Gluon confinement in quantum chromodynamics

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In the gauge $A_0 = 0$ in quantum chromodynamics, the vacuum and all physical states are invariant with respect to the residual symmetry. This invariance leads to confinement of colorless transverse gluons in the form of particles.

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In this letter we wish to show that in the $A_0 = 0$ in quantum chromodynamics all physical states, including the vacuum, are invariant with respect to the residual symmetry. The existence of this symmetry leads to the appearance of a complex operator representing the potential of the interaction between heavy quarks and to confinement of colorless gluons in the form of particles. The reason is that the symmetry of the "free" Hamiltonian in the in-representation is Abelian, while the initial complete Hamiltonian in non-Abelian. In electrodynamics the situation is fundamentally different: In the first place, in any gauge there is always a canonical transformation which eliminates from the Hamiltonian the interaction with nonphysical photons (longitudinal and scalar photons). Second, the symmetry of the complete Hamiltonian and that of the Hamiltonian in the in-representation are the same and are Abelian. In quantum chromodynamics in the $A_0 = 0$ gauge the Hamiltonian is

$$H = \int dx \left\{ \frac{1}{2} (\pi^a_\alpha (\mathrm{x}))^2 + \frac{1}{4} (F^a_{\alpha\beta} (\mathrm{x}))^2 + \Sigma\ \psi^* (a_\alpha (i \nabla_\alpha - g A_\alpha) - m\ ) \psi \right\}. \qquad (1)$$

Here $F_{\alpha\beta} = \partial_\alpha A^a_\beta - \partial_\beta A^b_\alpha + g f^{abc} A^b_\alpha A^c_\beta$; $\pi^q_\alpha(\mathrm{x}) = \dot{A}^a_\alpha(\mathrm{x})$; α_α are the Dirac matrices; the summation is over all flavors; $\alpha, \beta = 1, 2, 3$; and the operators obey the commutation relations

$$[\pi^a_\alpha(\mathrm{x}),\ A^b_\beta(\mathrm{y})]_- = -\ i\,\delta(\mathrm{x}-\mathrm{y})\delta_{\alpha\beta}\,\delta^{ab},$$

$$[\psi(\mathrm{x}),\ \psi^*(\mathrm{y})]_+ = \delta(\mathrm{x}-\mathrm{y}).$$

The operator

$$T^a(\mathrm{x}) = D^{ab}_\alpha \pi^b_\alpha(\mathrm{x}) + g j^a(\mathrm{x}),\ D^{ab}_\alpha = \delta^{ab}\partial_\alpha - g f^{abc} A^c_\alpha(\mathrm{x}) \qquad (2a)$$

commutes with H; here $j^a(\mathrm{x})\Sigma\psi^*(\mathrm{x})t^a\psi(\mathrm{x})$, and t^a are the generators of the representation by which the ψ transform. The operators T^a satisfy a non-Abelian algebra:

$$[T^a(\mathrm{x}),\quad T^b(\mathrm{y})]_- = i f^{abc}\,T^c(\mathrm{x})\,\delta(\mathrm{x}-\mathrm{y}). \qquad (2b)$$

It is not difficult to see that in this representation of the canonical commutation relations neither quarks nor gluons could be emitted as particles, because the vacuum is also invariant with respect to transformations of group (2b). There does exist, however, a canonical transformation U, nonequivalent from the unitary standpoint, which eliminates from H the interaction of the quarks with the longitudinal gluons. This transformation is found from the condition

$$U^{-1}\,T^a\,U \equiv \widetilde{T}^a = D^{ab}_\alpha\,\pi^b_\alpha$$

or

$$-\,i\;U^{-1} D^{ab}_\alpha \frac{\delta}{\delta A^b_\alpha(\mathrm{x})}\,U = g j^a(\mathrm{x}).$$

Hence,

$$U = \exp\{ig \int dx\, j^a(\mathrm{x})\,\lambda^a(\mathrm{x}\,|A)\},$$

$$U^{-1}\,\psi(\mathrm{x})\,U = \exp(ig\lambda^a(\mathrm{x}\,|A)\,t^a)\,\psi(\mathrm{x}) \equiv \Omega(\mathrm{x})\,\psi(\mathrm{x}). \qquad (3)$$

From (2) and (3) we find that Ω satisfied a Batalin-Fradkin equation[1]:

$$D^{ab}_\alpha\ \mathrm{tr}\, t^c \frac{\delta\,\Omega(\mathrm{y}\,|A)}{\delta A^b_\alpha(\mathrm{x})}\,\Omega^{-1}(\mathrm{y}|\,A) = i\delta(\mathrm{x}-\mathrm{y})\delta^{ac}.$$

Introducting the charge-density operator $\hat{\rho}$.

$$\partial_a \hat{\rho}(\mathbf{y} \mid \mathbf{x}) = \operatorname{tr} t^a \frac{\partial \Omega(\mathbf{y} \mid A)}{\partial A_a^b(\mathbf{x})} \Omega^{-1}(\mathbf{y} \mid A),$$

we find in the transformed Hamiltonian $\widetilde{H} = U^{-1}HU$ the operator form of the interaction law for heavy quarks:

$$\hat{V}^{ab}(\mathbf{x},\mathbf{y}) = g^2 \int dz\, \hat{\rho}^{ac}(\mathbf{x} \mid \mathbf{z})\, \Delta_z \hat{\rho}^{cb}(\mathbf{z} \mid \mathbf{y}).$$

