

New form of waves and solitons in a piezomagnetic antiferromagnet

I. V. Ioffe

A. F. Ioffe Physicotechnical Institute, Academy of Sciences of the USSR

(Submitted 20 May 1982)

Pis'ma Zh. Eksp. Teor. Fiz. **36**, No. 2, 33–35 (20 July 1982)

A new type of wave, which can arise in a plate made of a piezomagnetic antiferromagnet, is predicted. In these waves, the alternating magnetic field and the transverse (to the direction of propagation) displacement of the lattice differ from zero. It is shown that such a wave forms a soliton in the nonlinear limit.

PACS numbers: 75.70.Dp, 75.80. + q

1. Waves that have not been previously studied and that are related to oscillations of the transverse displacements of the crystal lattice u and of the magnetic field H can be produced in a piezomagnetic antiferromagnet plate whose axis of easy magnetization (z axis) is parallel to the surface of the plate. These waves propagate along the surface of the plate in a direction normal to the axis of easy magnetization (x axis); the displacement of the lattice is oriented along the axis of easy magnetization. The wavelength k^{-1} is much greater than the thickness of the plate $2d$ and the propagation velocity is close to the velocity of transverse sound s . If the plate is bounded on one or both sides by a crystal whose magnetic permeability is much greater than that of the antiferromagnet μ (case A below), then a soliton, which is described by the Korteweg-de Vries equation and which transforms in the linear approximation into the wave indicated, can arise in the plate. The soliton cannot arise in a plate bounded by media whose permeabilities are lower than or comparable to the permeability of the plate (case B).

We shall limit our analysis to the magnetostatic approximation, when $H = \nabla\varphi$, and we shall examine the elastic and magnetic properties of the antiferromagnet (except for the piezomagnetic properties) in the approximation of an isotropic solid.

2. The system of equations that describe this problem is

$$\operatorname{div}(\mu H + 4\pi m^\beta) = 0, \quad (1)$$

$$\rho \ddot{u}_z = \frac{\partial}{\partial x_i} (\sigma_{iz} + \sigma_{iz}^\beta) \quad (2)$$

and the expression for the piezomagnetic part of the free energy density is

$$F^\beta = -\beta \left[\frac{\partial u_z}{\partial x} \frac{\partial \varphi}{\partial y} \pm \frac{\partial u_z}{\partial y} \frac{\partial \varphi}{\partial x} \right] \quad (3)$$

(according to Ref. 1, the “+” sign is used for CoF_2 , MnF_2 , and FeF_2 , while the “−” sign is used for FeCO_3 ; ρ is the density; m^β is the piezomagnetic magnetic moment; σ_{ik} and σ_{ik}^β are the elastic and piezomagnetic stress tensors). The boundary conditions are determined by the absence of external forces and by the usual conditions for the magnetic field and the induction. Analysis shows that only lattice displacements and a magnetic field symmetrical with respect to y are possible. It is possible to obtain solutions that are antisymmetric with respect to y , but only if

$$\Gamma k d \gg 1; \quad \Gamma = 4\pi\beta^2\rho^{-1}s^{-2}. \quad (4)$$

Since $\Gamma \simeq 10^{-4}$, the last inequality is not consistent with the condition of weak damping of waves $\nu k \ll s$, where ν is the coefficient of viscosity. Let us substitute u , $\varphi = u(y)$, and $\varphi(y)\exp(ikx - i\omega t)$:

$$\begin{cases} u = u_1 \operatorname{ch} \kappa y \\ \dot{\varphi} = \varphi_1 \operatorname{sh} \kappa y + \varphi_2 \operatorname{sh} \kappa y \end{cases}, \quad (5)$$

$$\begin{cases} u = u_1 \cos \kappa y \\ \varphi = \varphi_1 \operatorname{sh} \kappa y \end{cases} \quad (6)$$

for the upper and lower signs in (3), respectively. Thus, from (1)–(3), for both signs in (3), we find for case A,

$$\kappa = \sqrt{\frac{\Gamma}{\mu}} k \left[1 + \frac{(kd)^2}{2} \right], \quad (7)$$

$$\omega = ks \left[1 - (kd_0)^2 \right], \quad d_0 = \sqrt{\frac{\Gamma}{\mu}} \frac{d}{2}$$

and for case B,

$$\begin{aligned} \kappa &= \sqrt{\frac{\Gamma}{\mu}} k \left[1 - \frac{(kd)}{\mu} \right], \\ \omega &= ks \left[1 + 2 \frac{(kd_0)}{\mu} \right]. \end{aligned} \quad (8)$$

As is evident from the dispersion equations, for $\Gamma kd \ll 1$ and $\nu k \ll s$, transverse displacement waves and magnetic field waves are possible.

3. In case A, from (7), after introducing for $|\partial u / \partial x| \ll 1$ an acoustic nonlinearity, we obtain, in accordance with Refs. 3 and 4, the Korteweg-de Vries equation

$$\frac{\partial u}{\partial t} s \frac{\partial u}{\partial x} + s d_0^2 \frac{\partial^3 u}{\partial x^3} - s \frac{\partial}{\partial x} \left(\frac{\partial u}{\partial x} \right)^2 = 0, \quad (9)$$

leading, with the usual zero conditions at infinity, to a soliton that moves with a velocity w ,

$$u \sim \text{ch}^{-2} \left(\sqrt{\frac{w-s}{s}} \frac{x - wt}{2d_0} \right).$$

The condition under which dissipation with respect to the two last terms in Eq. (9) can be ignored, taking into account that $\Gamma kd \ll 1$, $|ku| \ll 1$, and $w - s \ll s$, leads to the inequality

$$d \gg d_{\max} = \frac{\nu}{s} \sqrt{\frac{\mu}{\Gamma}},$$

which is easily satisfied, since $d_{\max} \cong 10^{-3}$ cm.

Waves can be excited either by charged particles which pass through the plate with a velocity $v > s$ (but, $v - s \ll s$) along the axis of easy magnetization or by applying an external magnetic field $H_y = H_0 \cos(kx - \omega t)$. In this case, the displacement amplitude is of the order of $\beta H_0 (\rho \nu s k)^{-1}$.

The observation of the waves and of the soliton is facilitated by the presence of a magnetic field wave, which propagates in air along the surface of the plate (we note that for the soliton to exist, the opposite side of the plate must be bounded by a crystal with a high magnetic permeability).

¹E. E. Dzyaloshinskii, Zh. Eksp. Teor. Fiz. **33**, 807 (1957) [Sov. Phys. JETP **6**, 621 (1958)].

²A. S. Borovik-Romanov, Zh. Eksp. Teor. Fiz. **38**, 1088 (1960) [Sov. Phys. JETP **11**, 725 (1960)].

³V. I. Karpman, Nelineinye volny v dispergiruyushchikh sredakh [Nonlinear Waves in Dispersive Media], Nauka, Moscow, 1973.

⁴G. W. Whitman, Linear and Nonlinear Waves, Wiley and Sons, New York, 1974; G. Widom, Linear and Nonlinear waves [Russian translation], Mir, Moscow, 1977.

Translated by M. E. Alferieff

Edited by S. J. Amoretty