Self-oscillatory solitons and Langmuir turbulence in a magnetized plasma

I. A. Kol'chugina, A. G. Litvak, and A. M. Sergeev
Institute of Applied Physics, Academy of Sciences of the USSR

(Submitted 19 April 1982)
Pis'ma Zh. Eksp. Teor. Fiz. **35**, No. 12, 510–514 (20 June 1982)

Nonlinear dissipative formations—self-oscillatory solitons—can play an important role in the dynamics of intense Langmuir turbulence in a magnetized plasma.

PACS numbers: 52.35.Mw, 52.35.Ra

Recent experiments on the interactions of electromagnetic waves[1,2] and charged-particle beams[3,4] with collisonless magnetized plasmas have shown convincingly that an intense plasma turbulence is excited an is important in dissipating the energy supplied to the plasma. The elementary "cell" of this turbulence is usually elongated along the magnetic field[1,2,4] and contains a burst of rf electric field which is spatially correlated with a region of an intense density perturbation. In this letter we will analyze the dynamic properties of the corresponding nonlinear formations which arise during the self-effects of strong plasma waves.

We adopt a simple model of a one-dimensional plasma turbulence (the one dimension is the x direction) in a constant, uniform magnetic field $\mathbf{H}_0 = H_0 \mathbf{z} \perp \mathbf{x}$. In this case, the following system of equations holds for the slow complex amplitude ψ of the rf electric field, $\vec{\mathscr{E}} = \mathscr{E}_0 \mathbf{x}(\psi e^{i\omega t} + \text{c.c.})$ with a frequency $\omega \gg \omega_{He} = eH_0/mc$ and for the average relative perturbation of the density of the medium, $\Delta N/N = n_0 n$:

$$-i\mu \frac{\partial \psi}{\partial \tau} + \frac{\partial^2 \psi}{\partial \xi^2} - n\psi = 0, \tag{1}$$

$$\frac{\partial^2 n}{\partial \tau^2} - \frac{\partial^2 n}{\partial \xi^2} + n = \frac{\partial^2}{\partial \xi^2} |\psi|^2 - \beta |\psi|^2. \tag{2}$$

Here $\tau = t\Omega_{LH}$, $\xi = x\Omega_{LH}/V_s$, $\mathscr{E}_0 = \sqrt{48\pi N t_e}\, \omega_{He}/\omega_{pe}$, $n_0 = 3\omega_{He}^2/\omega_{pe}^2$, $\mu = 2\omega_{pi}/3\omega_{He}$, $\beta = V_s^2/V_A^2 \ll 1$, $\Omega_{LH} = \sqrt{\omega_{He}\,\omega_{Hi}}$, V_s and V_A are the sound and Alfvén velocities, $\omega_{pe,i}$ and $\omega_{He,i}$ are the plasma frequencies (or Langmuir frequencies) and the cyclotron frequencies of the electrons and ions, respectively, and T_e and $r_{He} = \sqrt{T_e/M}/\omega_{He}$ are the temperature and gyroradius of the electrons.

It is easy to see that in an approximately steady state the spatial structure and magnitude of the perturbation of the charged-particle density depend on the scale dimension of the distribution of the rf field amplitude. For broad packets with $l > \beta^{-1/2}$ ($L > c/\omega_{pe}$, in dimensional variables), we can use the relation to (a) $n \simeq -\beta|\psi|^2$. The local coupling is also valid if $l < 1$ ($L < r_{He}$), but in this case it leads to a much more pronounced nonlinearity: (b) $n \simeq -|\psi|^2$. Finally, in the internal $1 < l < \beta^{-1/2}$ there is a definitely nonlocal dependence (c) $n \simeq \partial^2|\psi|^2/\partial\xi^2$. The variety of types of nonlinear coupling is reflected in the nature of the modulational instability of a uniform distribution of waves, where the scale dimension of the growing perturbation is determined by the initial amplitude ψ_0. The growth rate for this process, γ, can be found as a function of the wave number κ from the dispersion relation

$$(\mu^2\gamma^2 + \kappa^4)(1 + \gamma^2 + \kappa^2) = 2\psi_0^2\kappa^2(\beta + \kappa^2). \tag{3}$$