In the lowest-order approximation,

$$\hat{V}^{ab} = g^2 \delta^{ab} \mid \mathbf{x} - \mathbf{y} \mid^{-1}.$$

In the Hamiltonian H the quarks now interact with the transformed gluon field

$$B_a^a(\mathbf{x} \mid A) = \Omega^{-1}(\mathbf{x} \mid A) A_a \Omega(\mathbf{x} \mid A) + \frac{i}{\sigma} \Omega^{-1} \partial_a \Omega,$$

which satisfies the transverseness condition $\partial_a B_a = 0$ and commutes with $\widetilde{T}^a$:

$$[\widetilde{T}^a(\mathbf{x}), \; B_a^b(\mathbf{y} \mid A)]_- = 0. \quad (4)$$

For a Poincaré group to exist we need to construct, along the H, the 3-momentum operators $\mathscr{P}_a$. In contrast with H, the operators $\mathscr{P}_a$ are not gauge-invariant: $[T^a(\mathbf{x})\mathscr{P}_a]_- \neq 0$. Accordingly, a subspace of physical states should be defined such that

$$\langle \Psi_{\text{phys}}, [T^a(\mathbf{x}), \mathscr{P}_a]_- \Phi_{\text{phys}} \rangle = 0,$$

at least for all of the eigentransformations from group (2b)

We can show the "physical" fields B_a cannot be emitted as particles; i.e., they have no in (out) limits. A necessary condition for the emission of physical ψ and B_a particles is the existence at the formal level of a canonical transformation Λ ("half of an S matrix") which diagonalizes $\widetilde{H}$:

$$\Lambda^{-1} \widetilde{H} \Lambda = H_0, \quad (5)$$

where H_0 is bilinear in the operators representing the ψ and B_a fields. We then immediately conclude that there must exist a *non-Abelian* operator which satisfies the algebra in (2b),[1)]

$$T_0^a = \Lambda^{-1} T^a \Lambda, \quad (6)$$

and commutes with H_0 (we recall that H_0 also has Abelian symmetry). In this case the quarks and gluons may be emitted as particles. Actually, no such operator Λ exists (by virtue of Haag's theorem[2)]). Strictly speaking, emission in the form of particles implies the existence of H_0^{in} in the in (out) representation which describes the real (i.e., renormalized) particles. In this case, a non-Abelian operator $T_{0,\text{in}}^a$ which commutes with H_0^{in}, should exist, according to (6). It is impossible to construct such an operator, however (again by virtue of Haag's theorem). The operator $T_{0,\text{in}}^a$ is expressed terms of the in-operators with the help of the half-S-matrix. Expanding $T_{0,\text{in}}^a(x)$ in a series in the in-operators, we find that it can be constructed only up to terms $\sim g$; in the g^2 approximation, "surface" divergences which cannot be eliminated arise (after renormalization).[2] Some even stronger arguments for gluon confinement follow from (4). The ultimate reason for the gluon confinement lies in the fact that the particle in-states, which have Abelian symmetry, at the same time bear the indelible imprint of the symmetry of the original complete Hamiltonian, and these two symmetries are incompatible for gluons.[2)]

We therefore conclude that either we have $B^a_{\text{in}} = 0$, or this limit cannot be interpreted as an operator representing a one-particle state. Gluons (*colorless* gluons) do not exist as free particles in quantum chromodynamics. As for resolving the question of quark confinement, we note that the most important step here is to study the potential $\hat{V}^{ab}$ without using perturbation theory. At this point it is not clear which vacuum (quantum) configurations make a definite contribution; at any rate, they cannot be instanton configurations, since the introduction of an instanton topological structure would not alter the conclusions reached above [because of the invariance of the θ-vacuum with respect to the gauge eigentransformations generated by $\widetilde{T}^a(\mathbf{x})$]. The primary problem is thus to construct colorless physical states and objects of a nonlocal nature (strings, a $1/N$ expansion, etc.). We might note that in covariant gauges of the type $\partial^u A^a_\mu = 0$ there is also a residual symmetry, which leads to the confinement of both gluons and quarks.

[1)]In perturbartion theory, T^a_0 is an infinite series in powers of g. It follows from (5) and (6) that there must exist an Abelian operator that commutes with $\widetilde{H}$.

[2)]Feynman[3] has offered some strong arguments for gluon confinement in three-dimensional gluon dynamics, on the basis of the residual symmetry of the vacuum in the $A_0 = 0$ gauge.

[1]I. A. Batalin and E. S. Fradkin, Ann. Phys. **83**, 367 (1974).
[2]See, for example, A. Whiteman, Problems of the Relativistic Dynamics of Quantized Fields (Russ. transl. Nauka, Moscow, 1968, Part II).
[3]R. Feynman, Nucl. Phys. **B188**, 479 (1981).

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