes in the Doppler frequency shift and the frequency of the cyclotron harmonic.

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The interaction of charged particles with electromagnetic waves in plasmas under cyclotron-resonance conditions has attracted much interest in connection with the rf heating of plasmas and isotope separation.[1,2] Let us assume that a plasma is in inhomogeneous constant magnetic and electric fields $\mathbf{B}_0$ and $\mathbf{E}_0$, which are directed along the Oz and Oy axes, respectively. The electric field of the waves in the plasma is described by the potential

$$\varphi = \varphi_0(y)\cos(\omega t - k_z z - k_x x), \tag{1}$$

where ω and $\mathbf{k}$ are the frequency and wave vector of the wave. We assume that $B_0(y)$, $E_0(y)$, and the potential amplitude $\varphi_0(y)$ vary slowly along the Oy axis. Applying the averaging method of Ref. 3, we can write the nonrelativistic equations of motion of a charged particle in the form

$$\frac{d\bar{v}_z}{dt} = \frac{e}{m} k_z \varphi_0(\bar{y}) J_n(a)\sin\Phi_n, \quad \frac{d\bar{v}_\perp}{dt} = \frac{e}{m} K_x \varphi_0(\bar{y}) \frac{n}{a} J_n(a)\sin\Phi_n,$$

$$\frac{d\bar{\theta}}{dt} = -\omega_B(\bar{y}) + \frac{1}{2}\frac{dv_E(\bar{y})}{d\bar{y}} - \frac{ek_x\varphi_0(\bar{y})}{m\bar{v}_\perp} J'_n(a)\cos\Phi_n, \quad \frac{d\bar{z}}{dt} = \bar{v}_z, \tag{2}$$

$$\frac{d\bar{x}}{dt} = v_E(\bar{y}) - \frac{\bar{v}_\perp^2}{2\omega_B^2(\bar{y})}\frac{d\omega_B(\bar{y})}{d\bar{y}} \quad \frac{d\bar{y}}{dt} = -\frac{ek_x\varphi_0(\bar{y})}{m\omega_B(\bar{y})} J_n(a)\sin\Phi_n,$$

where $\mathbf{v}$ is the velocity of the particle, $v_\perp^2 = (v_x - v_E)^2 + v_y^2$, $v_E = c\,E_0/B_0$, θ is the azimuthal angle in velocity space, e and m are respectively the charge and mass of the particle, $\Phi_n = k_z\bar{z} + k_x\bar{x} - n\bar{\theta} - \omega t$, $J_n(a)$ and $J'_n(a)$ are the Bessel functions and their derivative with respect to a, $a = k_x\bar{v}_\perp/\omega_B(\bar{y})$, $\omega_B = eB_0/mc$, c is the speed of light in a vacuum, and n is the index of the cyclotron resonance harmonic. The superior bar denotes an average. In the derivation of (2) it was asumed that the gyroradius of the particle and the transverse wavelength are both small in comparison with the scale dimension of the transverse inhomogeneity. From (2) we find the approximate integrals of motion y_0 and V:

$$y_0 = y + \frac{k_x v_\perp^2}{2n\,\omega_B^2(y)}, \quad V = v_z - \frac{k_z v_\perp^2}{2n\omega_B(y)} \tag{3}$$

Here and below, the equations contain only average values, and we will omit the superior bar to simplify the notation. It follows from (3) that on the trajectories with $|a| \lesssim 1$ the increment in y does not exceed the gyroradius of the particle. For the resonant phase Φ_n we have the equation

$$\frac{d\Phi_n}{dt} = \Delta_n + \frac{nek_x\varphi_0(y)}{mv_\perp} J'_n(a)\cos\Phi_n, \tag{4}$$

where

$$\Delta_n = n\omega_B(y) + k_z v_z + k_x v_E(y) - \frac{n}{2}\frac{dv_E(y)}{dy} - \frac{k_x v_\perp^2}{2\omega_B^2}\frac{d\omega_B}{dy} - \omega. \tag{5}$$