In the case, $\psi_0 \ll 1$, the optimum inverse scale dimension for the instability is $\kappa_{\text{opt}} = \sqrt{\beta}\,\psi_0$, and the maximum value $\gamma = \gamma_{\max} = \beta\psi_0^2/\mu$ corresponds to a weak local nonlinearity, (a). As the amplitude approaches $\psi_0^* = 1/\sqrt{2}$, there is a sharp increase in the growth rate ($\gamma_{\max} \simeq \beta\psi_0^2/(\mu\sqrt{1-2\psi_0^2})$, and the instability spreads out to a broader range of wave numbers $\kappa_{\max} = \sqrt{2}\kappa_{\text{opt}} \simeq \psi_0\sqrt{2\beta/(1-2\psi_0^2)}$. The critical amplitude of the uniform field, ψ_0^*, may be called the threshold for the excitation of the low-frequency (in this example, the lower-hybrid) resonance in the self-effect of the plasma wave. If $\psi_0 \gg \psi_0^*$, we have $\kappa_{\max} = \sqrt{2}\psi_0 \gg 1$ (b); in this case, dispersion relation (3) becomes the familiar dispersion relation for plasma waves in an isotropic plasma which are unstable (with respect to self-modulation).

Depending on the form of the nonlinear coupling, we may distinguish among three types of Langmuir solitons, in which changes in the density of the medium determined by relations similar to (a), (b), and (c) lead to a localization of the rf field $\psi(\xi)$ acting on the plasma. Large-scale solitons ($l > \beta^{-1/2}$) and small-scale solitons ($l < 1$) are evidently similar to steady-state solitary waves in a medium having a local cubic nonlinearity and an isotropic plasma, while structures with intermediate scale dimensions are "cusp" solitons.[5-7] The scale dimension for a distribution of the type $\psi = \psi(\xi)e^{-iE\tau/\mu}$, $n = n(\xi)$, is related to the eigenvalue E ($l \simeq E^{-1/2}$), so that it is possible to draw a very instructive diagram illustrating the variety of solitons by plotting the rf energy (the number of quanta) in a soliton, $I = \int_{-\infty}^{+\infty} |\psi|^2 d\xi$, against the parameter E (Fig. 1).[1)] By analogy with the well-known condition for the stability of solitons,[8] it may be asserted that steady-state localized solutions with values of E corresponding to a descending part of the curve ($dI/dE < 0$) are unstable, at least with respect to slowly growing perturbations (with a scale time $\tau \gg 1$). Stable plasma-wave solitons with identical numbers of quanta thus exist in two different phases: large-scale ($E < E_1$) and small-scale ($E > E_2$) phases.[2)]

In a nonconservative system, solitons of one type may transform into the other type. Let us consider, for example, the extremely common situation in which energy is being pumped into a turbulence at small wave numbers (a plasma in a uniform external field, a plasma with a charged-particle beam, etc.), and for narrow spatial distributions a collisionless dissipation is important.[3)] A large-scale soliton, whose energy is increasing adiabatically slowly by virtue of an external energy source, increases in amplitude until the eigenvalue E reaches E_1 (follow the arows in Fig. 1). A further increase in the energy causes the solution to "hop" over to the stable brance with $E > E_2$. The conversion of a soliton of the first type into one of the second type, accompanied by a rapid increase in the field amplitude at the center of the formation and by an enrichment of the spectrum with high spatial harmonics, is of the nature of a one-dimensional collapse of plasma waves[6] and can be described in terms of the self-similar variables U, η: $\eta = \xi/(\tau_0 - \tau)^{2/5}$, $U(\eta) = |\psi|^2(\tau_0 - \tau)^{2/5}$. The most important property of this self-similar solution is that the number of quanta, I, is conserved throughout the collapse, which terminates in the formation of a stable, small-scale soliton. The behavior of the soliton may be affected predominantly by collisionless damping, which leads to a slow decrease in the amplitude and to a displacement along the $I(E)$ curve to the left, to the boundary of the instability interval, E_2. After a hop into the region of the large-scale solutions, the soliton returns to its original state, ending its evolution cycle. Consequently, at each instant, except during the hopping intervals, this distribution corresponds to a soliton which is adjusting itself slowly in a dissipative system. In view of the cyclic nature of the overall process, this localized formation might naturally be called a "self-oscillatory soliton."

