Conversion of an atomic electron into a positron and double β^+ decay

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The probability for the conversion of an atomic electron into a positron should exceed the probability for $2\beta^+$ decay in all nuclei. The calculated ratio of probabilities does not depend on the nuclear model; for most nuclei it is ~ 10 for conversion without neutrinos and $\sim 10^3$ for two-neutrino processes.

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The suggestion of a nonzero neutrino mass found in an experiment at the Institute of Theoretical and Experimental Physics[1] has reawakened interest in the neutrino-free 2β decay of nuclei as a source of information about the nature of the neutrino mass (see Refs. 2–4, for example). This decay might be one which results from the diagonal Majorana mass term in the neutrino mass matrix.

In cases of practical importance, $2\beta^-$ decay is more probable than $2\beta^+$ decay because of the Coulomb repulsion of the positrons from the daughter nucleus in the latter case. For this reason, the $2\beta^+$ decay attracted no interest until very recently. Despite its significantly lower probability, the $2\beta^+$ process may deserve study because it is simpler to identify. The positrons produced in the decay furnish four γ rays after their annihilation. The coincident detection of two positrons and four γ rays would be used as a highly reliable condition for selecting such events. The $2\beta^+$ decay was recently analyzed by Rosen[3] from this point of view.

If $2\beta^+$ decay occurs, then there should also be a conversion of an atomic electron into a positron, just as ordinary β^+ decay is accompanied by the capture of an atomic electron.[5] In this letter we wish to point out that the ratio of the probability for the conversion of an electron into a positron to the probability for $2\beta^+$ decay is essentially independent of the nuclear matrix elements, being determined by the atomic number and the energy release. In all real transitions, the conversion is significantly enhanced.

For allowed transitions the ratio of the rates of neutrino-free conversion and $2\beta^+$ decay can be written as follows for the case in which the transitions are caused by the Majorana neutrino mass:

$$\frac{W_{e^-\to\beta^+}}{W_{2\beta^+}} = \frac{2\frac{1}{\pi}\Delta^2 m_e^2 |\psi(0)|^2 f(\Delta)}{\frac{1}{2}\frac{1}{2\pi^3} m_e^5 \int_1^{\Delta-1} \epsilon_1^2(\Delta-\epsilon_1)^2 f(\epsilon_1) f(\Delta-\epsilon_1) d\epsilon_1}, \qquad (1)$$

where Δ is the difference between the masses of the initial and daughter nuclei, expressed in units of m_e; ϵ_1 is the positron energy, in the same units; $\psi(0)$ is the K-electron wave function at $r=0$; $|\psi(0)|^2 \approx (aZm_e)^3/\pi$; $f(\epsilon) = 2\pi aZ\,|\{\exp(2\pi aZ/\text{v}) - 1\}|$ is a factor which arises from the Coulomb repulsion of the positron from the daughter nucleus; $\text{v}_1 = (1-\epsilon_1^{-2})^{1/2}$; and $\text{v}_2 = [1-(\Delta-\epsilon_1)^{-2}]^{1/2}$. The factor of 2 in the numerator corresponds to the presence of two electrons in the K shell, while the factor of $\frac{1}{2}$ in the denominator results from the identity of the positrons. If transitions are caused by right-hand currents, expression (1) for the probability ratio will contain an additional factor of $\Delta^2/(\epsilon_1-\epsilon_2)^2$. On the average, this ratio is on the order of ten, and the difference in principle could be used to establish the nature of the neutrino-free 2β transition. In our opinion, however, the hypothesis of right-hand currents seems contrived.

In the case of two-neutrino processes, there is the possibility that both of the K electrons as well as the possibility that only a single K electron will be captured. The ratios of the probabilities of the three corresponding processes may be written

$$W_{2e^-\to 2\nu} : W_{e^-\to\beta^+ + 2\nu} : W_{2\beta^+ + 2\nu} = 16\pi^4|\psi(0)|^4\Delta^5$$

$$: 8\pi^2|\psi(0)|^2 \int_0^{\Delta-1} f(\Delta-w)(\Delta-w)^2 w^5 dw : \int_0^{\Delta-2} J(\Delta-w) w^5\, dw, \qquad (2)$$

