Stability of Rossby solitons

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The mechanism for the decay of individual vortices, excited in a rotating shallow fluid with constant depth, is investigated experimentally under conditions of approximate equilibrium between the Coriolis force and the hydrostatic pressure gradient, when the characteristic rotational frequency of the vortex is small compared to the rotational frequency of the fluid as a whole (so-called geostrophic equilibrium). It is shown that only Rossby solitons are stable, while all remaining vortices are unstable and decay into geostrophic flows.

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The Rossby soliton, which is an isolated vortex, or an anticyclone (local rise) in the shallow fluid layer which rotates around the vertical axis with a frequency greater than the characteristic rotational frequency of the vortex, was discovered experimentally and investigated in Ref. 1. According to theory,[2] the Rossby soliton is a particular solution of a nonlinear equation that admits many other vortical solutions. The following question arises: Is the Rossby soliton physically distinguished somehow from other vortices that are possible in the same system? The solution of this problem was the purpose for these experiments, which were carried out on the same setup as in Ref. 1. In this setup, a thin layer of fluid (water, water solution of nickel sulfate), having an approximately homogeneous depth (varying from 3 to 12 mm), is established in a vessel with a parabolic bottom profile (the diameter of the vessel is 28 cm and the height is 20 cm), which rotates around a vertical axis with frequency $\Omega_0/2\pi = 1.7$ Hz. The basic indications of a Rossby soliton, found in the experiments,[1] are as follows. 1) The Rossby soliton is an anticyclone and rotates opposite to the global rotation of the fluid. In it, the Coriolis force, which is directed toward the center of the vortex and which creates the rise, and the hydrostatic pressure gradient are approximately balanced; this is the so-called geostrophic equilibrium. The characteristic rotational frequency of the vortex is small compared to the rotational frequency of the system, so that the centrifugal force from the characteristic rotation is much smaller than Coriolis force. 2) The characteristic diameter of the soliton, 2a, defined as the distance between opposite points of its profile, at which the linear velocity of the characteristic rotation is maximum, is related to the Rossby radius r_R by the relation

$$2a \gtrsim 2.5 r_R \ , \tag{1}$$

$$r_R = (g^* H_0)^{1/2} / f_0 \ , \tag{2}$$

H_0 is the depth of the fluid, $g^* = g/\cos\alpha$, g is the acceleration of gravity, $f_0 = 2\Omega_0 \cos\alpha$ is the Coriolis parameter, and α is the angle between the vector Ω_0 and the normal to the surface of the fluid at the given location. For example, for $H_0 = 5$

mm, $r_R = 2.1$ cm. 3) The Rossby soliton drifts around the axis of a paraboloid relative to the fluid opposite to its motion as a whole with velocity V_{dr}, close to the Rossby velocity V_R (for more detail see below):

$$V_R = H_0 \Omega_0 \sin a . \qquad (3)$$

4) If the amplitude of the Rossby soliton is large enough, so that the linear velocity of the characteristic rotation V_{rot} on the slope of the vortex profile exceeds its drift velocity V_{dr}, then it drags behind it all fluid particles except those that are located on the periphery of the vortex and have a velocity less than V_{dr}. Dragging of fluid particles by the Rossby vortex is known to be observed if the rise ΔH of the vortex above the level of the surrounding fluid satisfies the (sufficient) condition $\Delta H \gtrsim 0.2 H_0$.

The motion of the fluid in the experiments examined is visualized with the help of miniature white paper circles, floating on the surface of the fluid against the background of the black bottom. The tracks of these test particles are photographed by a camera, rotating together with the paraboloid, i.e., stationary relative to the fluid as a whole. The rotational velocity of the vortex particles is determined from the length of the tracks and the exposure of the camera.

Since the other vortices must differ from the Rossby solitons by their dimensions and drift velocities, we have measured here the lifetimes and drift velocities of vortices-anticyclones of different dimensions, which were excited by "pumping disks" with different diameters (as far as cyclones are concerned, they are unstable for any dimensions; see Ref. 1). The results are presented in Figs. 1 and 2. The lifetime of a vortex τ is defined as the time interval from the time that the vortex is formed (its separation from the pumping disk) up to the time the vortex trajectories are opened and the particle tracks no longer follow closed trajectories around the axis of the vortex (see below).

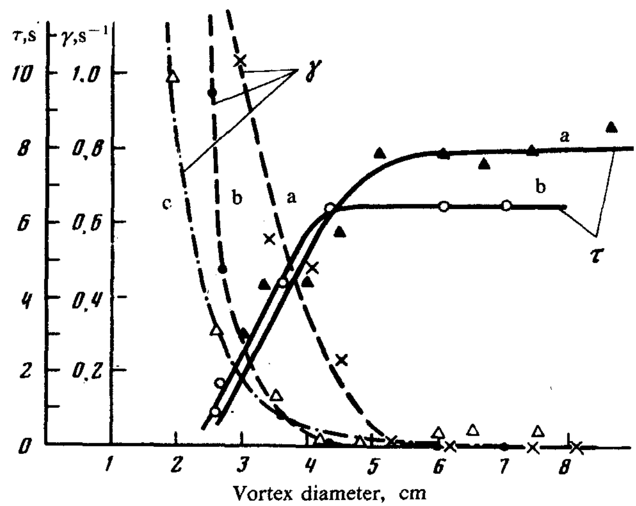

FIG. 1. Dependence of the lifetime τ and increment γ of the decay instability on the vortex diameter. (a) $H_0 = 5$ mm and 6 mm (water), (b) $H_0 = 3$ mm (water), (c) $H_0 = 3$ mm (water solution of nickel bisulfate).