This qualitative picture of the dynamics of a self-oscillatory soliton has been confirmed by numerical calculations from system (1), (2), supplemented with small nonconservative terms.[4)] The time dependence shown in Fig. 2 for the rf energy and the amplitude of an isolated soliton distribution ($\beta = 0.04$, $\mu = 0.2$) demonstrates that the repeating self-excited oscillation cycles are identical. These nonlinear dynamic formations may play an important role in the overall structure of the turbulence driven by an external source. In this case the lifetime of each self-oscillatory soliton is

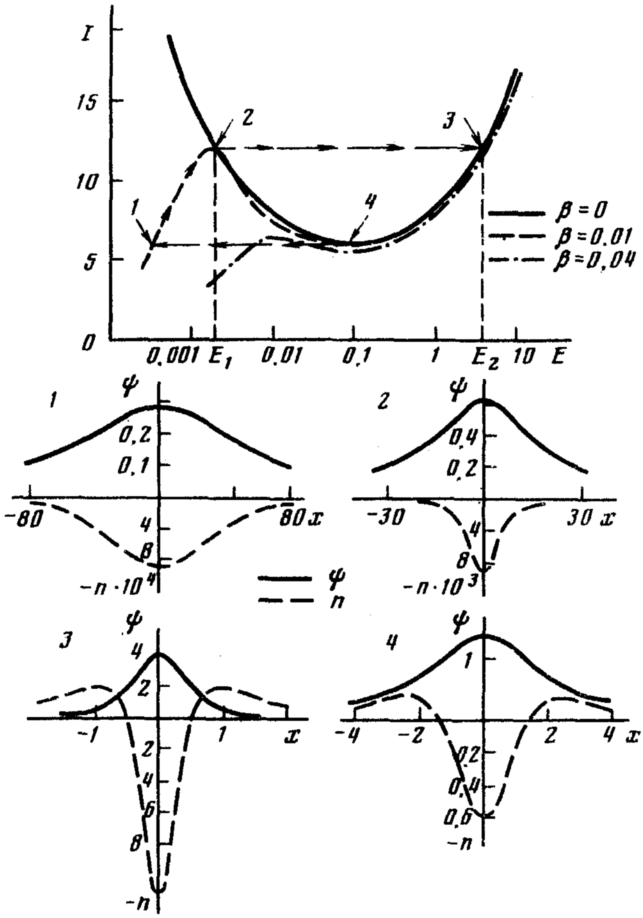

FIG. 1

limited because of the interaction of the solitons directly with each other or with free low-frequency motions in the medium. The numerical simulation, however, shows that the individual localized field packets exist for a few self-excited oscillation cycles,

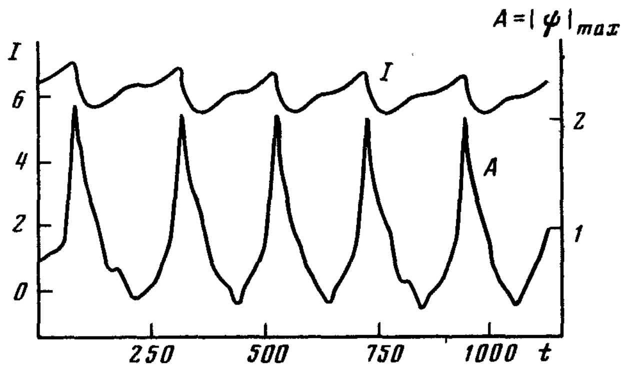

FIG. 2

providing a regular, periodic energy transfer in wave-number space from the source to the region of effective dissipation. If the turbulence region contains several self-oscillatory solitons, it may be possible to see isolated "dips" on the integrated (over space) time dependence of the rf energy—which is, on the whole, a random dependence. These dips correspond to the intervals over which the solitons exist in the small-scale phase.