In the case of a slight inhomogeneity we find from (2) and (4) the additional integral of motion W:

$$J_n(a)\cos\Phi_n + c_n\frac{a^2}{2} + q_n\frac{a^4}{4} = W, \tag{6}$$

where

$$c_n = \frac{m\omega_B(y_0)}{nk_x^2 e\varphi_0(y_0)}\left[n\,\omega_B(y_0) + k_z V + k_x v_E(y_0) - \frac{n}{2}\frac{d v_E(y_0)}{dy_0} - \omega\right], \tag{7}$$

$$q_n = \frac{k_z^2 m\,\omega_B^3(y_0)}{2n^2 k_x^4\, e\varphi_0(y_0)}(1-R), \tag{8}$$

$$R = -\frac{k_x^2}{k_z^2\,\omega_B(y_0)}\frac{d v_E(y_0)}{dy_0} - \frac{2nk_x}{k_z^2\,\omega_B(y_0)}\frac{d\omega_B(y_0)}{dy_0} \tag{9}$$

We assume the initial value $v_\perp = 0$. We then find the following expression for $v_{\perp\max}^{(n)}$ (the maximum transverse velocity) for the fundamental cyclotron resonance ($n = 1$), under the condition $|a| \ll 1$:

$$v_{\perp max}^{(1)} = 2\left|\frac{2ek_x\varphi_0(y_0)\,\omega_B(y_0)}{k_z^2\, m\,(1-R)}\right|^{\frac{1}{3}} \tag{10}$$

For the second harmonic ($n = 2$) we find, with $|a| \ll 1$,

$$v_{\perp max}^{(2)} = 2\frac{k_x}{k_z}\left|\frac{2e\varphi_0(y_0)}{m(1-R)}\right|^{1/2} \tag{11}$$

The highest attainable velocity increases rapidly in the limit $R \to 1$. If $k_x\, v_{\perp\max}^{(n)} \gtrsim |\omega_B|$, we cannot use expressions (10) and (11), and we can find the value of $v_{\perp\max}^{(n)}$ by setting $R = 1$ and $c_n = 0$ in (6). Figure 1 shows trajectories on the $(v_\perp, \Phi_n)$ phase plane for this case. Particles which start with small transverse velocities reach values of $v_\perp$ approaching

$$v_{\perp max}^{(n)} \approx a_{n1}\frac{|\omega_B|}{k_x}, \tag{12}$$

where a_{n1} is the first root of the Bessel function $J_n(a)$. If the field B_0 is uniform, and $E_0 = 0$, then Eqs. (10)–(12) correspond to the results given in Ref. 4. In the case of

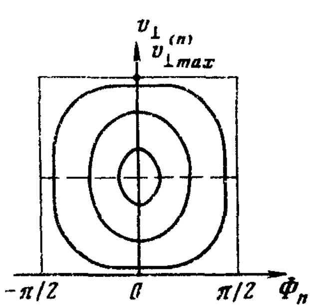

FIG. 1. Phase trajectories in the $(v_\perp, \Phi_n)$ plane with $c_n = 0$ and $q_n = 0$.

uniform fields B_0 and E_0, the value of $v_{\perp\max}^{(n)}$ turns out to be small for the ions in real experimental devices,[2] since the condition $k_z \gg \omega/c$ usually holds, and the condition for the resonance for ions is violated because of a change in v_z. In an inhomogeneous plasma with $R = 1$ the resonance is automatically maintained, since the changes in the Doppler frequency shift and the frequency of the cyclotron harmonic cancel out.

As an example we consider the motion of singly ionized nitrogen atoms in fields $E_0 \sim 1$ V/cm and $B_0 \sim 1.5$ kG, with inhomogeneity scale dimensions of 10 and 100 cm, respectively. With $k_x \sim 1$ cm^{-1} and $k_z \sim 0.1$ cm^{-1}, the terms associated with the inhomogeneity of E_0 and B_0 make a contribution of order unity to the parameter R. For the conditions corresponding to the self-resonant cyclotron acceleration mechanism in weak variable fields, we find the estimate $v_{\perp\max} \sim 4 \times 10^6$ cm/s.

This self-resonant acceleration of ions might be used to separate different ion components in devices with rotating plasmas. Waves which act on the ions arise in a natural manner in a rotating plasma because of its instabilities.[2] Since the resonance condition $\Delta_n \approx 0$ is a local condition, the energy separation of the ion components will be accompanied by a spatial separation.

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