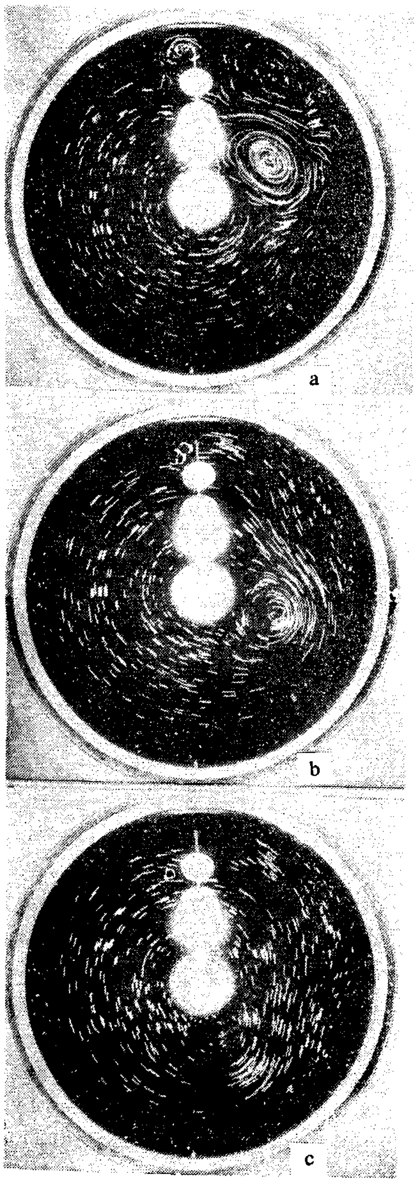

FIG. 2. Decay of a weakly unstable vortex into a flow. The view looking down on the paraboloid is shown, D is the pumping disk (see Ref. 1). The diameter of the vortex is about 4 cm and the instability is weakly manifested (see Fig. 1). The vessel with the water rotates together with the camera in a counterclockwise direction, while the vortex drifts relative to the fluid in a clockwise direction. The tracks of the white test particles, floating on the surface of the water and moving over the time of the exposure of the camera, can be seen against the background of the black bottom of the paraboloid. The vortex changes into a geostrophic flow while the condition $V_{\rm rot} > V_{\rm dr}$ is still satisfied. The time intervals between frames are equal to 3.5 s. The motion of the vortex particles gradually slows down due to the viscosity of the fluid.

Figure 1 shows the lifetime of the vortices as a function of their diameter d, defined analogously to the diameter of the Rossby vortices at the time the vortex can be viewed as being formed. Experimental data are presented for three cases: (a) $H_0 = 5$–6 mm (water), (b) $H_0 = 3$ mm (water), and (c) $H_0 = 3$ mm (water solution of nickel sulfate, whose viscosity is approximately 3 times greater than that of water). It is evident that if the diameter of the vortex exceeds some magnitude:

$$d \gtrsim (2.5 - 3) r_R \,, \qquad (4)$$

which increases with fluid depth, then the lifetime of the vortex τ reaches a maximum,

τ_{max}, which has a clear physical meaning: as a result of the viscosity of the fluid, the characteristic velocity of intrinsic rotation of the particles in the vortex decreases over the time τ_{max} to the magnitude of the drift velocity of the vortex and laminar opening of the vortex trajectories occurs.

The drift velocity of the vortices likewise increases appreciably with an increase in their diameter and over the range of conditions (4) almost reaches a maximum. The maximum drift velocity is determined by the depth of the fluid: by a dependence which differs from (3) only by a numerical factor ; $V_{dr} = V_R/2.4$. (The fact that $V_{dr} < V_R$ does not contradict the theory[2]: It stems from the finite nature of the ratio of the vortex diameter to the radius of the meridional curvature of the system[1] and can be explained quantitatively.)

Comparison of these experimental data with relations (1)–(3) leads to the conclusion that (in accordance with Ref. 1) vortices observed over the range of conditions (4) are Rossby solitons. They are distinguished by the fact that they have the longest lifetime and are stable. In contrast to them, vortices with a shorter diameter have a short lifetime (Fig. 1), which, as experiments showed, is determined by the instability relative to the structuring of the vortex in the flow; it is manifested in the fact that the divergence of the vortex trajectories is known to occur earlier than in a laminar regime, namely, when $V_{rot} > V_{dr}$ is still satisfied. This instability is demonstrated in Fig. 2, which shows photographs of a weakly unstable vortex, taken at three successive times with an interval of 3.5 s. It is evident that the characteristic rotation of the vortex changes into rotation around the axis of the paraboloid: The geostrophic vortex changes into a geostrophic flow. Unstable vortices [not corresponding to the range of conditions (4)] are not Rossby solitons. Figure 1 also shows the characteristic rate (increment) of decay of unstable vortices $\gamma = 1/\tau_u$, also plotted as a function of their diameter:

$$\frac{1}{\tau_u} = \frac{1}{\tau} - \frac{1}{\tau_{max}}. \tag{5}$$

It is evident from Figs. 1 and 2 that when the diameter of the vortices decreases by only approximately a factor of 2, as compared with the diameter of the Rossby soliton, the decay of vortices into a flow occurs over a time of the order of a single revolution of the paraboloid, much less than a single revolution of the vortex around its own axis.

Thus, we have demonstrated in this paper that Rossby solitons are distinguished from other geostrophic vortices primarily by their stability. It turns out that this stability is also manifested in the fact that the creation of an anticyclonic disturbance of a more or less arbitrary form over the range of conditions (1) and (4) leads to the formation of a (circular) Rossby soliton.

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