where $J(\Delta-w)$ is an integral of the same nature as in the denominator in (1); it corresponds to an energy release $\Delta - w$. Working from (1) and (2), we calculated the ratios of the probability for the conversion of an atomic electron and the probability for $2\beta^+$ decay, for both the neutrino-free transition and the two-neutrino emission. The results are listed in Table I. The transitions listed here exhaust the list of possible types of $2\beta^+$ decay. It follows from this table that the probability for the conversion of an atomic electron is considerably higher than that for neutrino-free $2\beta^+$ decay in all transitions of interest. We also note that the absolute value of the conversion probability depends only slightly on the energy release, being determined primarily by the magnitude of the nuclear matrix element. If two neutrinos are emitted, the conversion of an atomic electron is enhanced by three or more orders of magnitude in comparison with $2\beta^+$ decay.

We note further that neutrino-free double K capture would be of considerable interest if there were a pair of nuclei for which this process could go in a resonant manner, i.e., if the mass of the daughter atom in which electrons are excited from the

TABLE I.

Transition	%	Δ_K / m_e	$W_{e^- \to \beta^+} / W_{2\beta^+}$	$W_{2e^- \to 2\nu} / W_{e^- \to \beta^+ 2\nu} / W_{2\beta^+ 2\nu}$
$^{78}_{36}$Kr → Se	0,36	3,64	2.6	1900 : 580 : 1
$^{96}_{44}$Ru → Mo	5,7	3,33	13	$7.9 \cdot 10^4$: $5.8 \cdot 10^3$: 1
$^{106}_{48}$Cd → Pd	1,22	3.44	16	$1.1 \cdot 10^5$: $6.2 \cdot 10^3$: 1
$^{124}_{54}$Xe → Te	0,10	4,00	11	$2.9 \cdot 10^4$: $2.0 \cdot 10^3$: 1
$^{130}_{56}$Ba → Xe	0,10	3,045	115	$1.2 \cdot 10^7$: $1.4 \cdot 10^5$: 1
$^{136}_{58}$Ce → Ba	0,19	2,71	650	$8.2 \cdot 10^8$: $3.7 \cdot 10^6$: 1

K shell to an excited state coincided within about 10 eV with the mass of the initial atom in its ground state. The only possible pair would be $W_{74}^{180} \to Hf_{72}^{180}$, for which the mass difference is 155 ± 10 keV, which is approximately twice the binding energy of the K electron. The resonant transition might also go to an excited state of the daughter nucleus, in which case the difference between the masses of the atoms would have to coincide with the nuclear excitation energy, within the same tolerance. However, it appears unlikely that there are masses which are close enough to each other.

Accordingly, if an attempt is made to study $2\beta^+$ decay, it should be kept in mind that the conversion of an atomic electron is the more probable process, and a study of the transition should apparently begin with this process. Whether a neutrino-free process will be successfully observed is an open question. However, even if a process involving the emission of two neutrinos is detected, it would be quite interesting in itself, since double β transitions, even involving two neutrinos, have not yet been reliably detected. In this manner, the transition matrix element would be determined for the given pair of nuclei, and this information would make it possible to choose among different models for calculating the matrix elements. Finally, from an observation of this type it might be possible to determine the level of the background which the two-neutrino process represents with respect to the neutrino-free process.

If, for some pair of nuclei, both processes are observed—the conversion of an atomic electron into a positron and the $2\beta^+$ decay—the ratio of the probabilities of these processes can be used to identify the neutrino-free transition, in the spirit in which Rosen[3] used Pontecorvo's argument[6] regarding the ratio of probabilities for $2\beta^-$ transitions in ^{130}Te and ^{128}Te.

The conversion of an atomic electron can occur in far more nuclei than can $2\beta^+$ decay, since the latter process requires the expenditure of energy on the production of two electrons. Zdesenko's review[7] gives a nearly complete list of the nuclei in which an atomic electron can undergo conversion. Some of the isotopes in this list have a large natural abundance: 67.8% for ^{58}Ni and 15.9% for ^{92}Mo. These isotopes are also interesting for a practical study of the 2β-transition problem.

For all pairs of nuclei, the conversion of an atomic electron thus turns out to be more probable than $2\beta^+$ decay. The ratio of the probabilities for these processes, which is essentially independent of the nuclear model, is determined by characteristics such as the atomic number and the energy release.

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