In the case of self-excited oscillations with a broad energy-storage region and with rapid transfer of the energy to the particles of the medium, the average rate of dissipation of the rf-field quanta per unit length of the turbulence, Γ, can be estimated easily from the simple expression $\Gamma \simeq \nu_0 |a_0|^2$, where ν_0 is the growth rate of the pumping to the zeroth harmonic with the amplitude $|a_0|$ (in the calculation, $\nu_0 = 2 \times 10^{-3}$). Assuming that the average soliton field does not depend on E over the given averaging interval (i.e., assuming that the amplitude is inversely proportional to the width), we find the value $\Gamma \simeq 3\nu_0$, which is higher than the corresponding result for a system of Langmuir solitons in an isotropic plasma.

Finally, we note that a process analogous to that discussed here may occur during a self-effect of rf electromagnetic waves in a plasma due to the excitation of the ion cyclotron wave branch in a magnetized plasma. Evidence for this process has apparently emerged from a recent experiment[4] on the production of plasma turbulence by electron beams; in particular, it was found that there is a threshold field which is required for the ponderomotive perturbation of the density of the medium. A quantitative estimate of the threshold for the excitation of the ion cyclotron resonance, $\psi^*_{0IC} = \sqrt{24\pi N T_e}\, \omega_{Hi} / \omega_{pi}$, yields $\psi^*_{0IC} = 2$–5 V/cm, in good agreement with the experimental data.

[1)] The exact form of the $I(E)$ curve was found from the numerical simulation, but some of its parts (including the descending part) can be determined by a strictly analytic procedure.[6]

[2)] The region with the negative derivative disappears at $\beta > 0.051$.

[3)] For $\omega \gg \omega_{He}$, the rate (ν) of the Landua damping of small-scale rf harmonics can be found from the equations for an isotropic medium if $\nu > \omega_{He}$.

[4)] Equations (1) and (2) were solved numerically through an expansion in spatial harmonics. This approach allowed us to introduce some nonconservative terms with arbitrary spectral characteristics; in particular, we were able to simulate linear Landau damping in the equations for the rf field and for the density perturbations.

[1] W. Gekelman and R. L. Stenzel, Phys. Rev. Lett. **35**, 1708 (1975); W. Gekelman and R. L. Stenzel, Phys. Fluids **20**, 1316 (1977).

[2] S. N. Gromov, L. L. Pasechnik, and V. F. Semenyuk, Pis'ma Zh. Eksp. Teor. Fiz. **23**, 509 (1976) [JETP Lett. **23**, 464 (1976)].

[3] S. V. Antipov, M. V. Nezlin, and A. S. Trubnikov, Zh. Eksp. Teor. Fiz. **78**, 1743 (1980) [Sov. Phys. JETP **51**, 874 (1980)].

[4] P. J. Christiansen, V. K. Jain, and L. Stenflo, Phys. Rev. Lett. **46**, 1333 (1981).

[5] M. Porkolab and M. V. Goldman, Phys. Fluids **19**, 872 (1976).

[6] A. G. Litvak and A. M. Sergeev, Pis'ma Zh. Eksp. Teor. Fiz. **27**, 549 (1978) [JETP Lett. **27**, 517 (1978)].

[7] I. A. Kol'chugina, V. A. Mironov, and A. M. Sergeev, Pis'ma Zh. Eksp. Teor. Fiz. **31**, 333 (1980) [JETP Lett. **31**, 304 (1980)].

[8] A. A. Kolokolov, Izv. Vyssh. Uchebn. Zaved., Radiofiz. **17**, 1332 (1974).

Translated by Dave Parsons
Edited by S. J. Amoretty